17.3 Kinematics of rolling and sliding

17.3.1 A stone in a wheel. A round wheel rolls to the right. At time $t = 0$ it picks up a stone from the road. The stone is stuck in the edge of the wheel. You want to know the direction of the rock's motion just before and after it next hits the ground. Here are some candidate answers:

- When the stone approaches the ground its motion is tangent to the ground.
- The stone approaches the ground at angle x (you name it).
- When the stone approaches the ground its motion is perpendicular to the ground.
- The stone approaches the ground at various angles depending on the following conditions(...you list the conditions.)

Although you could address this question analytically, you are to try to get a clear answer by looking at computer generated plots. In particular, you are to plot the pebble's path for a small interval of time near when the stone next touches the ground. You should pick the parameters that make your case for an answer the strongest. You may make more than one plot.

Here are some steps to follow:

a) Assuming the wheel has radius R_w and the pebble is a distance R_p from the center (not necessarily equal to R_w). The pebble is directly below the center of the wheel at time $t = 0$. The wheel spins at constant clockwise rate ω. The x-axis is on the ground and $x(t = 0) = 0$. The wheel rolls without slipping. Using a clear well labeled drawing (use a compass and ruler or a computer drawing program), show that

$$x(t) = \omega t R_w - R_p \sin(\omega t)$$

$$y(t) = R_w - R_p \cos(\omega t)$$

b) Using this relation, write a program to make a plot of the path of the pebble as the wheel makes a little more than one revolution. Also show the outline of the wheel and the pebble itself at some intermediate time of interest. [Use any software and computer that pleases you.]

c) Change whatever you need to change to make a good plot of the pebble's path for a small amount of time as the pebble approaches and leaves the road. Also show the wheel and the pebble at some time in this interval.

d) In this configuration the pebble moves a *very* small distance in a small time so your axes need to be scaled. But make sure your x- and y- axes have the same scale so that the path of the pebble and the outline of the wheel will not be distorted.

e) How does your computer output buttress your claim that the pebble approaches and leaves the ground at the angles you claim?

f) Think of something about the pebble in the wheel that was not explicitly asked in this problem and explain it using the computer, and/or hand calculation and/or a drawing.

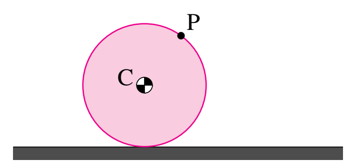
17.3.2 A uniform disk of radius r rolls at a constant rate without slip. A small ball of mass m is attached to the outside edge of the disk. What is the force required to hold the disk in place when the mass is above the center of the disk?

17.3.3 Rolling at constant rate. A round disk rolls on the ground at constant rate. It rolls $1\frac{1}{4}$ revolutions over the time of interest.

a) **Particle paths.** Accurately plot the paths of three points: the center of the disk C, a point on the outer edge that is initially on the ground, and a point that is initially half way between the former two points. [Hint: Write a parametric equation for the position of the points. First find a relation between ω and v_C. Then note that the position of a point is the position of the center plus the position of the point relative to the center.] Draw the paths on the computer, make sure x and y scales are the same.

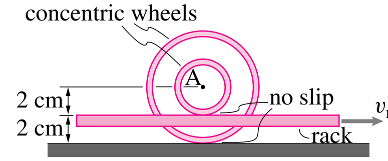
b) **Velocity of points.** Find the velocity of the points at a few instants in the motion: after $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1 revolution. Draw the velocity vector (by hand) on your plot. Draw the direction accurately and draw the lengths of the vectors in proportion to their magnitude. You can find the velocity by differentiating the position vector or by using relative motion formulas appropriately. Draw the disk at its position after one quarter revolution. Note that the velocity of the points is perpendicular to the line connecting the points to the ground contact.

c) **Acceleration of points.** Do the same as above but for acceleration. Note that the acceleration of the points is parallel to the line connecting the points to the center of the disk.



Problem 17.3.3

17.3.4 The concentric wheels are welded to each other and roll without slip on the rack and stationary support. The rack moves to the right at $v_r = 1\,\mathrm{m/s}$. What is the velocity of point A in the middle of the wheels shown?



Problem 17.3.4

17.3.5 Questions (a) - (e) refer to the cylinders in the configuration shown in the figure. Question (f) is closely related. Answer the questions in terms of the given quantities (and any other quantities you define if needed).

a) What is the speed (magnitude of velocity) of point c?

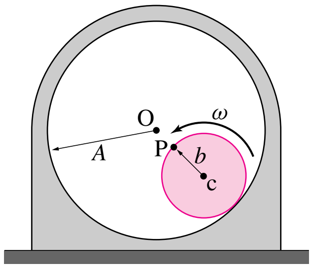
b) What is the speed of point P?

c) What is the magnitude of the acceleration of point c?

d) What is the magnitude of the acceleration of point P?

e) What is the radius of curvature of the path of the particle P at the point of interest?

f) In the special case of $A = 2b$ what is the curve which particle P traces (for all time)? Sketch the path.



Problem 17.3.5: A little cylinder (with outer radius b and center at point c) rolls without slipping inside a bigger fixed cylinder (with inner radius A and center at point O). The *absolute* angular velocity of the little cylinder ω is *constant*. P is attached to the outside edge of the little cylinder. At the instant of interest, P is on the line between O and c.