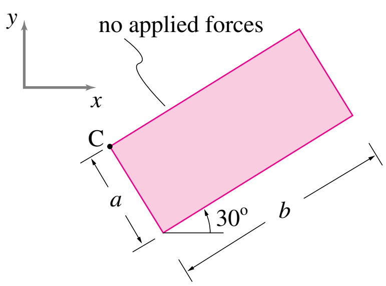
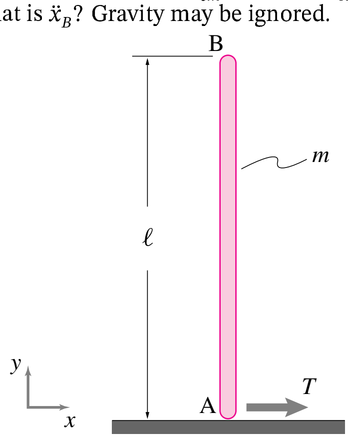
17.2 Dynamics of a rigid object

17.2.1 The uniform rectangle of width $a = 1\,\mathrm{m}$, length $b = 2\,\mathrm{m}$, and mass $m = 1\,\mathrm{kg}$ in the figure is sliding on the xy-plane with no friction. At the moment in question, point C is at $x_C = 3\,\mathrm{m}$ and $y_C = 2\,\mathrm{m}$. The linear momentum is $\vec{\boldsymbol{L}} = 4\hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}}\ (\mathrm{kg{\cdot}m/s})$ and the angular momentum about the center of mass is $\vec{\boldsymbol{H}}_{cm} = 5\hat{\boldsymbol{k}}(\mathrm{kg{\cdot}m/s^2})$. Find the acceleration of any point on the body that you choose. (Mark it.) [Hint: You have been given some redundant information.]



Problem 17.2.1

17.2.2 The vertical pole AB of mass m and length ℓ is initially, *at rest* on a frictionless surface. A tension T is suddenly applied at A. What is $\ddot{x}_{cm}$? What is $\ddot{\theta}_{AB}$? What is $\ddot{x}_B$? Gravity may be ignored.



Problem 17.2.2

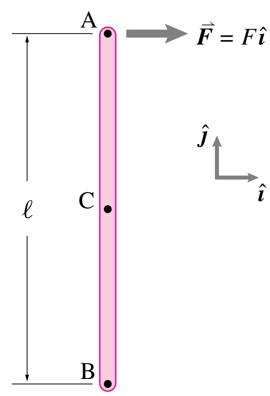
17.2.3 Force on a stick in space. 2-D . No gravity. A uniform thin stick with length ℓ and mass m is, at the instant of interest, parallel to the y axis and has no velocity and no angular velocity. The force $\vec{\boldsymbol{F}} = F\hat{\boldsymbol{\imath}}$ with $F > 0$ is suddenly applied at point A. The questions below concern the instant after the force $\vec{\boldsymbol{F}}$ is applied.

a) What is the acceleration of point C, the center of mass? *

b) What is the angular acceleration of the stick? *

c) What is the acceleration of the point A? *

d) (relatively harder) What additional force would have to be applied to point B to make point B's acceleration zero? *



Problem 17.2.3

17.2.4 A uniform thin rod of length ℓ and mass m stands vertically, with one end resting on a frictionless surface and the other held by someone's hand. The rod is released from rest, displaced slightly from the vertical. No forces are applied during the release. There is gravity.

a) Find the path of the center of mass.

b) Find the force of the floor on the end of the rod just before the rod is horizontal.

17.2.5 A uniform disk, with mass center labeled as point G, is sitting motionless on the frictionless xy plane. A massless peg is attached to a point on its perimeter. This disk has radius of 1 m and mass of 10 kg. A constant force of $F = 1000\,\mathrm{N}\boldsymbol{\imath}$ is applied to the peg for .0001 s (one ten-thousandth of a second).

a) What is the velocity of the center of mass of the center of the disk after the force is applied?

b) Assuming that the idealizations named in the problem statement are exact is your answer to (a) exact or approximate?

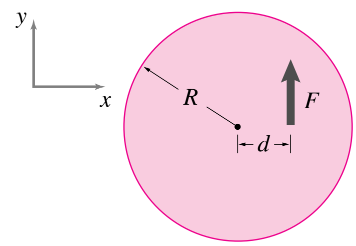
c) What is the angular velocity of the disk after the force is applied?

d) Assuming that the idealizations named in the problem statement are exact is your answer to (c) exact or approximate?

17.2.6 A uniform thin flat disc is floating in space. It has radius R and mass m. A force F is applied to it at a distance d from the center in the y direction. Treat this problem as two-dimensional.

a) What is the acceleration of the center of the disc? *

b) What is the angular acceleration of the disk? *



Problem 17.2.6

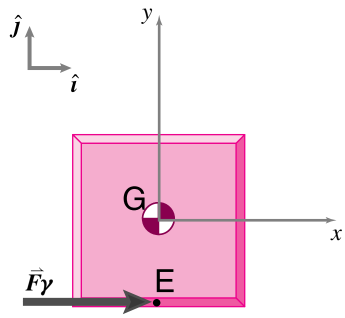
17.2.7 A uniform 1kg plate that is one meter on a side is initially at rest in the position shown. A constant force $\vec{\boldsymbol{F}} = 1\,\mathrm{N}\hat{\boldsymbol{\imath}}$ is applied at $t = 0$ and maintained henceforth. If you need to calculate any quantity that you don't know, but can't do the calculation to find it, assume that the value is given.

a) Find the position of G as a function of time (the answer should have numbers and units).

b) Find a differential equation, and initial conditions, that when solved would give θ as a function of time. θ is the counterclockwise rotation of the plate from the configuration shown.

c) Write computer commands that would generate a drawing of the outline of the plate at $t = 1\,\mathrm{s}$. You can use hand calculations or the computer for as many of the intermediate commands as you like. Hand work and sketches should be provided as needed to justify or explain the computer work.

d) Run your code and show clear output with labeled plots. Mark output by hand to clarify any points.

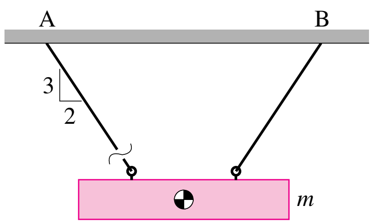


Problem 17.2.7

17.2.8 A uniform rectangular metal beam of mass m hangs symmetrically by two strings as shown in the figure.

a) Draw a free-body diagram of the beam and evaluate $\sum \vec{F}$.

b) Repeat (a) immediately after the left string is cut.



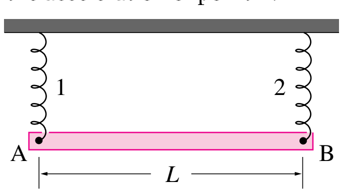
Problem 17.2.8

17.2.9 A uniform slender bar AB of mass m is suspended from two springs (each of spring constant K) as shown. Immediately after spring 2 breaks, determine

a) the angular acceleration of the bar,

b) the acceleration of point A, and

c) the acceleration of point B.



Problem 17.2.9

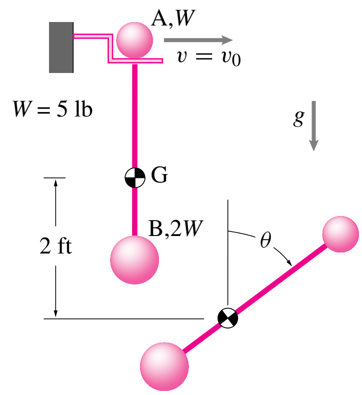
17.2.10 Two small spheres A and B are connected by a rigid rod of length $\ell = 1.0\,\text{ft}$ and negligible mass. The assembly is hung from a hook, as shown. Sphere A is struck, suddenly breaking its contact with the hook and giving it a horizontal velocity $v_0 = 3.0\,\text{ft/s}$ which sends the assembly into free fall. Determine the angular momentum of the assembly about its mass center at point G immediately after A is hit. After the center of mass has fallen two feet, determine:

a) the angle θ through which the rod has rotated,

b) the velocity of sphere A,

c) the total kinetic energy of the assembly of spheres A and B and the rod, and

d) the acceleration of sphere A.



Problem 17.2.10

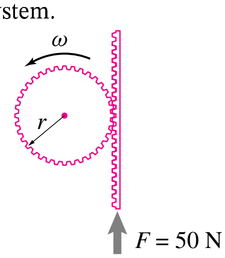
The next several problems concern Work, power and energy

17.2.11 Verify that the expressions for work done by a force F, $W = F\Delta S$, and by a moment M, $W = M\Delta\theta$, are dimensionally correct if ΔS and $\Delta\theta$ are linear and angular displacements respectively.

17.2.12 A uniform disc of mass m and radius r rotates with angular velocity $\omega\hat{\boldsymbol{k}}$. Its center of mass translates with velocity $\vec{\boldsymbol{v}} = \dot{x}\hat{\boldsymbol{\imath}} + \dot{y}\hat{\boldsymbol{\jmath}}$ in the xy-plane. What is the total kinetic energy of the disk?

17.2.13 Calculate the energy stored in a spring using the expression $E_P = \frac{1}{2}k\delta^2$ if the spring is compressed by 100 mm and the spring constant is 100 N/m.

17.2.14 In a rack and pinion system, the rack is acted upon by a constant force $F = 50\,\text{N}$ and has speed $v = 2\,\text{m/s}$ in the direction of the force. Find the input power to the system.

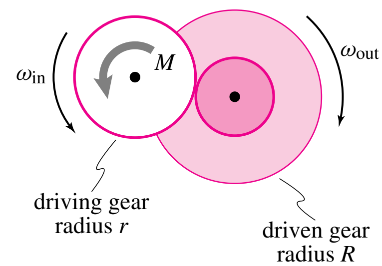


Problem 17.2.14

17.2.15 The driving gear in a compound gear train rotates at constant speed ω_0. The driving torque is M_{in}. If the driven gear rotates at a constant speed ω_{out}, find:

a) the input power to the system, and

b) the output torque of the system assuming there is no power loss in the system; i.e., power in = power out.

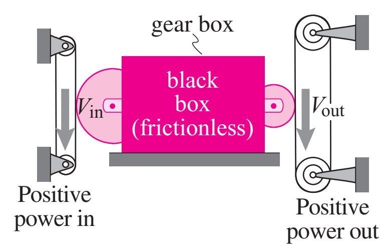


Problem 17.2.15

17.2.16 An elaborate frictionless gear box has an input and output roller with $V_{in} = const$. Assuming that $V_{out} = 7V_{in}$ and the force between the left belt and roller is $F_{in} = 3\,\text{lb}$:

a) What is F_{out} (draw a picture defining the signs of F_{in} and F_{out})?. *

b) Is F_{out} greater or less than the F_{in}? (Assume $F_{in} > 0$.) Why? *



Problem 17.2.16