17.1.1 A disk of radius R is hinged at point O at the edge of the disk, approximately as shown. It rotates counterclockwise with angular velocity $\dot{\theta} = \vec{\omega}$. A bolt is fixed on the disk at point P at a distance r from the center of the disk. A frame $x'y'$ is fixed to the disk with its origin at the center C of the disk. The bolt position P makes an angle ϕ with the x'-axis. At the instant of interest, the disk has rotated by an angle θ.

a) Write the position vector of point P relative to C in the $x'y'$ coordinates in terms of given quantities.

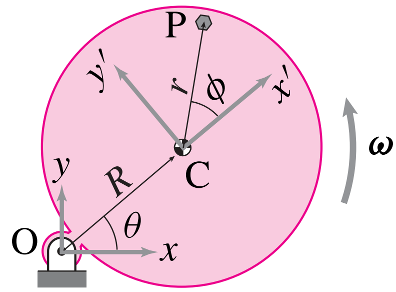
b) Write the position vector of point P relative to O in the xy coordinates in terms of given quantities.

c) Write the expressions for the rotation matrix $R(\theta)$ and the angular velocity matrix $\mathcal{S}(\vec{\omega})$.

d) Find the velocity of point P relative to C using $R(\theta)$ and the angular velocity matrix $\mathcal{S}(\vec{\omega})$.

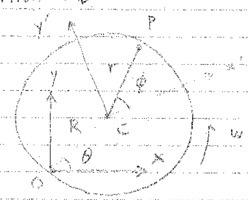
e) Using $R = 30\,\text{cm}$, $r = 25\,\text{cm}$, $\theta = 60°$, and $\phi = 45°$, find $[\vec{r}_{C/0}]_{xy}$, and $[\vec{r}_{P/0}]_{xy}$ at the instant shown.

f) Assuming that the angular speed is $\omega = 10\,\text{rad/s}$ at the instant shown, find $[\vec{v}_{C/0}]_{xy}$ and $[\vec{v}_{P/0}]_{xy}$ taking other quantities as specified above.



Problem 17.1

17.1 Solution



a) $\boxed{[\vec{r}_{P/C}]_{x'y'} = \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}}$

b) $[\vec{r}_{P/O}]_{xy} = [\vec{r}_{C/O}]_{xy} + [R(\theta)][\vec{r}_{P/C}]_{x'y'}$

$= \boxed{\begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}}$

c) $\boxed{R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}$ Rotation matrix

Book ⇒ pg. 669 $S(\vec{\omega}) = \dot{R}R^{-1} = \begin{bmatrix} -\dot{\theta}\sin\theta & -\dot{\theta}\cos\theta \\ \dot{\theta}\cos\theta & -\dot{\theta}\sin\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \dot{\theta}(-\sin\theta\cos\theta + \sin\theta\cos\theta) & -\dot{\theta}(\sin^2\theta + \cos^2\theta) \\ \dot{\theta}(\cos^2\theta + \sin^2\theta) & \dot{\theta}(\sin\theta\cos\theta - \sin\theta\cos\theta) \end{bmatrix}$

$= \boxed{\begin{bmatrix} 0 & -\dot{\theta} \\ \dot{\theta} & 0 \end{bmatrix}}$

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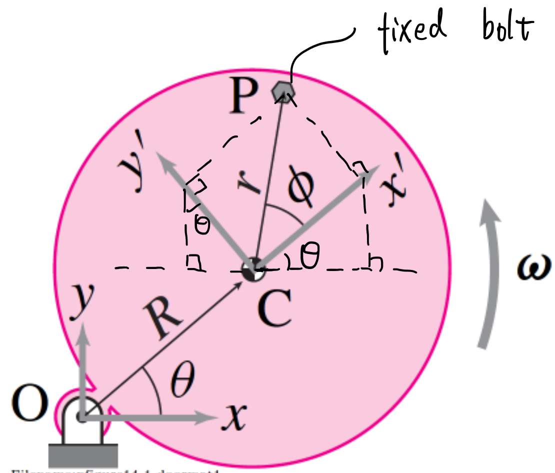
16.1.1

Thursday, April 11, 2019 14:17



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Problem 16.1.1

given: ω CCW

θ

ϕ

a) Find $[\vec{r}_{p/c}]_{x'y'}$

$$[\vec{r}_{p/c}]_{x'y'} = r * \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$

b) Find $[\vec{r}_{p/o}]_{xy}$

$$[\vec{r}_{p/o}]_{xy} = [\vec{r}_{c/o}]_{xy} + [\vec{r}_{p/c}]_{xy}$$

where $[\vec{r}_{p/c}]_{xy} = \begin{bmatrix} x_{p/c} \\ y_{p/c} \end{bmatrix} = \begin{bmatrix} x'_{p/c}\cos\theta - y'_{p/c}\sin\theta \\ x'_{p/c}\sin\theta + y'_{p/c}\cos\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x'_{p/c} \\ y'_{p/c} \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}_{R(\theta)} [\vec{r}_{p/c}]_{x'y'}$$

$$\therefore [\vec{r}_{p/o}]_{xy} = [\vec{r}_{c/o}]_{xy} + R(\theta)\cdot[\vec{r}_{p/c}]_{x'y'}$$

$$= R * \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} * r * \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

c) Find Rotation matrix $R(\theta)$ & angular velocity matrix $S(\vec{\omega})$

$R(\theta)$ see b) $R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ ← orthogonal

$$S(\vec{\omega}) = \dot{R}R^{-1} = \begin{bmatrix} -\dot{\theta}\sin\theta & -\dot{\theta}\cos\theta \\ \dot{\theta}\cos\theta & -\dot{\theta}\sin\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\dot{\theta} \\ \dot{\theta} & 0 \end{bmatrix}$$

d) Find $\vec{v}_{P/C}$ using $R(\theta)$ and $S(\vec{\omega})$

$$[\dot{r}_{P/C}]_{x'y'} = S(\vec{\omega})\, R(\theta)\, [\vec{r}_{P/C}]_{x'y'}$$

$$= \begin{bmatrix} 0 & -\dot{\theta} \\ \dot{\theta} & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$= \begin{bmatrix} -\dot{\theta}\,(r\cos\phi\sin\theta + r\cos\phi\cos\theta \\ \dot{\theta}\,(r\cos\phi\cos\theta - r\sin\phi\sin\theta \end{bmatrix}$$

e) Find $[\vec{r}_{C/O}]_{xy}$ and $[\vec{r}_{P/O}]_{xy}$

Given $R = 30\text{cm}$ $r = 25\text{cm}$ $\theta = 60°$ $\phi = 45°$

$$[\vec{r}_{C/O}]_{xy} = R * \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} 15 \\ 15\sqrt{3} \end{bmatrix} [\text{cm}]$$

$$[\vec{r}_{P/O}]_{xy} = R * \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} * r * \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 15\sqrt{3} \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{25\sqrt{2}}{2} \\ \frac{25\sqrt{2}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 8.53 \\ 50.13 \end{bmatrix} [\text{cm}]$$

f) Find $[\vec{v}_{C/O}]_{xy}$ and $[\vec{v}_{P/O}]_{xy}$

Given R, r, θ, ϕ as e) and $\omega = 10\,\text{rad/s}$

$$[\vec{v}_{C/O}]_{xy} = [\dot{\vec{r}}_{C/O}]_{xy} = \begin{bmatrix} \dot{R\cos\theta} \\ R\sin\theta \end{bmatrix} = \dot{\theta} \begin{bmatrix} -R\sin\theta \\ R\cos\theta \end{bmatrix} = \begin{bmatrix} -150\sqrt{3} \\ 150 \end{bmatrix} [\text{cm/s}]$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$[\vec{V}_{p/o}]_{xy} = [\dot{\vec{r}}_{p/o}]_{xy} = [\vec{V}_{c/o}]_{xy} + \dot{R}(\theta)\,[\vec{V}_{p/c}]_{x'y'}$$

$$= \begin{bmatrix} -150\sqrt{3} \\ 150 \end{bmatrix} + \dot{\theta} \begin{bmatrix} -\sin\theta & -\cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$= \begin{bmatrix} -150\sqrt{3} \\ 150 \end{bmatrix} + \begin{bmatrix} -241.48 \\ -64.70 \end{bmatrix} [\text{cm/s}] = \begin{bmatrix} -501.29 \\ 85.30 \end{bmatrix} [\text{cm/s}]$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Katherine Cao

P755

Point P relative to C in $x'y'$: $[\vec{r}_{P/C}]_{x'y'}$

$$\boxed{[\vec{r}_{P/C}]_{x'y'} = \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix} = r\begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}}$$

Point P relative to O in xy: $[\vec{r}_{P/O}]_{xy}$

$$[\vec{r}_{P/O}]_{xy} = [\vec{r}_{C/O}]_{xy} + [\vec{r}_{P/C}]_{xy}$$

$$[\vec{r}_{P/C}]_{xy} = [R(\theta)][\vec{r}_{P/C}]_{x'y'}$$

with $[R(\theta)] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$$[\vec{r}_{P/C}]_{xy} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} r \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$$

$$= r\begin{bmatrix} \cos\theta\cos\phi - \sin\theta\sin\phi \\ \sin\theta\cos\phi + \cos\theta\sin\phi \end{bmatrix}$$

$$= r\begin{bmatrix} \cos(\theta+\phi) \\ \sin(\theta+\phi) \end{bmatrix}$$

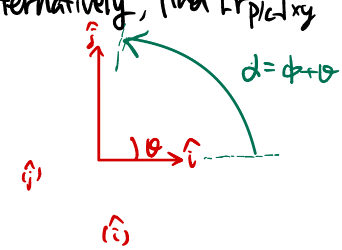
$$[\vec{r}_{C/O}]_{xy} = \begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix} = R\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$[\vec{r}_{P/O}]_{xy} = [\vec{r}_{C/O}]_{xy} + [\vec{r}_{P/C}]_{xy}$$

$$= R\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + r\begin{bmatrix} \cos(\theta+\phi) \\ \sin(\theta+\phi) \end{bmatrix}$$

$$\boxed{[\vec{r}_{P/O}]_{xy} = \begin{bmatrix} R\cos\theta + r\cos(\theta+\phi) \\ R\sin\theta + r\sin(\theta+\phi) \end{bmatrix}}$$

Alternatively, find $[\vec{r}_{P/C}]_{xy}$



$$[\vec{r}_{P/C}]_{xy} = \begin{bmatrix} r\sin\alpha \\ r\cos\alpha \end{bmatrix}$$

with $\alpha = \phi + \theta$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

See P755 16.8 & b) For $S(\vec{\omega})$ see P765

$$\boxed{R(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix} \quad S(\vec{\omega})=\begin{bmatrix}0 & -\omega\\ \omega & 0\end{bmatrix}}$$

Or, get $S(\vec{\omega})$ from

$$S(\vec{\omega})=\dot{R}(\theta)\,R(\theta)^{-1}$$

$$\dot{R}(\theta)=\begin{bmatrix}-\dot{\theta}\sin\theta & -\dot{\theta}\cos\theta\\ \dot{\theta}\cos\theta & -\dot{\theta}\sin\theta\end{bmatrix}$$

since $R(\theta)$ is orthogonal

$$R(\theta)^{-1}=R(\theta)^{T}=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$S(\vec{\omega})=\dot{R}(\theta)\,R(\theta)^{-1}$$

$$=\begin{bmatrix}-\dot{\theta}\sin\theta & -\dot{\theta}\cos\theta\\ \dot{\theta}\cos\theta & -\dot{\theta}\sin\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}0 & -\dot{\theta}\\ \dot{\theta} & 0\end{bmatrix}=\begin{bmatrix}0 & -\omega\\ \omega & 0\end{bmatrix}$$

see P765 for details

$$[\vec{v}_{P/C}]_{xy}=[\dot{\vec{r}}_{P/C}]_{xy}$$

$$=S(\vec{\omega})[\vec{r}_{P/C}^{\,ref}]_{xy}$$

$$=S(\vec{\omega})[\vec{r}_{P/C}]_{x'y'}$$

$$\boxed{=S(\vec{\omega})\,R(\theta)[\vec{r}_{P/C}]_{x'y'}}$$

$$=\begin{bmatrix}0 & -\omega\\ \omega & 0\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}r\cos\phi\\ r\sin\phi\end{bmatrix}$$

$$\boxed{=\begin{bmatrix}-\omega r\sin(\theta+\phi)\\ \omega r\cos(\theta+\phi)\end{bmatrix}}$$

$$[\vec{v}_{P/C}]_{xy}=[\dot{\vec{r}}_{P/C}]_{xy}$$

P is fixed in x'y' coordinate

$$S(\vec{\omega})=\dot{R}(\theta)\,R(\theta)^{-1}$$

$$\dot{R}(\theta)=S(\vec{\omega})\,R(\theta)$$

$$=\frac{d}{dt}\left(R(\theta)[\vec{r}_{P/C}]_{x'y'}\right)$$

$$=\dot{R}(\theta)[\vec{r}_{P/C}]_{x'y'}+R(\theta)[\cancel{\dot{\vec{r}}_{P/C}}]_{x'y'}^{\,0}$$

$$=S(\vec{\omega})R(\theta)[\vec{r}_{P/C}]_{x'y'}$$

(Slightly more detailed math)

Given $R=30$ cm $r=25$ cm

$\theta=60^\circ$ $\phi=45^\circ$

Find $[\vec{r}_{C/O}]_{xy}$, $[\vec{r}_{P/O}]_{xy}$

from b)

$$[\vec{r}_{C/O}]_{xy}=\begin{bmatrix}R\cos\theta\\ R\sin\theta\end{bmatrix}=R\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$$

$$=\begin{bmatrix}30\cos(60^\circ)\\ 30\sin(60^\circ)\end{bmatrix}\text{cm}=\boxed{\begin{bmatrix}15\\ 15\sqrt{3}\end{bmatrix}\text{cm}}$$

$$[\vec{r}_{P/O}]_{xy}=\begin{bmatrix}R\cos\theta+r\cos(\theta+\phi)\\ R\sin\theta+r\sin(\theta+\phi)\end{bmatrix}$$

$$=\begin{bmatrix}30\cos(60^\circ)+25\cos(60^\circ+45^\circ)\\ 30\sin(60^\circ)+25\sin(60^\circ+45^\circ)\end{bmatrix}\text{cm}=\boxed{\begin{bmatrix}8.53\\ 50.1\end{bmatrix}\text{cm}}$$

$$[\vec{v}_{C/O}]_{xy}=S(\vec{\omega})I[\vec{r}_{C/O}]_{xy}=S(\vec{\omega})[\vec{r}_{C/O}]_{xy}$$

$$=\begin{bmatrix}0 & -\omega\\ \omega & 0\end{bmatrix}\begin{bmatrix}R\cos\theta\\ R\sin\theta\end{bmatrix}=\begin{bmatrix}-\omega R\sin\theta\\ \omega R\cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}-(10\,\text{rad/s})(30\,\text{cm})\sin(60^\circ)\\ (10\,\text{rad/s})(30\,\text{cm})\cos(60^\circ)\end{bmatrix}=\boxed{\begin{bmatrix}-150\sqrt{3}\\ 150\end{bmatrix}\text{cm/s}}$$

$$[\vec{v}_{P/O}]_{xy}=S(\vec{\omega})I[\vec{r}_{P/O}]_{xy}=S(\vec{\omega})[\vec{r}_{P/O}]_{xy}$$

$$=\begin{bmatrix}0 & -\omega\\ \omega & 0\end{bmatrix}\begin{bmatrix}R\cos\theta+r\cos(\theta+\phi)\\ R\sin\theta+r\sin(\theta+\phi)\end{bmatrix}$$

$$=\boxed{\begin{bmatrix}-501\\ 85.3\end{bmatrix}\text{cm/s}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

d) $$[\dot{\vec{r}}_{P/C}]_{x'y'} = S(\vec{\omega})\, R(\theta)\, [\vec{r}_{P/C}]_{x'y'}$$

$$= \begin{bmatrix} 0 & -\dot{\theta} \\ \dot{\theta} & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$= \begin{bmatrix} -\dot{\theta}\sin\theta & -\dot{\theta}\cos\theta \\ \dot{\theta}\cos\theta & -\dot{\theta}\sin\theta \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -\dot{\theta}\,(r\cos\phi\sin\theta + r\cos\phi\cos\theta) \\ \dot{\theta}\,(r\cos\phi\cos\theta - r\sin\phi\sin\theta) \end{bmatrix}}$$

e) $R = 30\,\text{cm}, \quad r = 25\,\text{cm}, \quad \theta = 60^\circ, \quad \phi = 45^\circ$

$$[\vec{r}_{C/O}]_{xy} = \begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix} = \begin{bmatrix} 30\cos 60^\circ \\ 30\sin 60^\circ \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 15 \\ 15\sqrt{3} \end{bmatrix} [\text{cm}]}$$

$$[\vec{r}_{P/O}]_{xy} = \begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 15\sqrt{3} \end{bmatrix} + \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 25\cos 45^\circ \\ 25\sin 45^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 15\sqrt{3} \end{bmatrix} + 25 \begin{bmatrix} \sqrt{2}/4 - \sqrt{6}/4 \\ \sqrt{6}/4 + \sqrt{2}/4 \end{bmatrix} [\text{cm}]$$

$$= \boxed{\begin{bmatrix} 8.53 \\ 50.13 \end{bmatrix} [\text{cm}]}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

f) $\omega = 10\ rad/s$

$$[\vec{v}_{C/O}]_{xy} = [\dot{\vec{r}}_{C/O}]_{xy} = \begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix}^{\cdot} = \dot{\theta}\begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix}$$

$$= \dot{\theta}\,[\vec{r}_{C/O}]_{xy} = (10)\begin{bmatrix} 15 \\ 15\sqrt{3} \end{bmatrix} = \boxed{\begin{bmatrix} 150 \\ 150\sqrt{3} \end{bmatrix} [cm/s]}$$

$$[\vec{v}_{P/O}]_{xy} = \dot{\theta}\,[\vec{r}_{P/O}]_{xy} = (10)\begin{bmatrix} 8.53 \\ 50.13 \end{bmatrix}$$

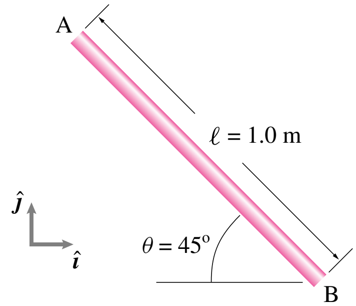
$$= \boxed{\begin{bmatrix} 85.3 \\ 500.13 \end{bmatrix} [cm/s]}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

17.1.2 A uniform rigid rod AB of length $\ell = 1\,\text{m}$ rotates at a constant angular speed ω about an unknown fixed point. At the instant shown, the velocities of the two ends of the rod are $\vec{v}_A = -1\,\text{m/s}\,\hat{\imath}$ and $\vec{v}_B = 1\,\text{m/s}\,\hat{\jmath}$.

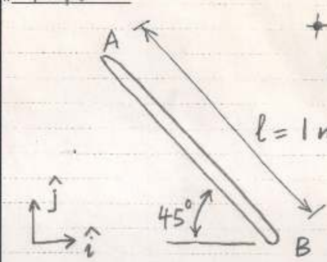
a) Find the angular velocity of the rod.

b) Find the center of rotation of the rod.



Problem 17.2

4.40 2



+ P? center of rotation (unknown)

Given : rod rotates at constant rate about unknown fixed point "P"

$\underline{V}_A = -1\,\hat{i}$ m/s , $\underline{V}_B = 1$ m/s $\hat{j}$

Find : $\underline{\omega}$, center of rotation P

Note : KINEMATICS prob., FBD not reqd.

a) $\underline{V}_B = \underline{V}_A + \underline{V}_{B/A} = \underline{V}_A + \underline{\omega} \times \underline{r}_{B/A}$

or, $\underline{V}_{B/A} = \underline{V}_B - \underline{V}_A = \underline{\omega} \times \underline{r}_{B/A}$

or $(1\hat{i} + 1\hat{j})\,(\text{m/s}) = \underline{\omega} \times \left(\dfrac{\hat{i} - \hat{j}}{\sqrt{2}}\right)(\text{m})$

ASSUME $\underline{\omega} = \omega\hat{k}$, planar prob.

$\sqrt{2}(1\hat{i} + 1\hat{j})(\text{m/s}) = \omega(\hat{j} + \hat{i})\,(\text{m})$

$\Rightarrow \omega = 1\,\text{s}^{-1}$, $\boxed{\underline{\omega} = \sqrt{2}\,\hat{k}\ \text{s}^{-1}}$

b) $\underline{V}_P = \underline{V}_A + \underline{\omega} \times \underline{r}_{P/A} = \underline{0}$

Let $\underline{r}_{P/A} = a\hat{i} + b\hat{j}$

$-1\hat{i}\,(\text{m/s}) + \sqrt{2}\,\hat{k} \times (a\hat{i} + b\hat{j})\ \text{s}^{-1}$

$= -1\hat{i}\ (\text{m/s}) + \sqrt{2}a\,\hat{j}\,\text{s}^{-1} - \sqrt{2}b\,\hat{i}\,\text{s}^{-1} = \underline{0}$

$\Rightarrow \boxed{a = 0\ \text{m},\ b = -\dfrac{1}{\sqrt{2}}\ \text{m}}$

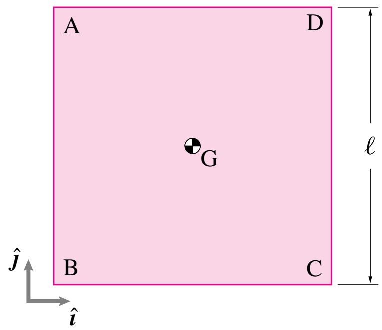
$\boxed{\underline{r}_P = \underline{r}_A - \dfrac{1}{\sqrt{2}}\,\hat{j}\ \text{m}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

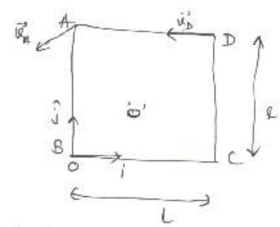
17.1.3 A square plate ABCD rotates at a constant angular speed about an unknown point in its plane. At the instant shown, the velocities of the two corner points A and D are $\vec{v}_A = -2\,\text{ft/s}(\hat{\imath} + \hat{\jmath})$ and $\vec{v}_D = -(2\,\text{ft/s})\hat{\imath}$, respectively.

a) Find the center of rotation of the plate.

b) Find the acceleration of the center of mass of the plate.



Problem 17.3



Co ordinates of A $(0, l)$; $l\hat{j}$

" " D (l, l) ; $l(\hat{i} + \hat{j})$

Let the " " O' [center] ; $(a\hat{i} + b\hat{j})$

Center O' has zero velocity.

Position vectors of points A and D w.r.t. O' are $\vec{O'A}$ & $\vec{O'D}$ respectively.

$\vec{O'A} \perp \vec{V_A}$ $\qquad \vec{O'A} = \vec{OA} - \vec{OO'} = (-a)\hat{i} + (l-b)\hat{j}$

$\vec{O'D} \perp \vec{V_D}$ $\qquad \vec{O'D} = \vec{OD} - \vec{OO'} = (l-a)\hat{i} + (l-b)\hat{j}$

$$\vec{O'A} \cdot \vec{V_A} = 0 \Rightarrow [-a\hat{i} + (l-b)\hat{j}] \cdot [-2\hat{i} - 2\hat{j}] = 0$$

$$\Rightarrow 2a - 2l + 2b = 0$$

$$\Rightarrow \boxed{a - l + b = 0}$$

$$\vec{O'D} \cdot \vec{V_D} = 0 \Rightarrow [(l-a)\hat{i} + (l-b)\hat{j}] \cdot [-2\hat{i}] = 0$$

$$\Rightarrow -2l + 2a = 0$$

$$\Rightarrow \boxed{l = a}$$

$\therefore b = 0$

(a) $\therefore$ Center O' is at $l\hat{i}$.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

(b) Angular velocity of the plate? Lets $\omega = \omega_0 \hat{k}$ (rotating in XY plane)

$$\vec{V}_D = \vec{V}_A + \vec{\omega} \times \vec{r}_{D/A}$$

$$\Rightarrow -2\hat{i} = -2(\hat{i}+\hat{j}) + \omega_0\hat{k} \times (L\hat{i})$$

$$\Rightarrow 0 = -2\hat{j} + (-\omega_0 l)\hat{j}$$

$$\Rightarrow 2 + \omega_0 l = 0$$

$$\Rightarrow \omega_0 = -\frac{2}{l} \text{ rad/sec}$$

velocity of center of mass?

$$\vec{V}_G = \vec{V}_A + \vec{\omega} \times \vec{r}_{G/A}$$

$$= (-2\hat{i} - 2\hat{j}) + \left(-\frac{2}{L}\right)\hat{k} \times \left(\frac{L}{2}\hat{i} - \frac{L}{2}\hat{j}\right)$$

$$= (-2\hat{i} - 2\hat{j} - \hat{j} - \hat{i}$$

$$= -3(i+j)$$

Acceleration of the center of mass?

$$\vec{a}_G = \vec{a}_{O'} + \vec{a}_{G/O'}$$

$$= \underbrace{\vec{a}_{O'}}_{0} + \underbrace{\dot{\vec{\omega}} \times \vec{r}_{G/O'}}_{0} - \omega^2 \vec{r}_{G/O'}$$

[const. angular velocity: given]

$$\Rightarrow \vec{a}_G = -\omega^2 \vec{r}_{G/O'}$$

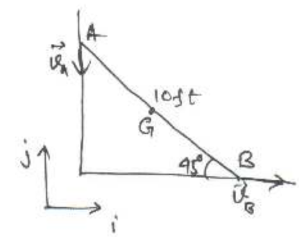
$$\Rightarrow \vec{a}_G = -\left(\frac{2}{l}\right)^2\left[-\frac{l}{2}\hat{i} + \frac{l}{2}\hat{j}\right]$$

$$= \frac{2}{l}\left[\hat{i} - \hat{j}\right]$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

17.1.4 A ten foot ladder is leaning between a floor and a wall. The top of the ladder is sliding down the wall at one foot per second. (The foot is simultaneously sliding out on the floor). When the ladder makes a 45 degree angle with the vertical what is the speed of the midpoint of the ladder?

6.1.4



$\vec{v}_A = (-1\ ft/sec)\hat{j}$

$\vec{v}_B = v\hat{i}$

Angular velocity of ladder ?

$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$

$\Rightarrow v\hat{i} = -1\hat{j} + \omega_0\hat{k} \times \left\{ \frac{10}{\sqrt{2}}\hat{i} - \frac{10}{\sqrt{2}}\hat{j} \right\}$

$\Rightarrow v\hat{i} = -1\hat{j} + \omega_0 \cdot \frac{10}{\sqrt{2}} \left\{ \hat{j} + \hat{i} \right\}$

$(\omega_0)\frac{10}{\sqrt{2}} - 1 = 0$

$\Rightarrow \omega_0 = \frac{\sqrt{2}}{10}$ rad/sec

Velocity of mid point G :

$\vec{V}_G = \vec{V}_A + \vec{\gamma}_{G/A}$

$= \vec{V}_A + \vec{\omega} \times \vec{r}_{G/A}$ $\quad \vec{r}_{G/A} = \frac{10}{\sqrt{2}}(i - j)$

$= -1\hat{j} + \frac{\sqrt{2}}{10}\hat{k} \times \frac{10}{\sqrt{2}}(i - j)$

$= -\hat{j} + \hat{k} \times (\hat{i} - \hat{j})$

$= -\hat{j} + (\hat{j} + \hat{i})$

$= \hat{i}$

$\Rightarrow \vec{V}_G = 1$ ft/sec along +ve x direction.

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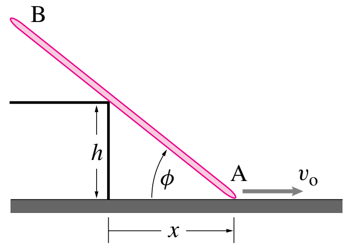
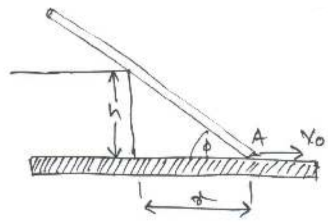
17.1.5 The slender rod AB rests against the step of height h, while end "A" is moved along the ground at a constant velocity v_o. Find $\dot{\phi}$ and $\ddot{\phi}$ in terms of x, h, and v_o. Is $\dot{\phi}$ positive or negative? Is $\ddot{\phi}$ positive or negative?



Problem 17.5

16.1.5



$$\tan\phi = \frac{h}{x}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{h}{x}\right)$$

$$\Rightarrow \frac{d\phi}{dt} = \frac{1}{1+\left(\frac{h}{x}\right)^2} \times \frac{x\cdot\cancelto{0}{\frac{dh}{dt}} - h\frac{dx}{dt}}{x^2}$$

$$= \frac{1}{x^2+h^2} \times -h\frac{dx}{dt}$$

$$\Rightarrow \frac{d\phi}{dt} = \frac{-hv_o}{x^2+h^2}$$

Denominator $\rightarrow$ +ve

$v_o \rightarrow$ +ve

$h \rightarrow$ +ve

$\therefore \dot{\phi} \rightarrow$ −ve.

$$\frac{d^2\phi}{dt^2} = \frac{-hv_o\left[-2x\frac{dx}{dt}\right]}{(x^2+h^2)^2} = \frac{2hxv_o^2}{(x^2+h^2)^2}$$

$\Rightarrow \ddot{\phi} \Rightarrow$ +ve.

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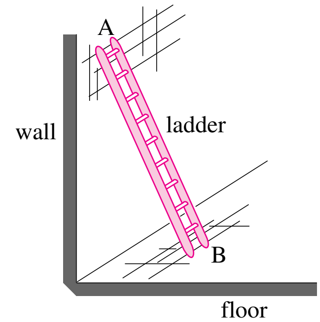
17.1.6 Consider the motion of a rigid ladder which can slide on a wall and on the floor as shown in the figure. The point A on the ladder moves parallel to the wall. The point B moves parallel to the floor. Yet, at a given instant, both have velocities that are consistent with the ladder rotating about some special point, the center of rotation (COR). Define appropriate dimensions for the problem.

a) Find the COR for the ladder when it is at some given position (and moving, of course). [Hint: if a point A is 'going in circles' about another point C, that other point C must be in the direction perpendicular to the motion of A.

b) As the ladder moves, the COR changes with time. What is the set of points on the plane that are the COR's for the ladder as it falls from straight up to lying on the floor?



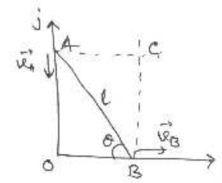
Problem 17.6

CHAPTER 16

⑤

Q 16.1.6

Length of ladder l.



$\vec{v}_A \Rightarrow$ along $-\hat{j}$

$\vec{v}_B \Rightarrow$ along $\hat{i}$

Lines perpendicular to the velocities $\vec{v}_A$ & $\vec{v}_B$ intersect at C.

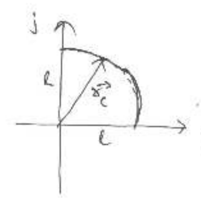
Hence C is center of rotation [COR]

co-ordinates of C w.r.t. origin O are $(l\cos\theta, l\sin\theta)$

(a) $\therefore \vec{r}_C = l\left[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right]$

As the ladder falls from straight up to lying on the floor θ varies from 90° to 0°.

b) The locus of center C will be quarter of a circle present in the first quadrant of co-ordinate axes.



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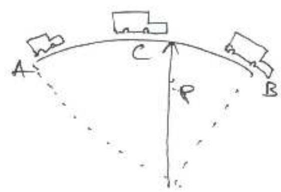
17.1.7 A car driver on a very boring highway is carefully monitoring her speed. Over a one hour period, the car travels on a curve with constant radius of curvature, $\rho = 25$ mi, and its speed increases uniformly from 50 mph to 60 mph. What is the acceleration of the car half-way through this one hour period, in path coordinates?

Q
16.1.7

6



Acceleration in path coordinates:

$$\bar{a} = a_t\hat{e}_t + a_n\hat{e}_n$$

$$a_t = \ddot{s} \qquad a_n = \frac{\dot{s}^2}{\rho}$$

$$\dot{s}_A = 50 \text{ mph}$$

$$\dot{s}_B = 60 \text{ mph}$$

$$\dot{s}_B = \dot{s}_A + a_t T$$

$$\Rightarrow a_t = \frac{60-50}{1} = 10 \text{ miles/hour}^2$$

Total distance travelled

$$s_{total} = \dot{s}_A T + \frac{1}{2} a_t T^2$$
$$= 50 \times 1 + \frac{1}{2} \times 10 \times 1^2$$
$$= 55 \text{ miles.}$$

Velocity at half way:

$$\dot{s}_C^2 = \dot{s}_A^2 + 2 \cdot a_t s_{1/2}$$
$$= 50^2 + 2 \cdot 10 \cdot \frac{55}{2}$$
$$= 2500 + 550$$
$$= 3050$$

Acceleration at C:

$$\bar{a} = a_t\hat{e}_t + a_n\hat{e}_n$$
$$= 10\hat{e}_t + \frac{3050}{25}\hat{e}_n$$
$$= 10\hat{e}_t + 122\hat{e}_n \quad \text{miles/hr}^2$$

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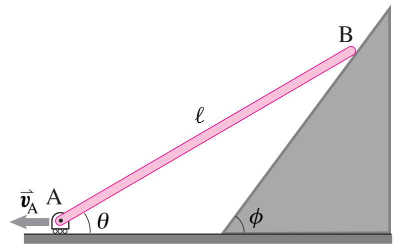
17.1.8 The bar AB shown in the figure is 1 m long. The end A of the bar is dragged to the left on the horizontal floor at a constant speed $v_0 = 0.2\,\mathrm{m/s}$. The other end of the bar drags along the inclined plane that makes an angle $\phi = 60°$ with the horizontal.

a) Does the end B of the rod also have a constant speed? Does it imply that the rod has a constant angular velocity? Guess your answer and write down your reasoning. No calculations.

b) Find an expression for the angular velocity of the rod in terms of θ, ϕ, ℓ, and v_0.

c) Find the velocity of end B when $\theta = 30°$.

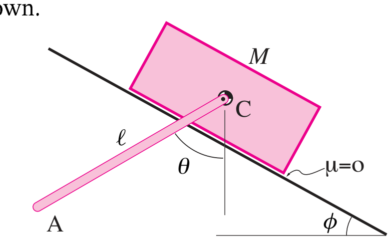


Problem 17.8

Answer:
(b) $\vec{\omega} = -\frac{v_0 \sin\phi}{\ell \cos(\phi-\theta)}\hat{\boldsymbol{k}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

17.1.10 A rod AC of length $\ell = 2\,\mathrm{m}$ is mounted on a cart at point C. The cart slides down under gravity on a frictionless surface inclined at $\phi = 45°$ from the horizontal. Assume that bar has negligible mass compared to that of the cart. The cart is released from rest and it reaches the position shown in $t = 2\,\mathrm{s}$. At the instant shown, $\theta = 60°$, and rod AC has an angular velocity $\vec{\omega} = 0.5\,\mathrm{rad/s}\hat{\boldsymbol{k}}$ and angular acceleration $\vec{\alpha} = 1\,\mathrm{rad/s^2}\hat{\boldsymbol{k}}$. Find the velocity and acceleration of the center of mass of the rod at the instant shown.

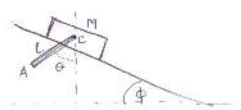


Problem 17.10

Q

16.1.10

(11)



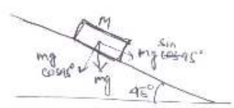
Acceleration of the cart $= g/\sqrt{2}$

velocity of the cart at the position shown

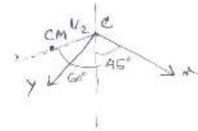
$v_c = u + at$

$= 0 + \frac{g}{\sqrt{2}} \cdot 2$

$\Rightarrow v_c = \sqrt{2}g$ m/s



The co-ordinate axes are choosen as shown.



Velocity of the center of mass of the rod:

$\vec{v}_{cm} = \vec{v}_c + \vec{\omega}_{rod} \times \vec{r}_{cm/c}$

Here $\vec{v}_c = \sqrt{2}g\,\hat{i}$ m/s

$\vec{\omega}_{rod} = 0.5\,\hat{k}$ rad/s

$\vec{r}_{cm/c} = \frac{\ell}{2}\left[-\cos 75°\,\hat{i} + \sin 75°\,\hat{j}\right]$

$= \frac{\ell}{2}\left[-0.258\,\hat{i} + 0.966\,\hat{j}\right]$

$= -0.258\,\hat{i} + 0.966\,\hat{j}$ m $\quad [\because \ell = 2\,\mathrm{m}]$

$\therefore \vec{v}_{cm} = \sqrt{2}g\,\hat{i} + 0.5\hat{k} \times \left[-0.258\,\hat{i} + 0.966\,\hat{j}\right]$

$= 13.87\,\hat{i} + 0.129\,(-\hat{j}) + 0.483\,(-\hat{i})$

$= \left[13.387\,\hat{i} - 0.129\,\hat{j}\right]$ m/s

Acceleration of the center of mass of the rod:

$\vec{a}_{cm} = \vec{a}_c + \vec{\alpha} \times \vec{r}_{cm/c} + \vec{\omega}_{rod} \times (\vec{\omega}_{rod} \times \vec{r}_{cm/c})$

$= 1\hat{k} \times \left[-0.258\,\hat{i} + 0.966\,\hat{j}\right] + 0.5\,\hat{k} \times \left[0.5\hat{k} \times (-0.258\,\hat{i} + 0.966\,\hat{j})\right]$

$= -0.258\,\hat{j} - 0.966\,\hat{i} + 0.5\,\hat{k} \times \left[-0.129\,\hat{j} - .483\,\hat{i}\right]$

$= -0.258\,\hat{j} - 0.966\,\hat{i} + 0.0648\,\hat{i} - 0.2415\,\hat{j}$

$= -0.9012\,\hat{i} - 0.4995\,\hat{j}$ m/s²

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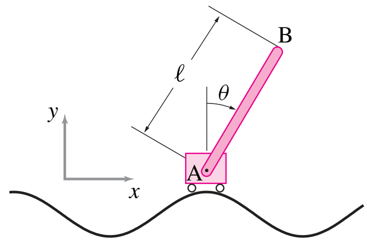
17.1.11 A vertical bar pendulum AB of length $\ell = 4\,\text{ft}$ is mounted on a cart at point A. The cart has a motor that drives the bar pendulum back and forth about the vertical position such that $\theta(t) = \theta_0 \sin \omega t$ where $\theta_0 = \pi/6$ and $\omega = \pi\,\text{rad/s}$. The cart moves at a constant speed $v_0 = 2\,\text{ft/s}$ on a sinusoidal track represented by $y(x) = y_0 \sin \lambda x$ where $y_0 = 0.5\,\text{ft}$ and $\lambda = \pi/4/\,\text{ft}$.

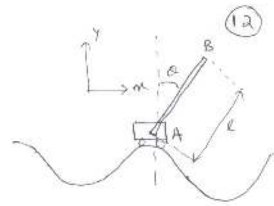
a) Find the velocity of point B when $\theta = 0$.

b) Find and plot the position of point B for one full oscillation of the bar, assuming the bar starts from its leftmost angular position (that is, $\theta = -\theta_0$) when the cart is on one of the crests of the track.



Problem 17.11

16.1.11



$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{AB}$$

$$\cancel{= 2\hat{i} + \vec{\omega}_{AB} \times \vec{r}_{AB}}$$

$\theta(t) = \theta_0 \sin \omega t$

$\Rightarrow \dot{\theta}(t) = (\theta_0 \omega) \cos \omega t$

$\Rightarrow \omega_{AB} = \frac{\pi^2}{6} \cos \pi t$

Here at $t = 0$ $\theta(t) = -\theta_0 = -\frac{\pi}{6}$

$\therefore \theta(t) = \theta_0 \sin(\omega t - \frac{\pi}{2}) = \frac{\pi}{6} \sin(\pi t - \frac{\pi}{2})$

Now ω_{AB} modifies to

$$\vec{\omega}_{AB} = \frac{\pi^2}{6} \cos(\pi t - \frac{\pi}{2})\,\hat{k}$$

(a) velocity of point B when $\theta = 0$?

time at $\theta = 0$?

$0 = \frac{\pi}{6} \sin(\pi t - \frac{\pi}{2})$

$\Rightarrow \pi t - \frac{\pi}{2} = 0 \Rightarrow t = \frac{1}{2}$ sec.

$\vec{\omega}_{AB}\Big|_{t=\frac{1}{2}} = \frac{\pi^2}{6}$ rad/s

$\vec{r}_{AB}\Big|_{\theta=0} = 4\hat{j}$ ft

At $t = \frac{1}{2}$, horizontal distance covered by the cart

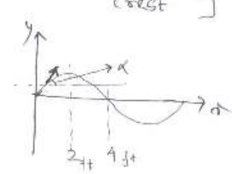
$= v_0 \times \frac{1}{2} = 2 \times \frac{1}{2} = 1$ ft.

y-co-ordinate of the sinusoidal track at $x = 1$ ft.

$y(x) = y_0 \sin(\lambda x + \frac{\pi}{2})$ [$\because$ the cart starts from a crest]

$= (0.5) \sin(\frac{\pi}{4} + \frac{\pi}{2}) = \frac{1}{2\sqrt{2}}$ ft

At point $(1, \frac{1}{2\sqrt{2}})$ slope (direction of $\vec{v}_A$):



$y'(x)\Big|_{x=1} = y_0 \lambda \cos(\lambda x + \frac{\pi}{2})$

$= (0.5 \times \frac{\pi}{4}) \cos(\frac{\pi}{4} \times 1 + \frac{\pi}{2})$

$= -0.2776$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Q

16.1.11 contd...

(13)

$$\tan^{-1}(\alpha) = -0.2776$$
$$\Rightarrow \alpha = -15.51^\circ$$

$$v_A \cos\alpha = 2 \text{ ft/sec}$$
$$\Rightarrow v_A \cos(15.51) = 2$$
$$\Rightarrow v_A = 2.076 \text{ ft/sec.}$$

$$\therefore \vec{v}_A = (v_A \cos\alpha)\hat{i} + (v_A \sin\alpha)\hat{j}$$
$$= 2\hat{i} - 0.55\hat{j}$$

$\vec{v}_B$?

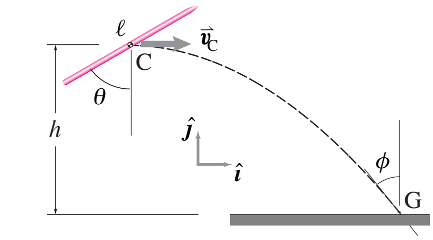
$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{AB}$$
$$= (2\hat{i} - 0.55\hat{j}) + \frac{\pi^2}{6}.\cos\left(\pi.\frac{1}{2} - \frac{\pi}{2}\right)\hat{k} \times (4\hat{j})$$
$$= 2i - 0.55\hat{j} - \frac{\pi^2}{6} \times 4\hat{i}$$
$$= (-4.58\hat{i} - 0.55\hat{j}) \text{ ft/s}$$

(b) Locus of point B:

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$
$$\vec{r}_A = 2t\hat{i} + 0.5\sin\left[\frac{\pi}{4}(2t) + \frac{\pi}{2}\right]\hat{j}$$
$$= 2t\hat{i} + \frac{1}{2}\sin\frac{\pi}{2}(t+1)\hat{j}$$
$$\vec{r}_{B/A} = AB\sin\theta\hat{i} + AB\cos\theta\hat{j}$$
$$\cancel{= 4\left[\frac{\pi}{6}\sin\left(\pi t - \frac{\pi}{2}\right)\right.}$$
$$= 4\sin\left[\frac{\pi}{6}\sin\left(\pi t - \frac{\pi}{2}\right)\right]\hat{i} + 4\cos\left[\frac{\pi}{6}\cos\left(\pi t - \frac{\pi}{2}\right)\right]\hat{j}$$
$$= 4\sin\left[-\frac{\pi}{6}\cos(\pi t)\right]\hat{i} + 4\cos\left[\frac{\pi}{6}\sin(\pi t)\right]\hat{j}$$

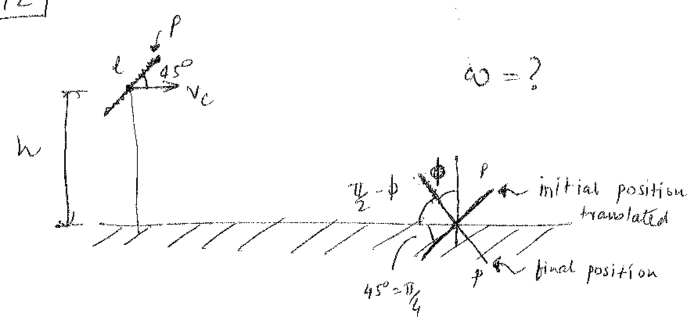
17.1.12 The center of mass of a javelin travels on a more or less parabolic path while the javelin rotates during its flight. In a particular throw, the velocity of the center of mass of a javelin is measured to be $\vec{v}_C = 10\,\mathrm{m/s}\hat{\imath}$ when the center of mass is at its highest point $h = 6\,\mathrm{m}$. As the javelin lands on the ground, its nose hits the ground at G such that the javelin is almost tangent to the path of the center of mass at G. Neglect the air drag and lift on the javelin.

a) Given that the javelin is at an angle $\theta = 45°$ at the highest point, find the angular velocity of the javelin. Assume the angular velocity is constant during the flight and that the javelin makes less than a full revolution.



Problem 17.12

14.12



$\omega = ?$

- Ignore the length of the javelin in calculation
- Consider linear motion, solve for ϕ and time to hit ground (t)
- Solve for angular speed using ϕ and t.

For projectile motion:

$$h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 6}{10}}$$

$$\Rightarrow t = 1.1\ s$$

$$\phi = \tan^{-1}\left(\frac{V_x}{V_y}\right) = \tan^{-1}\left(\frac{V_c}{gt}\right) = \tan^{-1}\left(\frac{10}{10 \times 1.1}\right)$$

$$\Rightarrow \phi = 0.7378$$

Total angle turned from figure above $= \frac{\pi}{4} + \left(\frac{\pi}{2} - \phi\right)$

$$= 1.62$$

Angular velocity, $\omega = \frac{\text{Angle turned}}{\text{time taken}} = \frac{1.62}{1.1}$

$$\boxed{\omega = 1.47\ \text{rad/s}}$$

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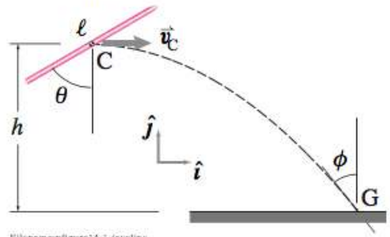
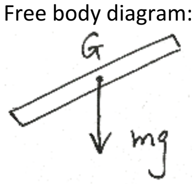


Problem 16.1.12

Free body diagram:



Linear momentum balance: $m\vec{a} = \sum \vec{F} = -mg\hat{\jmath}$

Take the moment that the center of mass is at its highest point (point C) as t=0 and integrate the LMB, we get

$$v_x = const. = v_x(0) = \vec{v}_C \cdot \hat{\imath} = 10\ m/s$$
$$v_y = \int -g\,dt = -gt + v_y(0) = -gt + \vec{v}_C \cdot \hat{\jmath} = -gt$$

$$y = \int -v_y\,dt = -\tfrac{1}{2}gt^2 + y(0) = -\tfrac{1}{2}gt^2 + h$$

The time the javelin gets to point G is

$$y = -\tfrac{1}{2}gt^2 + h = 0 \rightarrow t_G = \sqrt{\tfrac{2h}{g}} = 1.106\ s$$

At t_G, $v_x = 10\ m/s, v_y = -gt = -\sqrt{2gh} = -10.84\ m/s$

$$\phi = atan2(|v_x|, |v_y|) = 0.7451\ rad$$

We are given that the angular velocity of the javelin is constant, so the angular velocity is

$\omega = \frac{\pi-(\theta+\phi)}{t_G} = 1.456\ rad/s$ (clockwise) or

$\omega = \frac{\theta+\phi}{t_G} = 1.383\ rad/s$ (counterclockwise)

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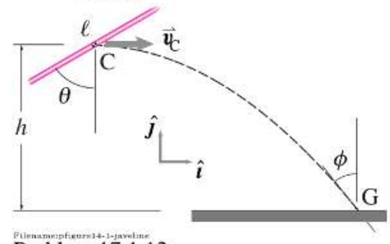
MAE2030 HW P54 (17.1.12) Solution Duan Li

17.1.12 The center of mass of a javelin travels on a more or less parabolic path while the javelin rotates during its flight. In a particular throw, the velocity of the center of mass of a javelin is measured to be $\vec{v}_C = 10\,\text{m/s}\hat{\imath}$ when the center of mass is at its highest point $h = 6\,\text{m}$. As the javelin lands on the ground, its nose hits the ground at G such that the javelin is almost tangent to the path of the center of mass at G. Neglect the air drag and lift on the javelin.

a) Given that the javelin is at an angle $\theta = 45°$ at the highest point, find the angular velocity of the javelin. Assume the angular velocity is constant during the flight and that the javelin makes less than a full revolution.



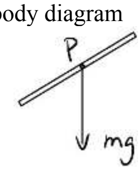
Problem 17.1.12

Given: $\vec{v}_C(t=0)$, h, $\theta(t=0)$, constant angular velocity

Define time $t = 0$ to be when the center of mass is at point C

Find: angular velocity

Free body diagram



Define point P to be the center of mass of the javelin

Linear momentum balance

$$m\vec{a} = \sum \vec{F} = -mg\hat{\jmath}$$

Initial conditions

$$v_x(0) = \vec{v}_C \cdot \hat{\imath} = 10 \text{ m/s}$$
$$v_y(0) = \vec{v}_C \cdot \hat{\jmath} = 0 \text{ m/s}$$
$$y(0) = h = 6 \text{ m}$$

Solve the LMB

$$\{m\vec{a} = -mg\hat{\jmath}\} \cdot \hat{\imath} \rightarrow ma_x = 0$$
$$v_x = \int a_x\,dt = \int 0\,dt = C_1$$
$$v_x(0) = C_1 = 10 \text{ m/s}$$
$$\rightarrow v_x = 10 \text{ m/s}$$

$$\{m\vec{a} = -mg\hat{\jmath}\} \cdot \hat{\jmath} \rightarrow ma_y = -mg$$
$$v_y = \int a_y\,dt = \int -g\,dt = -gt + C_2$$
$$v_y(0) = C_2 = 0 \text{ m/s}$$
$$\rightarrow v_y = -gt$$

$$y = \int v_y\,dt = -\frac{1}{2}gt^2 + C_3$$

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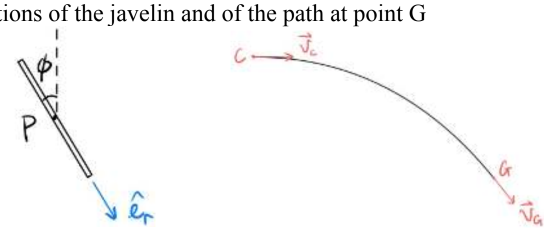
$y(0) = C_3 = h$
→ $y = -\frac{1}{2}gt^2 + h$

The time the javelin gets to point G is
$y = -\frac{1}{2}gt_G^2 + h = 0\, m$
→ $t_G = 1.11$ s

The velocity at point G
$v_x(t_G) = 10$ m/s $= v_{Gx}$
$v_y(t_G) = -gt_G = -10.8$ m/s $= v_{Gy}$
→ $\vec{v}_G = v_{Gx}\hat{\imath} + v_{Gy}\hat{\jmath}$

Directions of the javelin and of the path at point G

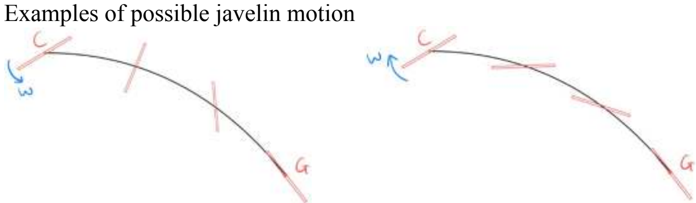


The direction of the javelin at point G
$\hat{e}_r = \sin\phi\,\hat{\imath} - \cos\phi\,\hat{\jmath}$

The tangent direction of the path at point G is $\frac{\vec{v}_G}{|\vec{v}_G|}$
(Velocity is always tangent to the path.)

According to the problem, at point G, the javelin is tangent to the path, so
$\hat{e}_r \times \vec{v}_G = \vec{0}$
$\sin\phi\, v_{Gy}\hat{k} + \cos\phi\, v_{Gx}\hat{k} = \vec{0}$
→ $\phi = \arctan(-\frac{v_{Gx}}{v_{Gy}}) = 0.745$ rad

Examples of possible javelin motion



According to the problem, the angular velocity of the javelin is constant and the javelin makes less than a full revolution, so there are 4 possible cases (shown below).
The javelin motion is drawn by denoting the two ends A and B moving to new locations A' and B'.

Case 1. the javelin rotates CCW by a small angle

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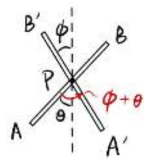
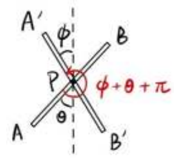


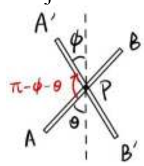
$\vec{\omega}_1 = \frac{\phi+\theta}{t_G}\hat{k} = 1.38 \text{ rad/s } \hat{k}$

Case 2. the javelin rotates CCW by a large angle



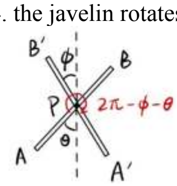
$\vec{\omega}_2 = \frac{\phi+\theta+\pi}{t_G}\hat{k} = 4.21 \text{ rad/s } \hat{k}$

Case 3. the javelin rotates CW by a small angle



$\vec{\omega}_3 = \frac{\pi-\phi-\theta}{t_G}(-\hat{k}) = -1.45 \text{ rad/s } \hat{k}$

Case 4. the javelin rotates CW by a large angle



$\vec{\omega}_4 = \frac{2\pi-\phi-\theta}{t_G}(-\hat{k}) = -4.28 \text{ rad/s } \hat{k}$

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