17.4 Mechanics of contact

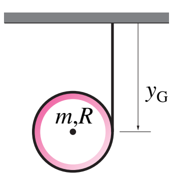
17.4.1 A uniform disc of mass m and radius r rolls without slip at constant rate. What is the total kinetic energy of the disk?

17.4.2 A non-uniform disc of mass m and radius r rolls without slip at constant rate. The center of mass is located at a distance $\frac{r}{2}$ from the center of the disc. What is the total kinetic energy of the disc when the center of mass is directly above the center of the disc?

17.4.3 Falling hoop. A bicycle rim (no spokes, tube, tire, or hub) is idealized as a hoop with mass m and radius R. G is at the center of the hoop. An inextensible string is wrapped around the hoop and attached to the ceiling. The hoop is released from rest at the position shown at $t = 0$.

a) Find y_G at a later time t in terms of any or all of m, R, g, and t.

b) Does G move sideways as the hoop falls and unrolls?

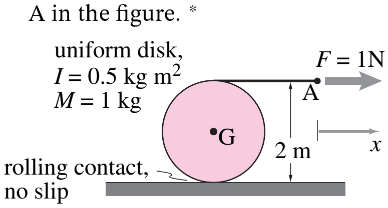


Problem 17.4.3

17.4.4 A uniform disk with radius R and mass m has a string wrapped around it. The string is pulled with a force F. The disk rolls without slipping.

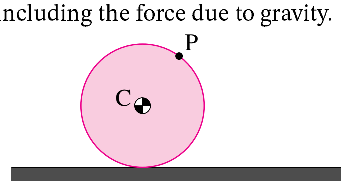
a) What is the angular acceleration of the disk, $\vec{\alpha}_{Disk} = -\ddot{\theta}\hat{k}$? Make any reasonable assumptions you need that are consistent with the figure information and the laws of mechanics. State your assumptions. *

b) Find the acceleration of the point A in the figure. *



Problem 17.4.4

17.4.5 If a pebble is stuck to the edge of the wheel in problem 17.3.3, what is the maximum speed of the pebble during the motion? When is the force on the pebble from the wheel maximum? Draw a good FBD including the force due to gravity.

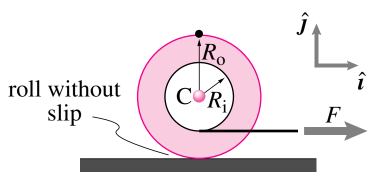


Problem 17.4.5

17.4.6 Spool Rolling without Slip and Pulled by a Cord. The light-weight spool is nearly empty but a lead ball with mass m has been placed at its center. A force F is applied in the horizontal direction to the cord wound around the wheel. Dimensions are as marked. Coordinate directions are as marked.

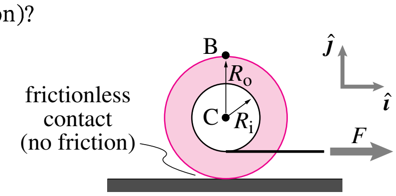
a) What is the acceleration of the center of the spool? *

b) What is the horizontal force of the ground on the spool? *



Problem 17.4.6

17.4.7 A film spool is placed on a very slippery table. Assume that the film and reel (together) have mass distributed the same as a uniform disk of radius R_i. What, in terms of R_i, R_o, m, g, $\hat{\imath}$, $\hat{\jmath}$, and F are the accelerations of points C and B at the instant shown (the start of motion)?



Problem 17.4.7

17.4.8 Again, Spool Rolling without Slip and Pulled by a Cord. Reconsider the spool from problem 17.4.6. This time, a force F is applied vertically to the cord wound around the wheel. In this case, what is the acceleration of the center of the spool? Is it possible to pull the cord at some angle between horizontal and vertical so that the angular acceleration of the spool or the acceleration of the center of mass is zero? If so, find the angle in terms of R_i, R_0, m, and F.

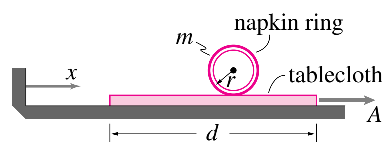
17.4.9 A napkin ring lies on a black velvet tablecloth. The thin ring (of mass m, radius r) rolls without slip as a mischievous child pulls the tablecloth (mass M) out with acceleration $\vec{a}_A$. The ring starts at the right end ($x = d$). You can make a reasonable physical model of this situation with an empty soda can and a piece of paper on a flat table.

a) What is the ring's acceleration as the tablecloth is being withdrawn?

b) How far has the tablecloth moved to the right from its starting point $x = 0$ when the ring rolls off its left-hand end?

c) Clearly describe the subsequent motion of the ring. Which way does it end up rolling at what speed?

d) Would your answer to the previous question be different if the ring slipped on the cloth as the cloth was being pulled out?



Problem 17.4.9

17.4.10 A block of mass M is supported by two rollers (uniform cylinders) each of mass m and radius r. They roll without slip on the block and the ground. A force F is applied in the horizontal direction to the right, as shown in the figure. Given F, m, r, and M, find:

a) the acceleration of the block,

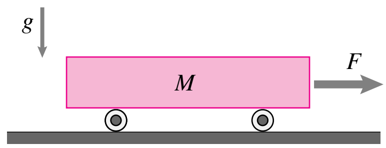
b) the acceleration of the center of mass of this block/roller system,

c) the reaction at the wheel bases,

d) the force of the right wheel on the block,

e) the acceleration of the wheel centers, and

f) the angular acceleration of the wheels.

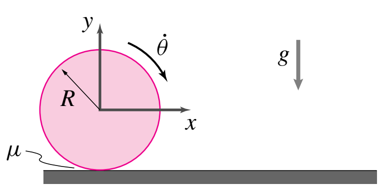


Problem 17.4.10

17.4.11 Dropped spinning disk. 2-D . A uniform disk of radius R and mass m is gently dropped onto a surface and doesn't bounce. When it is released it is spinning clockwise at the rate $\dot{\theta}_0$. The disk skids for a while and then is eventually rolling.

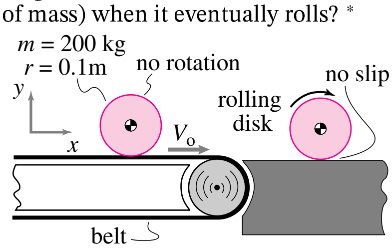
a) What is the speed of the center of the disk when the disk is eventually rolling (answer in terms of g, μ, R, $\dot{\theta}_0$, and m)? *

b) In the transition from slipping to rolling, energy is lost to friction. How does the amount lost depend on the coefficient of friction (and other parameters)? How does this loss make or not make sense in the limit as $\mu \to 0$ and the dissipation rate $\to$ zero? *



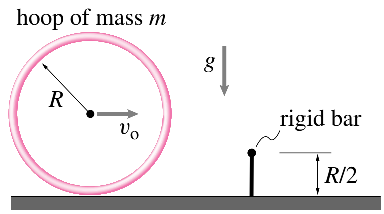
Problem 17.4.11

17.4.12 Disk on a conveyor belt. A uniform metal cylinder with mass of 200 kg is carried on a conveyer belt which moves at $V_0 = 3\,\mathrm{m/s}$. The disk is not rotating when on the belt. The disk is delivered to a flat hard platform where it slides for a while and ends up rolling. How fast is it moving (i.e. what is the speed of the center of mass) when it eventually rolls? *



Problem 17.4.12

17.4.13 A rigid hoop with radius R and mass m is rolling without slip so that its center has translational speed v_o. It then hits a narrow bar with height $R/2$. When the hoop hits the bar suddenly it sticks and doesn't slide. It does hinge freely about the bar, however. The gravitational constant is g. How big is v_o if the hoop just barely rolls over the bar? *



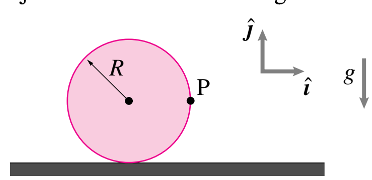
Problem 17.4.13

17.4.14 2-D rolling of an unbalanced wheel. A wheel, modeled as massless, has a point mass (mass $= m$) at its perimeter. The wheel is released from rest at the position shown. The wheel makes contact with coefficient of friction μ.

a) What is the acceleration of the point P just after the wheel is released if $\mu = 0$?

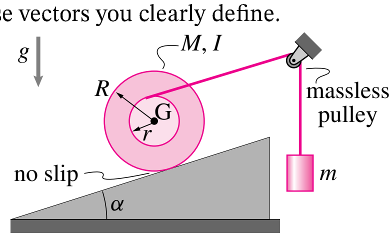
b) What is the acceleration of the point P just after the wheel is released if $\mu = 2$?

c) Assuming the wheel rolls without slip (no-slip requires, by the way, that the friction be high: $\mu = \infty$) what is the velocity of the point P just before it touches the ground?



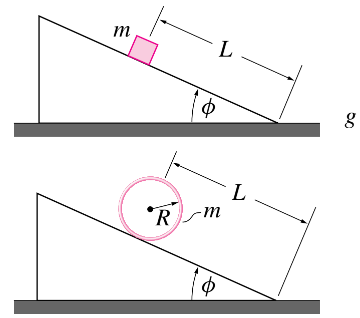
Problem 17.4.14

17.4.15 Spool and mass. A reel of mass M and moment of inertia $I_{zz}^{cm} = I$ rolls without slipping upwards on an incline with slope-angle α. It is pulled up by a string attached to mass m as shown. Find the acceleration of point G in terms of some or all of M, m, I, R, r, α, g and any base vectors you clearly define.



Problem 17.4.15

17.4.16 Two objects are released on two identical ramps. One is a sliding block (no friction), the other a rolling hoop (no slip). Both have the same mass, m, are in the same gravity field and have the same distance to travel. It takes the sliding mass 1 s to reach the bottom of the ramp. How long does it take the hoop? [Useful formula: "$s = \frac{1}{2}at^2$"]



Problem 17.4.16

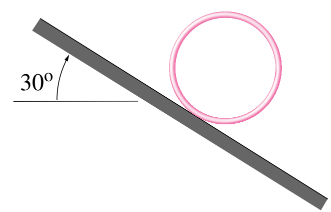
17.4.17 The hoop is rolled down an incline that is 30° from horizontal. It does not slip. It does not fall over sideways. It is let go from rest at t = 0.

a) At $t = 0^+$ what is the acceleration of the hoop center of mass?

b) At $t = 0^+$ what is the acceleration of the point on the hoop that is on the incline?

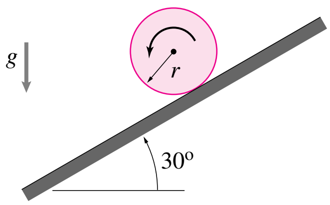
c) At $t = 0^+$ what is the acceleration of the point on the hoop that is furthest from the incline?

d) After the hoop has descended 2 vertical meters (and traveled an appropriate distance down the incline) what is the acceleration of the point on the hoop that is (at that instant) furthest from the incline?



Problem 17.4.17

17.4.18 A uniform cylinder of mass m and radius r rolls down an incline *without slip*, as shown below. Determine: (a) the angular acceleration α of the disk; (b) the minimum value of the coefficient of friction μ that will insure no slip.



Problem 17.4.18

17.4.19 Race of rollers. A uniform disk with mass M_0 and radius R_0 is allowed to roll down the quite slip-resistant ($\mu = 1$) 30° ramp shown. It is raced against four other objects (A, B, C and D), one at a time. Who wins the races, or are there ties? First try to construct any plausible reasoning. Good answers will be based, at least in part, on careful use of equations of mechanics. *

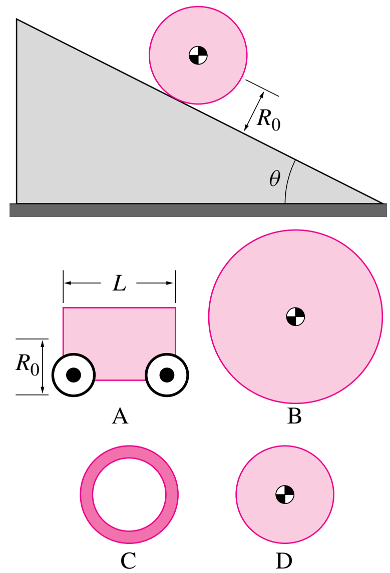
a) Block A has the same mass and has center of mass a distance R_0 from the ramp. It rolls on massless wheels with frictionless bearings.

b) Uniform disk B has the same mass ($M_B = M_0$) but twice the radius ($R_B = 2R_0$).

c) Hollow pipe C has the same mass ($M_C = M_0$) and the same radius ($R_C = R_0$).

d) Uniform disk D has the same radius ($R_D = R_0$) but twice the mass ($M_D = 2M_0$).

Can you find a round object which will roll as fast as the block slides? How about a massless cylinder with a point mass in its center? Can you find an object which will go slower than the slowest or faster than the fastest of these objects? What would they be and why? (This problem is harder.)



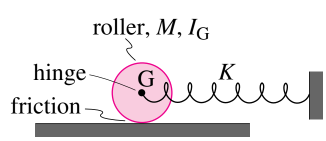
Problem 17.4.19

17.4.20 A roller of mass M and polar moment of inertia about the center of mass I_G is connected to a spring of stiffness K by a frictionless hinge as shown in the figure. Consider two kinds of friction between the roller and the surface it moves on:

1. Perfect slipping (no friction), and
2. Perfect rolling (infinite friction).

a) What is the period of oscillation in the first case?

b) What is the period of oscillation in the second case?

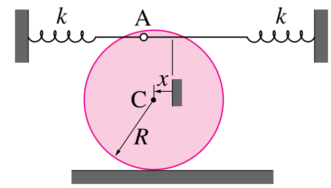


Problem 17.4.20

17.4.21 A uniform cylinder of mass m and radius R rolls back and forth without slipping through small amplitudes (i.e., the springs attached at point A on the rim act linearly and the vertical change in the height of point A is negligible). The springs, which act both in compression and tension, are unextended when A is directly over C.

a) Determine the differential equation of motion for the cylinder's center.

b) Calculate the natural frequency of the system for small oscillations.

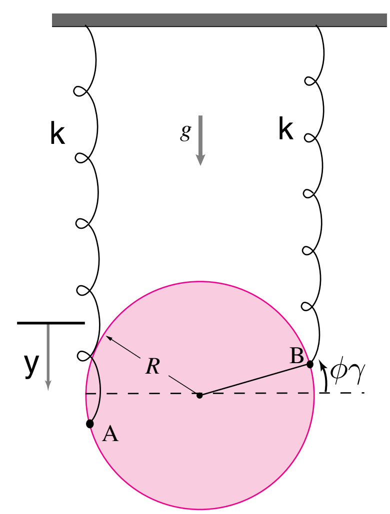


Problem 17.4.21

17.4.22 Hanging disk, 2-D. A uniform thin disk of radius R and mass m hangs in a gravity field g from a pair of massless springs each with constant k. In the static equilibrium configuration the springs are vertical and attached to points A and B on the right and left edges of the disk. In the equilibrium configuration the springs carry the weight, the disk counter-clockwise rotation is $\phi = 0$, and the downwards vertical deflection is $y = 0$. Assume throughout that the center of the disk only moves up and down, and that ϕ is small so that the springs may be regarded as vertical when calculating their stretch ($\sin\phi \approx \phi$ and $\cos\phi \approx 1$).

a) Find $\ddot{\phi}$ and $\ddot{y}$ in terms of some or all of $\phi, \dot{\phi}, y, \dot{y}, k, m, R$, and g.

b) Find the natural frequencies of vibration in terms of some or all of k, m, R, and g.



Problem 17.4.22

17.4.23 A disk rolls in a cylinder. For all of the problems below, the disk rolls without slip and rocks back and forth due to gravity.

a) **Sketch.** Draw a neat sketch of the disk in the cylinder. The sketch should show all variables, coordinates and dimension used in the problem.

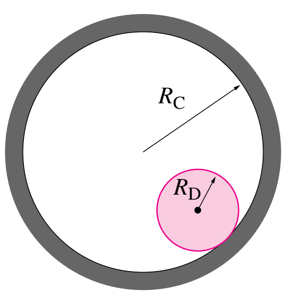
b) **FBD.** Draw a free-body diagram of the disk.

c) **Momentum balance.** Write the equations of linear and angular momentum balance for the disk. Use the point on the cylinder which touches the disk for the angular momentum balance equation. Leave as unknown in these equations variables which you do not know.

d) **Kinematics.** The disk rolling in the cylinder is a *one*-degree-of-freedom system. That is, the values of only *one* coordinate and its derivatives are enough to determine the positions, velocities and accelerations of all points. The angle that the line from the center of the cylinder to the center of the disk makes from the vertical can be used as such a variable. Find all of the velocities and accelerations needed in the momentum balance equation in terms of this variable and its derivative. [Hint: you'll need to think about the rolling contact in order to do this part.]

e) **Equation of motion.** Write the angular momentum balance equation as a single second order differential equation.

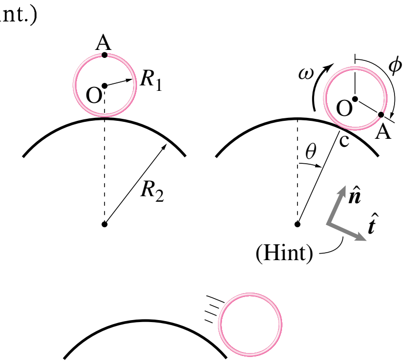
f) **Simple pendulum?** Does this equation reduce to the equation for a pendulum with a point mass and length equal to the radius of the cylinder, when the disk radius gets arbitrarily small? Why, or why not?



Problem 17.4.23: A disk rolls without slip inside a bigger cylinder.

17.4.24 A uniform hoop of radius R_1 and mass m rolls from rest down a semi-circular track of radius R_2 as shown. Assume that no slipping occurs. At what angle θ does the hoop leave the track and what is its angular velocity ω and the linear velocity $\bar{v}$ of its center of mass at that instant? If the hoop slides down the track without friction, so that it does not rotate, will it leave at a smaller or larger angle θ than if it rolls without slip (as above)? Give a qualitative argument to justify your answer.

HINT: Here is a geometric relationship between angle ϕ hoop turns through and angle θ subtended by its center when no slipping occurs: $\phi = [(R_1 + R_2)/R_2]\theta$. (You may or may not need to use this hint.)



Problem 17.4.24