16.4.1 An object consists of a massless bar with two attached masses m_1 and m_2. The object is hinged at O.

a) What is the moment of inertia of the object about point O (I_{zz}^O)?

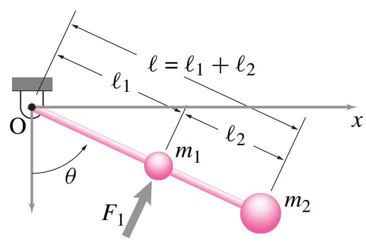
b) Given θ, $\dot{\theta}$, and $\ddot{\theta}$, what is $\vec{\boldsymbol{H}}_{/O}$, the angular momentum about point O?

c) Given θ, $\dot{\theta}$, and $\ddot{\theta}$, what is $\dot{\vec{\boldsymbol{H}}}_{/O}$, the rate of change of angular momentum about point O?

d) Given θ, $\dot{\theta}$, and $\ddot{\theta}$, what is T, the total kinetic energy?

e) Assume that you don't know θ, $\dot{\theta}$ or $\ddot{\theta}$ but you do know that F_1 is applied to the rod, perpendicular to the rod at m_1. What is $\ddot{\theta}$? (Neglect gravity.)

f) If F_1 were applied to m_2 instead of m_1, would $\ddot{\theta}$ be bigger or smaller?

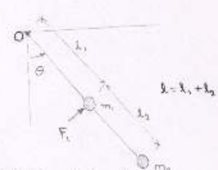


Problem 16.1

Answer:
(a) $I_{zz}^{cm} = m_1\ell_1^2 + m_2\ell^2$.
(b) $\vec{\boldsymbol{H}}_{/O} = \dot{\theta}(m_1\ell_1^2 + m_2\ell^2)\hat{\boldsymbol{k}}$.
(c) $\dot{\vec{\boldsymbol{H}}}_{/O} = \ddot{\theta}(m_1\ell_1^2 + m_2\ell^2)\hat{\boldsymbol{k}}$.
(d) $E_K = \frac{\dot{\theta}^2}{2}(m_1\ell_1^2 + m_2\ell^2)\hat{\boldsymbol{k}}$.
(e) $\ddot{\theta} = \frac{F_1\ell_1}{m_1\ell_1^2+m_2\ell^2}$
(f) $\ddot{\theta}$ will be bigger if F_1 is applied to m_2 instead of m_1.

a) Moment of inertia about 'O'

$I^O = m_1 l_1^2 + m_2(l_1+l_2)^2$



b) Angular momentum about 'O': $\underline{H}_{/O}$

$\underline{H}_{/O} = \int \underline{r}\times\underline{v}\,dm = \int \underline{r}\times(\underline{\omega}\times\underline{r})dm = \underline{\omega}\int r^2 dm = I_{zz}^O\,\omega\hat{\underline{k}}$

$\Rightarrow \underline{H}_{/O} = \dot{\theta} I_{zz}^O \hat{\underline{k}} = \dot{\theta}\hat{\underline{k}}\int r^2 dm = \dot{\theta}\,[m_1 l_1^2 + m_2(l_1+l_2)^2]\,\hat{\underline{k}}$

c) Rate of change of angular momentum, $\dot{\underline{H}}_{/O}$

$\dot{\underline{H}}_{/O} = \int \underline{r}\times\underline{a}\,dm = \int \underline{r}\times[\dot{\omega}\hat{\underline{k}}\times\underline{r} + \underbrace{\underline{\omega}\times(\underline{\omega}\times\underline{r})}_{-\omega^2\underline{r}}]\,dm$

$= \int \underline{r}\times(\dot{\omega}\hat{\underline{k}}\times\underline{r})\,dm = \dot{\underline{\omega}}\int r^2 dm = I_{zz}^O\,\dot{\omega}\hat{\underline{k}}$

$\Rightarrow \dot{\underline{H}}_{/O} = \ddot{\theta}\hat{\underline{k}} I_{zz}^O = \ddot{\theta}(m_1 l_1^2 + m_2 l^2)\hat{\underline{k}}$

d) Total Kinetic energy: T

$T = \frac{1}{2}\underline{\omega}\cdot[I^{cm}\underline{\omega}] + \frac{1}{2}m v_{cm}^2 = \frac{1}{2}m_1|\underline{v}_1|^2 + \frac{1}{2}m_2|\underline{v}_2|^2$

$\underline{v}_1 = \underline{\omega}\times\underline{r}_1 = l_1\dot{\theta}\hat{e}_\theta$ and $\underline{v}_2 = \underline{\omega}\times\underline{r}_2 = l\dot{\theta}\hat{e}_\theta$

$\Rightarrow T = \frac{1}{2}m_1 l_1^2\dot{\theta}^2 + \frac{1}{2}m_2 l^2\dot{\theta}^2 = \frac{1}{2}(m_1 l_1^2 + m_2 l^2)\dot{\theta}^2$

e) $\ddot{\theta} = ?$ Using Angular Momentum Balance about 'O'

$\sum \underline{M}_{/O} = \dot{\underline{H}}_{/O}$

$\{l_1\hat{e}_r \times F_1\hat{e}_\theta = \ddot{\theta}(m_1 l_1^2 + m_2 l^2)\hat{\underline{k}}\}$

$\{\}\cdot\hat{\underline{k}} \Rightarrow F_1 l_1 = \ddot{\theta}(m_1 l_1^2 + m_2 l^2)$

$\Rightarrow \ddot{\theta} = \frac{F_1 l_1}{m_1 l_1^2 + m_2 l^2}$

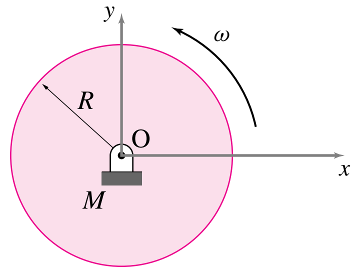
f) From the above expression, if F_1 is applied to m_2 instead of m_1, $\ddot{\theta}$ would be bigger.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.2 A uniform circular disc rotates at constant angular speed ω about the origin, which is also the center of the disc. It's radius is R. It's total mass is M.

a) What is the total force and moment required to hold it in place (use the origin as the reference point of angular momentum and torque).

b) What is the total kinetic energy of the disk?

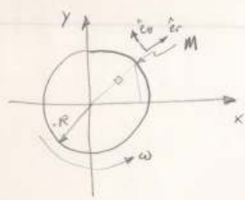


Problem 16.2

Answer:
(a) $\vec{F} = 0\,\mathrm{N}$.
(b) $E_K = \frac{1}{4} M R^2 \omega^2$.

uniform spinning disk

4.65



$\hat{\jmath}$ $\hat{\imath}$

Find reaction force $\underline{F}$
Find total kinetic Energy

$\sum F = \int_{\substack{\text{all}\\ \text{mass}}} \underline{a}\, dm$

FBD

LMB

$\underline{F} = \rho \int_0^{2\pi}\int_0^R -\omega^2 r\, \hat{e}_r\, r\, dr\, d\theta \qquad \rho = \frac{M}{\pi R^2}$

$\hat{e}_r$ not constant w.r.t. θ

$\underline{F} = \rho \int_0^{2\pi}\int_0^R -\omega^2 r(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})\, r\, dr\, d\theta$

$= \rho \int_0^R -\omega^2 r^2 dr \; \underbrace{\left[\sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath}\right]\Big|_0^{2\pi}}_{0}$

$\boxed{\underline{F} = 0}$

can also see the $\sum F = m_{tot}\, \underbrace{a_{cm}}_{0}$

$\therefore F = 0$ ✓

$K.E. = \frac{1}{2}\int_{\text{all mass}} (\underline{v}\cdot\underline{v})\, dm$

$= \frac{1}{2}\int_0^{2\pi}\int_0^R (\omega r\,\hat{e}_\theta)\cdot(\omega r\,\hat{e}_\theta)\; \rho\, r\, dr\, d\theta$

$= \frac{1}{2}\omega^2 \rho\, 2\pi\, \frac{R^4}{4} \qquad \rho = \frac{M}{\pi R^2}$

$= \frac{1}{2}\omega^2 \frac{M}{\pi R^2}\, 2\pi\, \frac{R^4}{4}$

$\boxed{K.E. = \frac{1}{4} M R^2 \omega^2}$

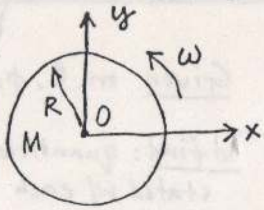
can also see that $K.E. = \frac{1}{2} I \omega^2 \qquad I = \frac{1}{2} M R^2$

$= \frac{1}{4} M R^2 \omega^2$ ✓

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

3

#4.65



Given: uniform disk rotates at const. speed. mass M, radius R

Find a) Total force & moment reqd. to hold it in place
b) total K.E. of disk

FBD: assume gravity acts along $-y$

a)

Left hand sides of momentum balance equations are:

$$\Sigma \underline{F} = \underline{R} - mg\,\hat{j}$$

$$\Sigma \underline{M}_{/O} = \underline{T}$$

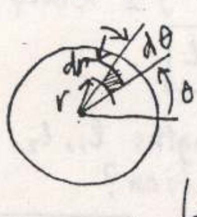
Right hand sides?

Now see example on page 258 of text.

$$\dot{\underline{L}} = \underline{0}, \quad \dot{\underline{H}}_{/O} = \underline{0}$$

$$\therefore \boxed{\underline{R} = mg\,\hat{j} \quad \text{and} \quad \underline{T} = \underline{0}}$$

b) total KE by integration $\int \frac{1}{2}|\underline{v}|^2 dm$



mass/unit area $= \dfrac{M}{\pi R^2}$

$$dm = r\,dr\,d\theta \cdot \frac{M}{\pi R^2}$$

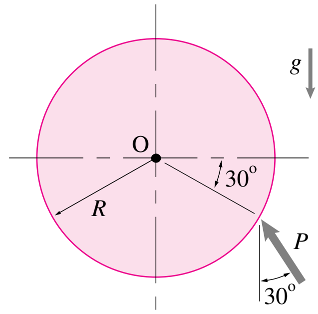
$$|\underline{v}|^2 = \omega^2 r^2$$

$$KE = \frac{1}{2}\int_0^R \int_0^{2\pi} \frac{M}{\pi R^2} \cdot \omega^2 r^2 \cdot r\,d\theta\,dr$$

$$= \frac{1}{\cancel{2}}\frac{\cancel{2}\pi M}{\pi R^2}\omega^2 \int_0^R r^3\,dr = \boxed{\frac{1}{4}MR^2\omega^2}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

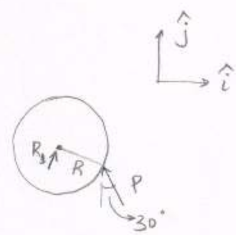
16.4.3 The hinged disk of mass m (uniformly distributed) is acted upon by a force P shown in the figure. Determine the *initial* angular acceleration and the reaction forces at the pin O.



Problem 16.3

Answer:
$\ddot{\theta}_0 = \frac{P}{mR}$ and $\vec{\boldsymbol{R}}_1 = P(\sin 30\hat{\boldsymbol{\imath}} - \cos 30\,\hat{\boldsymbol{\jmath}})$.

15.4.3



conservation of angular momentum

$$\sum \underline{M}_{/o} = \dot{\underline{H}}_{/o}$$

$$\sum \underline{M}_{/o} = (P\cos 30^\circ \cdot R\cos 30^\circ)\hat{k} - (P\sin 30^\circ \cdot R\sin 30^\circ)\hat{k}$$

$$\dot{\underline{H}}_{/o} = \ddot{\theta}\, I^{cm}\hat{k} = PR(\cos^2 30^\circ - \sin^2 30^\circ)\hat{k}$$

$$\ddot{\theta} = \frac{PR\cos 60^\circ}{I^{cm}} = \frac{PR\cos 60^\circ}{\frac{mR^2}{2}} = \frac{P}{mR}$$

$$\sum F = \dot{\underline{L}} = R_1 + P\cos 30\,\hat{j} - P\sin 30\,\hat{i} = \dot{\underline{L}}$$

$$\dot{\underline{L}} = m\underline{a}_{cm}\ ; \quad a_{cm} = \overset{0}{\underline{\omega}} \times (\underline{\omega} \times r_{cm/o}) + (\dot{\omega} \times \overset{0}{r_{cm/o}})$$

$$\therefore\ a_{cm} = 0$$

$$\therefore\ \underline{R}_1 = P\sin 30\,\hat{i} - P\cos 30\,\hat{j}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

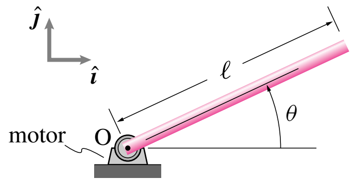
16.4.4 A motor turns a bar. A uniform bar of length ℓ and mass m is turned by a motor whose shaft is attached to the end of the bar at O. The angle that the bar makes (measured counter-clockwise) from the positive x axis is $\theta = 2\pi\ t^2/\mathrm{s}^2$. Neglect gravity.

a) Draw a free-body diagram of the bar.

b) Find the force acting on the bar from the motor and hinge at $t = 1\,\mathrm{s}$.

c) Find the torque applied to the bar from the motor at $t = 1\,\mathrm{s}$.

d) What is the power produced by the motor at $t = 1\,\mathrm{s}$?

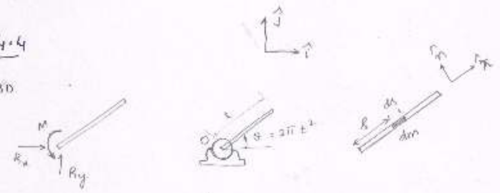


Problem 16.4

Answer:
(b) $\vec{\boldsymbol{R}}_{\mathrm{BM}} = -\frac{4\pi m\ell}{2}(4\pi\hat{\boldsymbol{\imath}} - \hat{\boldsymbol{\jmath}})$.
(c) $M = \frac{4\pi\ \mathrm{rad/s^2}}{3} m\ell^2$.
(d) $P = \frac{16\pi^2\ \mathrm{s}^{-3}}{3} m\ell^2$.

15·4·4

a) FBD



$\Sigma \underline{F} = \dot{\underline{L}} = m a_{cm} = \int a_{dm} \cdot dm$

$= \int \left[(\dot{\omega} \hat{k} \times s\hat{\lambda}) + (\underline{\omega} \times (\underline{\omega} \times s\hat{\lambda})) \right] \frac{m}{l} ds$

$= \frac{m}{l} \int_0^l (\dot{\omega} s \hat{n} - s\omega^2 \hat{\lambda}) ds$

$= \frac{m}{l} \dot{\omega} \frac{l^2}{2} \hat{n} - \frac{m}{l} \cdot \frac{l^2}{2} \omega^2 \hat{\lambda}$

$\frac{m}{l} = \frac{dm}{ds}$

$\hat{\lambda} = \cos\theta \hat{i} + \sin\theta \hat{j}$

$\hat{n} = -\sin\theta \hat{i} + \cos\theta \hat{j}$

$\theta = 2\pi t^2$

$\dot{\theta} = 4\pi t$

$\ddot{\theta} = 4\pi$

b) $R_x \hat{i} + R_y \hat{j} = \left(-\frac{1}{2} ml \dot{\omega} \sin\theta - \frac{1}{2} ml \omega^2 \cos\theta\right)\hat{i} + \left(\frac{1}{2} ml \dot{\omega} \cos\theta - \frac{1}{2} ml \omega^2 \sin\theta\right)\hat{j}$

$R_x \hat{i} + R_y \hat{j}$ at $t = 1$ is

$R_x \hat{i} + R_y \hat{j} = \left(-\frac{1}{2} ml (4\pi)^2\right)\hat{i} + \left(\frac{1}{2} ml\, 4\pi\right)\hat{j}$

$\Sigma M_o = \dot{H}_o$

c) $M\hat{k} = \int r_{/o} \times a\ dm$

$M\hat{k} = \int s\hat{\lambda} \times \left[\dot{\underline{\omega}} \times s\hat{\lambda} + \underline{\omega} \times (\underline{\omega} \times s\hat{\lambda})\right] \frac{m}{l} ds$

$M\hat{k} = \int_0^l \dot{\omega} s^2 \hat{k} \cdot \frac{m}{l} ds = m\dot{\omega} \frac{l^2}{3} \hat{k}$

$\Rightarrow M = \frac{4\pi}{3} m l^2$

d) $P = \omega \cdot M$ at $t = 1$

$P = 4\pi \hat{k} \cdot \frac{4\pi}{3} m l^2 \hat{k}$

$P = \frac{16\pi^2}{3} m l^2$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.5 A physical pendulum. A swinging stick is sometimes called a 'physical' pendulum. Take the 'body', the system of interest, to be the whole stick.

a) Draw a free-body diagram of the system.

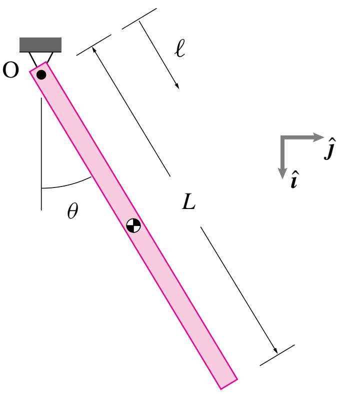
b) Write the equation of angular momentum balance for this system about point O.

c) Evaluate the left-hand-side as explicitly as possible in terms of the forces showing on your Free-Body Diagram.

d) Evaluate the right hand side as completely as possible. You may use the following facts:

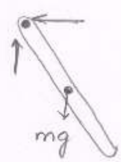
$$\begin{aligned}\vec{v} &= l\dot{\theta}\cos\theta\,\hat{\boldsymbol{\jmath}} + -l\dot{\theta}\sin\theta\,\hat{\boldsymbol{\imath}}\\ \vec{a} &= -l\dot{\theta}^2\left[\cos\theta\,\hat{\boldsymbol{\imath}} + \sin\theta\,\hat{\boldsymbol{\jmath}}\right]\\ &\quad + l\ddot{\theta}\left[\cos\theta\,\hat{\boldsymbol{\jmath}} - \sin\theta\,\hat{\boldsymbol{\imath}}\right]\end{aligned}$$

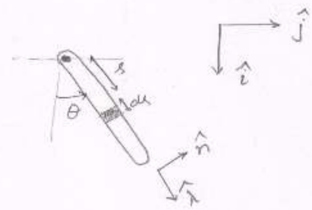
where ℓ is the distance along the pendulum from the top, θ is the angle by which the pendulum is displaced counter-clockwise from the vertically down position, $\hat{\boldsymbol{\imath}}$ is vertically down, and $\hat{\boldsymbol{\jmath}}$ is to the right. You will have to set up and evaluate an integral.



Problem 16.5

Answer:
(d) $\sum \vec{\boldsymbol{M}}_O = -\frac{\ell}{2}mg\sin\theta\hat{\boldsymbol{k}} = \dot{\vec{\boldsymbol{H}}}_{/O} = \frac{m\ell^2}{3}\ddot{\theta}\hat{\boldsymbol{k}}$.





$\sum \underline{M}_{/o} = \dot{\underline{H}}_{/o}$

$\hat{\lambda} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\hat{n} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

LHS $= -\frac{l}{2}\, mg \sin \hat{k}$

RHS $= \dot{\underline{H}}_{/o} = \int (\underline{r}_{/o} \times \underline{a})\, dm$

$\underline{a} = \dot{\omega}\hat{k} \times s\hat{\lambda} + \underline{\omega} \times (\underline{\omega} \times s\hat{\lambda})$

$\therefore\ \dot{\underline{H}}_{/o} = \int s^2 \ddot{\theta}\hat{k}\, \frac{m}{l}\, ds = \frac{m}{l}\ddot{\theta}\,\frac{l^3}{3}\hat{k}$

$\therefore\ \frac{m}{l}\cdot\ddot{\theta}\,\frac{l^3}{3} = -\frac{l}{2}\, mg \sin\theta$

$\ddot{\theta} = -\frac{3g}{2l}\sin\theta$

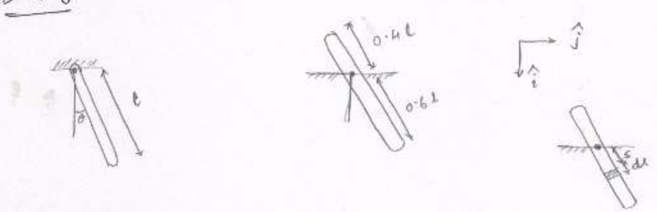
16.4.6 A uniform one meter bar is hung from a hinge that is at the end. It is allowed to swing freely. $g = 10\,\mathrm{m/s^2}$.

a) What is the period of small oscillations for this pendulum?

b) Suppose the rod is hung 0.4 m from one end. What is the period of small oscillations for this pendulum? Can you explain why it is longer or shorter than when it is hung by its end?

Answer:
(a) $T = 1.62\,\mathrm{s}$.
(b) The period of oscillations is longer if the rod is hung at 0.4 m because the torque due to weight of the portion above the hinge negates the torque due to the portion below the hinge. One way to see it is that if the bar is hinged at mid, net torque about the hinge becomes zero leading to infinitely large period of oscillation.

15·4·6



From previous problem

$$\ddot\theta + \frac{3g}{2l}\sin\theta = 0$$

for small θ

$$\omega^2 = \frac{3g}{2l}$$

$$\frac{2\pi}{T} = \sqrt{\frac{3g}{2l}}$$

$$T = 2\pi\sqrt{\frac{2l}{3g}}$$

$$T = 2\pi\sqrt{\frac{2}{30}}$$

T = 1.62 seconds

$$\sum \vec M_{/o} = 0.3l\,\hat\lambda \times 0.6 \times mg\,\hat\imath + (-0.2\hat\lambda)\times 0.4\,mg\,\hat\imath$$

$$= (-0.3l\times 0.6mg\sin\theta + 0.2\times 0.4\,l\,mg\sin\theta)\,\hat k$$

$$= -0.1\,mgl\sin\theta\,\hat k$$

$$\dot{\underline{H}}_{/o} = \int_0^{0.6} \vec r_{/o}\times\underline{a}\,dm + \int_0^{0.4}\vec r_{/o}\times\underline{a}\,dm$$

$$= \int_0^{0.6}\ddot\theta s^2\hat k\,\frac{m}{l}ds + \int_0^{0.4}\ddot\theta s^2\hat k\,\frac{m}{l}ds$$

$$= \ddot\theta\,\frac{m}{l}\cdot\frac{s^3}{3}\Big|_0^{0.6} + \ddot\theta\,\frac{m}{l}\cdot\frac{s^3}{3}\Big|_0^{0.4}$$

$$= \ddot\theta\,\frac{m}{3l}\left[0.6^3 + 0.4^3\right]$$

From $\sum M_{/o} = \dot{\underline{H}}_{/o}$

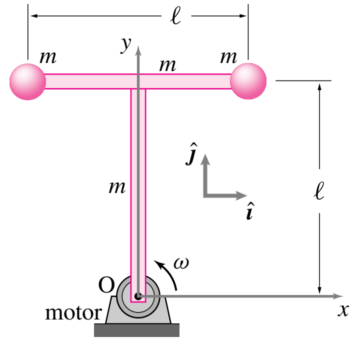
$$\ddot\theta(0.0933)m = -0.1\,mgl\sin\theta$$

$$\therefore\ \omega^2 = \frac{0.1g}{0.0933} \Rightarrow T = 1.9195$$

The period is longer partly due to moment caused by the portion of the bar above the hinge

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.7 Motor turns a dumbbell. Two uniform bars of length ℓ and mass m are welded at right angles. At the ends of the horizontal bar are two more masses m. The bottom end of the vertical rod is attached to a hinge at O where a motor keeps the structure rotating at constant rate ω (counter-clockwise). What is the net force and moment that the motor and hinge cause on the structure at the instant shown?



Problem 16.7

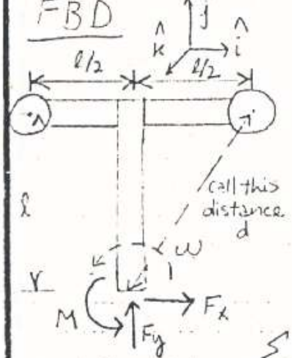
Answer:
$F_x = 0, F_y = \frac{7\ell m\omega^2}{2}, \vec{\boldsymbol{M}}_O = 0\hat{\boldsymbol{k}}$.

4.66 2 rods, 2 point masses, all of mass m. Apparatus rotating at $\underset{\sim}{\omega} = \omega \hat{k}$.
Find reaction forces & moments at point O.

CONTINUED

4.66 continued p. 5

FBD



I'll ignore gravity.

LMB:

$\sum \underset{\sim}{F} = \sum m_i \underset{\sim}{a}_i = m_{tot} \underset{\sim}{a}_{cm}$

I'll use $\sum m_i \underset{\sim}{a}_i$; you can find the center of mass and use $m_{tot} \underset{\sim}{a}_{cm}$ as well.

$\sum \underset{\sim}{F} = F_x \hat{i} + F_y \hat{j}$;

$\sum m_i \underset{\sim}{a}_i = m\omega^2 (\sum \underset{\sim}{R}_i)$

where I have used the fact that $a = \omega^2 R$.

$\therefore \sum m_i \underset{\sim}{a}_i = m\omega^2 \left(-\frac{\ell}{2}\hat{i} - \ell\hat{i} + d\left(\frac{\frac{\ell}{2}\hat{i} - \ell\hat{j}}{2}\right) + d\left(\frac{-\frac{\ell}{2}\hat{i} - \ell\hat{j}}{2}\right)\right)$

m, ω are the same for each mass.

$\therefore \sum m_i \underset{\sim}{a}_i = -\frac{7\ell m\omega^2}{2}\hat{j} = F_x\hat{i} + F_y\hat{j}$

$\therefore \boxed{F_x = 0, \quad F_y = -\frac{7\ell m\omega^2}{2}}$

Any applied moment? AMB tells us that

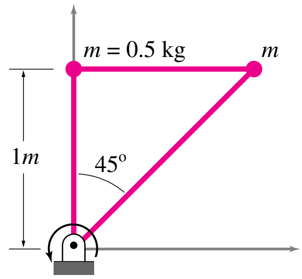
$\sum \underset{\sim}{M}_{/O} = M\hat{k} = \dot{\underset{\sim}{H}}_{/O} = \sum \underset{\sim}{r}_{/O} \times m\underset{\sim}{a} = \underset{\sim}{0}$

b/c constant-rate, 2-d circular motion. $\underset{\sim}{r}_O$ and $\underset{\sim}{a}$ point in the same direction for every mass, so $\underset{\sim}{r}_{/O} \times \underset{\sim}{a} = 0$.

$\Rightarrow \boxed{M = 0}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.8 The structure shown in the figure consists of two point masses connected by three rigid, massless rods such that the whole structure behaves like a rigid body. The structure rotates counterclockwise at a constant rate of 60 rpm. At the instant shown, find the force in each rod.



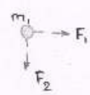
Problem 16.8

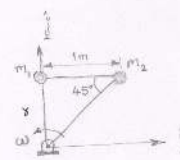
Answer:
$F_1 = 0\,\text{N}$, $F_2 = 2\pi^2\,\text{N}$, and $F_3 = 2\sqrt{2}\pi^2\,\text{N}$.

15.4.8 : Force in each rod:

Given $m = 0.5$ kg and $\omega = 60$ rpm

Applying L.M.B on m_1 : $\Sigma F = m_1 a_1$





$$a_1 = \cancel{\dot{\omega}}^{0} \times r + \omega \times (\omega \times r)$$

$$a_1 = \omega \hat{k} \times (\omega \hat{k} \times r \hat{j})$$

$$a_1 = \omega \hat{k} \times (-r\omega \hat{i})$$

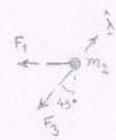
$$a_1 = -r\omega^2 \hat{j}$$

$$\therefore \quad \{F_1 \hat{i} - F_2 \hat{j} = -m r \omega^2 \hat{j}\}$$

$\{\}\cdot\hat{i} \Rightarrow F_1 = 0$ N

$\{\}\cdot\hat{j} \Rightarrow F_2 = m r \omega^2 = 0.5 \times 1 \times \left[60 \times \frac{2\pi}{60}\right]^2 = 2\pi^2$ N

Applying L.M.B on m_2 : $\Sigma F = m_2 a_2$



$$a_2 = \cancel{\dot{\omega}}^{0} \times r + \omega \times (\omega \times r)$$

$$a_2 = \omega \hat{k} \times (\omega \hat{k} \times \sqrt{2} r \hat{\lambda})$$

$$a_2 = -\sqrt{2} r \omega^2 \hat{\lambda}$$

$$\hat{\lambda} = \cos 45\, \hat{i} + \sin 45\, \hat{j}$$

$$\therefore \quad \{-F_1 \hat{i} - F_3 \hat{\lambda} = m_2 (-\sqrt{2} r \omega^2) \hat{\lambda}\}$$

$\{\}\cdot\hat{j} \Rightarrow -F_3 \sin 45^\circ = -m_2 \sqrt{2} r \omega^2 \sin 45$

$\Rightarrow F_3 = 0.5 \cdot \sqrt{2} \cdot 1 \cdot \left(60 \times \frac{2\pi}{60}\right)^2$

$\Rightarrow F_3 = 2\sqrt{2}\pi^2$ N

$\{\}\cdot\hat{i} \Rightarrow -F_1 - F_3 \cos 45 = -m_2 \sqrt{2} r \omega^2 \cos 45$

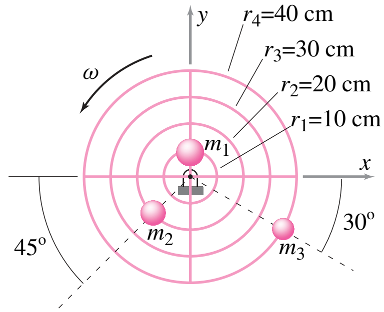
$\Rightarrow F_1 = 0$ N

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.9 Balancing a system of rotating particles. A wire frame structure is made of four concentric loops of massless and rigid wires, connected to each other by four rigid wires presently coincident with the x and y axes. Three masses, $m_1 = 200\,\text{grams}$, $m_2 = 150\,\text{grams}$ and $m_3 = 100\,\text{grams}$, are glued to the structure as shown in the figure. The structure rotates counterclockwise at a constant rate $\dot{\theta} = 5\,\text{rad/s}$. There is no gravity.

a) Find the net force exerted by the structure on the support at the instant shown.

b) You are to put a mass m at an appropriate location on the third loop so that the net force on the support is zero. Find the appropriate mass and the location on the loop.



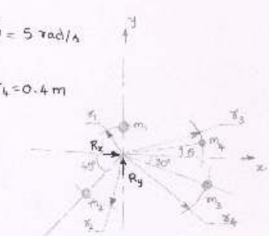
Problem 16.9

Answer:
(a) (a) $\vec{F} = 0.33\,\text{N}\hat{\imath} - 0.54\,\text{N}\,\hat{\jmath}$.
(b) The net force on the support becomes zero if we put $m = 0.84\,\text{kg}$ on $0.3\,\text{m}$ radius loop at $\theta = 302.33°$.

Given $m_1 = 0.2\,\text{Kg}$; $m_2 = 0.15\,\text{Kg}$; $m_3 = 0.1\,\text{Kg}$; $\dot{\theta} = 5\,\text{rad/s}$

$R_x = ?$ $R_y = ?$ $r_1 = 0.1\,\text{m}$, $r_2 = 0.2\,\text{m}$; $r_3 = 0.3\,\text{m}$; $r_4 = 0.4\,\text{m}$

Applying L.M.B on the structure:



$$\Sigma \underline{F} = m\underline{a}$$

$$\Rightarrow R_x\hat{i} + R_y\hat{j} = m_1\underline{a}_1 + m_2\underline{a}_2 + m_3\underline{a}_3$$

$$\underline{a}_1 = (\dot{\underline{\omega}} \times \underline{r}_1) + (\underline{\omega} \times (\underline{\omega} \times \underline{r}_1))$$

$$\Rightarrow \underline{a}_1 = -\dot{\theta}^2 \underline{r}_1 \;;\; \underline{a}_2 = -\dot{\theta}^2 \underline{r}_2 \;;\; \underline{a}_3 = -\dot{\theta}^2 \underline{r}_4$$

$$\Rightarrow \{R_x\hat{i} + R_y\hat{j} = m_1(-\dot{\theta}^2\, 0.1\text{m}\,\hat{j}) + m_2(\dot{\theta}^2\, 0.2\text{m})(\cos 45\,\hat{i} + \sin 45\,\hat{j}) + m_3(\dot{\theta}^2\, 0.4\text{m})(-\cos 30\,\hat{i} + \sin 30\,\hat{j})\}$$

$\{\}\cdot\hat{i} \Rightarrow R_x = -0.336\,\text{N}$

$\{\}\cdot\hat{j} \Rightarrow R_y = 0.530\,\text{N}$

$\therefore$ Net force exerted by the structure on support, $\underline{R} = 0.336\hat{i} - 0.530\hat{j}$ (N)

b) Find m_4 on r_3 at θ to make $\underline{R} = \underline{0}$

Applying L.M.B on the structure:

$$\Sigma \underline{F} = m\underline{a}$$

$$\Rightarrow \underline{0} = 0\hat{i} + 0\hat{j} = -(m_1\underline{a}_1 + m_2\underline{a}_2 + m_3\underline{a}_3) + m_4\underline{a}_4$$

(or) $$\underline{0} = -(R_x\hat{i} + R_y\hat{j}) + m_4\underline{a}_4$$

$$\Rightarrow \{+R_x\hat{i} + R_y\hat{j} = m_4(-\dot{\theta}^2 r_3[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}])\}$$

$\{\}\cdot\hat{i} \Rightarrow -0.336\,\text{N} = -m_4 r_3 \dot{\theta}^2 \cos\theta$ N

$\{\}\cdot\hat{j} \Rightarrow 0.530\,\text{N} = -m_4 r_3 \dot{\theta}^2 \sin\theta$ N

$\Rightarrow \tan\theta = -1.579 \Rightarrow \theta = -57.67°$ and $m_4 = 0.0837\,\text{Kg}$

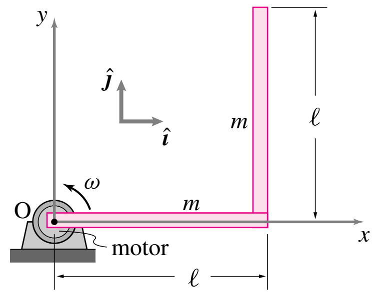
$\therefore$ In order to make net force on the support zero, a mass of $0.0837\,\text{Kg}$ is to be placed on $0.3\,\text{m}$ loop at $302.33°$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.10 Motor turns a bent bar. Two uniform bars of length ℓ and uniform mass m are welded at right angles. One end is attached to a hinge at O where a motor keeps the structure rotating at a constant rate ω (counterclockwise). What is the net force and moment that the motor and hinge cause on the structure at the instant shown.

a) neglecting gravity

b) including gravity.

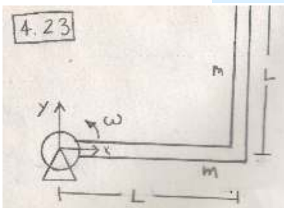


Problem 16.10: A bent bar is rotated by a motor.

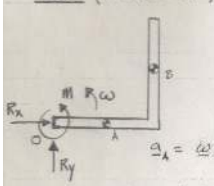
Answer:
(a) Net force: $\vec{F}_{net} = -(\frac{3m\omega^2 L}{2})\hat{\imath} - (\frac{m\omega^2 L}{2})\hat{\jmath}$, Net moment: $\vec{M}_{net} = \vec{0}$.
(b) Net force: $\vec{F}_{net} = -(\frac{3m\omega^2 L}{2})\hat{\imath} + (2mg - \frac{m\omega^2 L}{2})\hat{\jmath}$, Net moment: $\vec{M}_{net} = \frac{3mgL}{2}\hat{k}$.

4.23

Motor Turns a Bent Bar rotating at constant rate ω



FBD (NO GRAVITY) $r_A = \frac{L}{2}\hat{\imath}$ $r_B = L\hat{\imath} + \frac{L}{2}\hat{\jmath}$ $\omega = \omega\hat{k}$



$\Sigma F = ma$

$R_x\hat{\imath} + R_y\hat{\jmath} = m(\omega \times (\omega \times r_A)) + m(\omega \times (\omega \times r_B))$

$a_A = \omega \times (\omega \times r_A) = \omega \times (\frac{\omega L}{2}\hat{\jmath}) = -\frac{\omega^2 L}{2}\hat{\imath}$

$a_B = \omega \times (\omega \times r_B) = \omega \times (\omega L\hat{\jmath} - \frac{\omega L}{2}\hat{\imath}) = -\omega^2 L\hat{\imath} - \frac{\omega^2 L}{2}\hat{\jmath}$

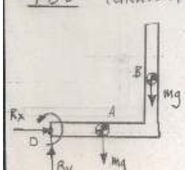
$\boxed{R_x = -\frac{3m\omega^2 L}{2}\hat{\imath}}$ $\boxed{R_y = -\frac{m\omega^2 L}{2}\hat{\jmath}}$

$\Sigma M_{/O} = \dot{H}_{/O}$ $M\hat{k} = (r_A \times m a_A) + (r_B \times m a_B)$

$$M\hat{k} = \bar{0} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ L & \frac{L}{2} & 0 \\ -\omega^2 L & -\frac{\omega^2 L}{2} & 0 \end{vmatrix} = \bar{0} + \bar{0}$$

$\boxed{M = 0}$

FBD (GRAVITY) $\Sigma F = ma$



$R_x\hat{\imath} + R_y\hat{\jmath} - 2mg\hat{\jmath} = m(\omega \times (\omega \times r_A)) + m(\omega \times (\omega \times r_B))$

$\boxed{R_x = -\frac{3m\omega^2 L}{2}\hat{\imath}}$ $\boxed{R_y = (2mg - \frac{\omega^2 m L}{2})\hat{\jmath}}$

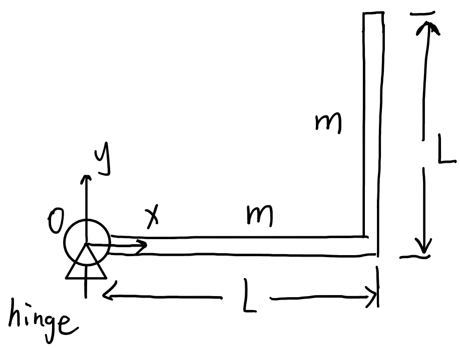
$\Sigma M_{/O} = \dot{H}_{/O}$ $M\hat{k} - \frac{mgL}{2}\hat{k} - mgL\hat{k} = (r_A \times m a_A) + (r_B \times m a_B)$

$\boxed{M = \frac{3mgL}{2}\hat{k}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

15.4.10

Sunday, March 31, 2019 5:59 PM



Moter Turns a Bent Bar rotating at constant rate ω

$\vec{r}_A = \frac{L}{2}\hat{i}$

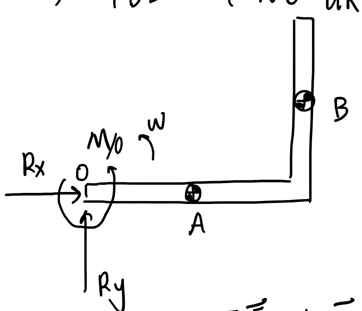
$\vec{r}_B = L\hat{i} + \frac{L}{2}\hat{j}$

$\vec{w} = w\hat{k}$

Net force at O = ?

Moment that the motor and hinge causes at O = ?

a) FBD (NO GRAVITY)



$\Sigma \vec{F} = m\vec{a}$

$$R_x\hat{i} + R_y\hat{j} = m(\underbrace{\vec{w}\times(\vec{w}\times\vec{r}_A)}_{\vec{a}_A}) + m(\underbrace{\vec{w}\times(\vec{w}\times\vec{r}_B)}_{\vec{a}_B})$$

$\vec{a}_A = \vec{w}\times(\vec{w}\times\vec{r}_A) = -w^2\vec{r}_A \qquad \vec{a}_A = -\frac{w^2L}{2}\hat{i}$

$\vec{a}_B = \vec{w}\times(\vec{w}\times\vec{r}_B) = -W^2\vec{r}_B \qquad \vec{a}_B = -w^2L\hat{i} - \frac{w^2L}{2}\hat{j}$

$\boxed{\vec{R}_x = -\frac{3mW^2L}{2}\hat{i}} \qquad \boxed{\vec{R}_y = -\frac{mw^2L}{2}\hat{j}}$

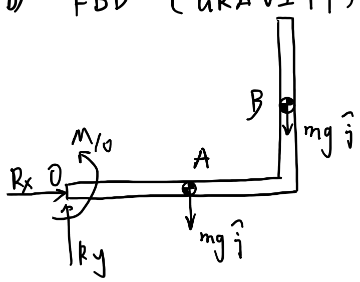
$$\Sigma \vec{M}_{/O} = \dot{\vec{H}}_{/O}$$

$$M_O\hat{k} = (\vec{r}_A \times m\vec{a}_A) + (\vec{r}_B \times m\vec{a}_B)$$

$$= \vec{r}_A \times m(-\omega^2\vec{r}_A) + \vec{r}_B \times m(-\omega^2\vec{r}_B) = \vec{0}$$

$$\boxed{M_O = \vec{0}}$$

b) FBD (GRAVITY)



Net force at O = ?
Moment that the motor and hinge causes at O = ?

$$\Sigma \vec{F} = m\vec{a}$$

$$R_x\hat{i} + R_y\hat{j} - 2mg\hat{j} = m\vec{a}_A + m\vec{a}_B$$

$$\boxed{R_x = -\frac{3m\omega^2 L}{2}\hat{i}} \qquad \boxed{R_y = \left(2mg - \frac{\omega^2 m L}{2}\right)\hat{j}}$$

$$\Sigma \vec{M}_{/O} = \dot{\vec{H}}_{/O}$$

$$M_O\hat{k} - \frac{mgL}{2}\hat{k} - mgL\hat{k} = (\vec{r}_A \times m\vec{a}_A) + (\vec{r}_B \times m\vec{a}_B)$$

$$= \vec{r}_A \times m(-\omega^2\vec{r}_A) + \vec{r}_B \times m(-\omega^2\vec{r}_B) = \mathbf{0}$$

$$\Rightarrow \quad \boxed{\vec{M} = \frac{3}{2}mgL\hat{k}}$$

15.2.35

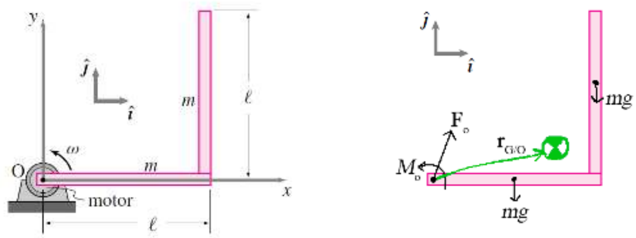


Figure 15.2.1: (a) A bent bar is rotated by a motor. (b) Free body diagram.

Two uniform bars of length ℓ and uniform mass m are welded at right angles. One end is attached to a hinge at O where a motor keeps the structure rotating at a constant rate ω (counterclockwise); see Fig. 15.2.1(a). What is the net force and moment that the motor and hinge cause on the structure at the instant shown, when

(a) neglecting gravity, or

(b) including gravity?

Net force

The free body diagram of the system is shown in Fig. 15.2.1(b). The linear momentum balance equation for the system is

$$\vec{\mathbf{F}}_O - 2mg\,\hat{\mathbf{j}} = m\vec{\mathbf{a}}_{1/O} + m\vec{\mathbf{a}}_{2/O}, \tag{15.2.1}$$

where the acceleration of each mass (circular motion) is

$$\vec{\mathbf{a}}_{i/O} = -\omega^2\vec{\mathbf{r}}_{i/O}. \tag{15.2.2}$$

We have $\vec{\mathbf{r}}_{1/O} = (\ell/2)\hat{\mathbf{i}}$ and $\vec{\mathbf{r}}_{2/O} = \ell\hat{\mathbf{i}} + (\ell/2)\hat{\mathbf{j}}$, so after simplifying

$$m\vec{\mathbf{a}}_{1/O} + m\vec{\mathbf{a}}_{2/O} = -\frac{m\ell\omega^2}{2}\left(3\hat{\mathbf{i}} + \hat{\mathbf{j}}\right), \tag{15.2.3}$$

we retrieve the net force at the hinge as

$$\vec{\mathbf{F}}_O = 2mg\hat{\mathbf{j}} - \frac{m\ell\omega^2}{2}\left(3\hat{\mathbf{i}} + \hat{\mathbf{j}}\right). \tag{15.2.4}$$

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Note that this force is nonzero even if we neglect gravity, and that the direction of it will change in general as the hinge rotates the bars. The magnitude of the net force changes as the bars rotate; however, if we neglect gravity it does not change and remains constant at

$$F_{\mathrm{O}}\big|_{g=0} = \frac{m\ell\omega^2\sqrt{10}}{2}. \tag{15.2.5}$$

This problem can also be done using the center of mass.

Net moment

The gravitational forces cause a net moment of $-3mg\ell/2$ about O. The angular momentum balance equation about O is then

$$M_{\mathrm{O}} - \frac{3mg\ell}{2} = \dot{H}_{/\mathrm{O}}. \tag{15.2.6}$$

The rate of change of the angular momentum $\dot{H}_{/\mathrm{O}}$ is zero since ω is constant, so the net moment from the motor is then

$$M_{\mathrm{O}} = \frac{3mg\ell}{2}. \tag{15.2.7}$$

Note that the moment is zero if we neglect gravity.

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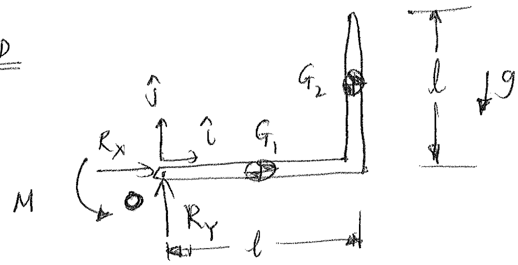
13.112

FBD



Given: $m, \ell, g, \omega, \alpha = 0$

Find: R_x, R_Y, M

We will solve the general case involving gravity and than reduce to no gravity case by putting gravity $=0$.

AMB$/_0$ $\quad \vec{M}_{/0} = \dot{\vec{H}}_{/0}$

$$\vec{r}_{G_1/0} \times mg(-\hat{j}) + \vec{r}_{G_2/0} \times mg(-\hat{j}) + M\hat{k} = \vec{r}_{G_1/0} \times m\vec{a}_{G_1} + \vec{r}_{G_2/0} \times m\vec{a}_{G_2}$$

$$\therefore \quad \frac{\ell}{2}\hat{i} \times mg(-\hat{j}) + \left(\ell\hat{i} + \frac{\ell}{2}\hat{j}\right) \times mg(-\hat{j}) + M\hat{k} = \vec{0}$$

why?

$\left\{ \text{Because } \begin{array}{l} a_{G_1} = -\omega^2 r_{G_1/0} \\ a_{G_2} = -\omega^2 r_{G_2/0} \end{array} \right\}$

$\{AMB_{/0}\} \cdot k$

$$-\frac{3}{2}mg\ell + M = 0$$

$$\Rightarrow \quad M = \frac{3}{2}mg\ell \qquad \text{— (I)}$$

LMB

$$R_x\hat{\imath} + R_y\hat{\jmath} - mg\hat{\jmath} - mg\hat{\jmath} = m\vec{a}_{G_1} + m\vec{a}_{G_2}$$

But $\vec{a}_{G_1} = \vec{a}_{G_1/0} = -\omega^2 r_{G_1/0} = -\omega^2\left\{\frac{l}{2}\hat{\imath}\right\}$

$$\vec{a}_{G_2} = \vec{a}_{G_2/0} = -\omega^2 r_{G_2/0} = -\omega^2\left\{l\hat{\imath} + \frac{l}{2}\hat{\jmath}\right\}$$

Thus

$$R_x\hat{\imath} + \{R_y - 2mg\}\hat{\jmath} = m\left\{-\frac{3\omega^2 l}{2}\hat{\imath} - \frac{\omega^2 l}{2}\hat{\jmath}\right\}$$

$\{LMB\}\cdot\hat{\imath}$

$$R_x = -\frac{3m\omega^2 l}{2} \qquad \text{— II}$$

$\{LMB\}\cdot\hat{\jmath}$

$$R_y = 2mg - \frac{m\omega^2 l}{2} \qquad \text{— III}$$

a) Neglect gravity

Put $g = 0$ in I, II, III

$$\boxed{\begin{aligned} M &= 0 \\ R_x &= -\frac{3m\omega^2 l}{2} \\ R_y &= -\frac{m\omega^2 l}{2} \end{aligned}}$$

b) Including gravity

From I, II, III

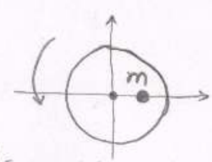
$$\boxed{\begin{aligned} M &= \frac{3}{2}mgl \\ R_x &= -3m\omega^2 l/2 \\ R_y &= 2mg - \frac{3m\omega^2 l}{2} \end{aligned}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.11 A uniform disk of mass M and radius R rotates about a hinge O in the xy-plane. A point mass m is fixed to the disk at a distance $R/2$ from the hinge. A motor at the hinge drives the disk/point mass assembly with constant angular acceleration α. What torque at the hinge does the motor supply to the system?

Answer:
$T = \frac{\alpha R^2}{4}(2M + m)$.

15·4·11



α = const

$$\sum \underline{M}_{/c} = \dot{\underline{H}}_{/o}$$

$$T\hat{K} = \dot{\underline{H}}_{/o} = \ddot{\theta}\, \underline{I}^{o}\, \hat{k}$$

$$\left\{ T\hat{k} = \alpha \left[\frac{MR^2}{2} + m\left(\frac{R}{2}\right)^2 \right] \hat{K} \right\}$$

$$\{\ \} \cdot \hat{K}$$

$\Rightarrow$

$$T = \frac{\alpha R^2}{4}(2M + m)$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.12 A rigid rod of length ℓ and total mass m is held fixed at one end and whirled around in circular motion at a constant rate ω in the horizontal plane. Ignore gravity.

a) Find the tension in the rod as a function of r, the radial distance from the center of rotation to any desired location on the rod.

b) Where does the maximum tension occur in the rod? .

c) At what distance from the center of rotation does the tension drop to half its maximum value? .

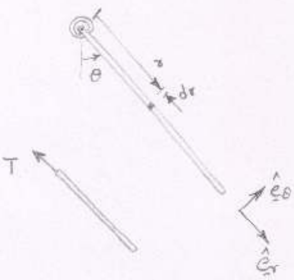
Answer:
(a) $T(r) = \frac{m\omega^2}{2L}(L^2 - r^2)$
(b) at $r = 0$; i.e., at the center of rotation
(c) $r = L/\sqrt{2}$

a) Find the tension in the rod.

Given $\dot{\theta} = \text{Constant} = \omega$

Let the rod length be 'L'.

Applying L.M.B on the rod at r



$\frac{m}{L} = \frac{dm}{dr}$

$$\sum \underline{F} = m\underline{a}$$

$$-T\hat{e}_r = \int \underline{\omega} \times (\underline{\omega} \times r\hat{e}_r)\, dm$$

$$= \int_r^L \omega\hat{k} \times (\omega\hat{k} \times r\hat{e}_r)\, \frac{m}{L}\, dr$$

$$= \frac{m}{L} \int_r^L -\omega^2 r\, dr\, \hat{e}_r$$

$$= -\frac{m\omega^2}{L} \cdot \frac{r^2}{2}\Big|_r^L \hat{e}_r$$

$$\left\{-T\hat{e}_r = -\frac{m\omega^2}{2L}(L^2 - r^2)\hat{e}_r\right\}$$

$\{\}\cdot\hat{e}_r \Rightarrow$ Tension in the rod, $T = \frac{m\omega^2}{2L}(L^2 - r^2)$ //

b) Tension keeps increasing as r decreases. Hence, maximum tension occurs at $r = 0$ and is, $T_{max} = \frac{mL\omega^2}{2}$ //

c) Tension drops to $\frac{T_{max}}{2}$ at $r = ?$

$$\frac{\cancel{m}L\omega^2}{2\cdot 2} = \frac{\cancel{m}\omega^2}{2L}(L^2 - r^2)$$

$$\Rightarrow \frac{L^2}{2} = L^2 - r^2$$

$$\Rightarrow r^2 = \frac{L^2}{2}$$

$$(\text{or})\quad r = \frac{L}{\sqrt{2}}$$ //

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

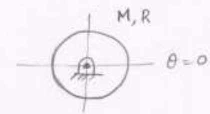
16.4.13 A thin uniform circular disc of mass M and radius R rotates in the xy plane about its center of mass point O. Driven by a motor, it has rate of change of angular speed proportional to angular position, $\alpha = k\theta^{5/2}$. The disc starts from rest at $\theta = 0$.

a) What is the rate of change of angular momentum about the origin at $\theta = \frac{\pi}{3}$ rad?

b) What is the torque of the motor at $\theta = \frac{\pi}{3}$ rad?

c) What is the total kinetic energy of the disk at $\theta = \frac{\pi}{3}$ rad?

Answer:
(a) $\dot{\vec{H}}_{/O} = \left(k\frac{\pi}{3}\right)^{2.5}\left(\frac{MR^2}{2}\right)\hat{\boldsymbol{k}}$.
(b) $T = k\left(\frac{\pi}{3}\right)^{2.5}\left(\frac{MR^2}{2}\right)$.
(c) $E_K = k^2\left(\frac{\pi}{3}\right)^{3.5}\left(\frac{MR^2}{2}\right)$.



$\ddot{\theta} = K\theta^{5/2}$

$\sum M_{/o} = \dot{\underline{H}}_{/o} \quad , \theta = \frac{\pi}{3}$

a) $\dot{\underline{H}}_{/o} = K\theta^{5/2}\,\frac{MR^2}{2}\,\hat{k}$

$\Rightarrow \dot{\underline{H}}_{/o} = K\left(\frac{\pi}{3}\right)^{5/2}\frac{MR^2}{2}$

b) $T = K\left(\frac{\pi}{3}\right)^{5/2}\frac{MR^2}{2}$

c) $E_K = \frac{1}{2}I\omega^2$

we know that

$\ddot{\theta} = K\theta^{5/2}$

$\ddot{\theta} = K\left(\frac{\pi}{3}\right)^{5/2}$

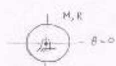
$d\dot{\theta} = K\left(\frac{\pi}{3}\right)^{5/2}dt$

$\dot{\theta} = K\left(\frac{\pi}{3}\right)^{5/2}t$

$\theta = \frac{K}{2}\left(\frac{\pi}{3}\right)^{5/2}t^2$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

15.4.13



$\ddot{\theta} = K\theta^{5/2}$

$\sum M_{/0} = \dot{\underline{H}}_{/0} \quad , \theta = \frac{\pi}{3}$

a) $\dot{\underline{H}}_{/0} = K\theta^{5/2}\frac{MR^2}{2}\hat{k}$

$\Rightarrow \dot{\underline{H}}_{/0} = K\left(\frac{\pi}{3}\right)^{5/2}\frac{MR^2}{2}$

b) $T = K\left(\frac{\pi}{3}\right)^{5/2}\frac{MR^2}{2}$

c) $E_K = \frac{1}{2}I\omega^2$

we know that

$\ddot{\theta} = K\theta^{5/2}$

$\ddot{\theta} = K\left(\frac{\pi}{3}\right)^{5/2}$

$d\dot{\theta} = K\left(\frac{\pi}{3}\right)^{5/2}dt$

$\dot{\theta} = K\left(\frac{\pi}{3}\right)^{5/2}t$

$\theta = \frac{K}{2}\left(\frac{\pi}{3}\right)^{5/2}t^2$

$\frac{\pi}{3} = \frac{K}{2}\left(\frac{\pi}{3}\right)^{5/2}t^2$

$t = \left[\frac{2}{K}\left(\frac{\pi}{3}\right)^{-3/2}\right]^{1/2}$

$\dot{\theta} = \sqrt{2}K\left(\frac{\pi}{3}\right)^{7/4}$

$\therefore K.E = \frac{1}{2}I\omega^2$

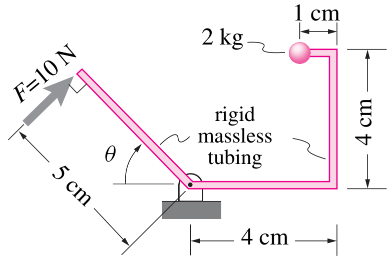
$= \frac{1}{2}I\left[\sqrt{2}K\left(\frac{\pi}{3}\right)^{1/4}\right]^2$

$= \frac{1}{2}I \times 2K^2\left(\frac{\pi}{3}\right)^{7/2}$

$= \frac{MR^2}{2}K^2\left(\frac{\pi}{3}\right)^{7/2}$

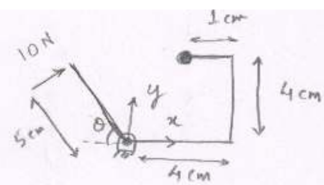
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.14 Neglecting gravity, calculate $\alpha = \dot{\omega} = \ddot{\theta}$ at the instant shown for the system in the figure.



Problem 16.14

Answer:
$\ddot{\theta} = 100$ rev/sec.



$\sum \underline{F} = m\underline{a}$

$\sum F = 10\cos\theta\,\hat{j} + 10\sin\theta\,\hat{i} + R_x\hat{i} + R_y\hat{j}$

$m\underline{a} = 2[\ddot{\theta}\cdot 5\hat{e}_\theta - \dot{\theta}^2 5\hat{e}_r]$

$= 2[\ddot{\theta}(-4\hat{i} + 3\hat{j}) - \dot{\theta}^2(3\hat{i} + 4\hat{j})]$

$= (-8\ddot{\theta} - 3\dot{\theta}^2)\hat{i} + (\ddot{\theta} - 4\dot{\theta}^2)\hat{j}$

$R_x + 10\sin\theta = -8\ddot{\theta} - 3\dot{\theta}^2$

$R_y + 10\cos\theta = 6\ddot{\theta} - 4\dot{\theta}^2$

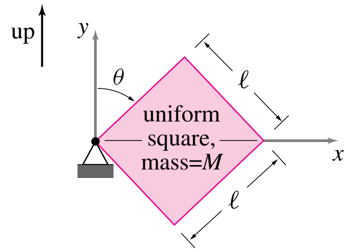
$\sum M_{/0} = \dot{\underline{H}}_{/0} \Rightarrow -50\ N\cdot cm\,\hat{k} = \dot{\underline{H}}_{/c} = -50\times 10^{-2}\ Nm$

$\dot{\underline{H}}_{/0} = \ddot{\theta} I \hat{k} = \ddot{\theta}\cdot 2[5]^2\hat{k} = 50\ddot{\theta}\times 10^{-4}$

$\ddot{\theta} = -100$ cycles/sec^2

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.15 The uniform square shown is released from rest at $t = 0$. What is $\alpha = \dot{\omega} = \ddot{\theta}$ immediately after release?



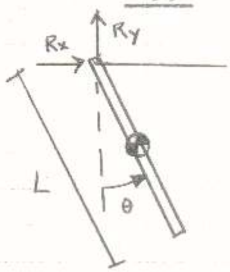
Problem 16.15

Answer:
$\ddot{\theta} = -\frac{3g}{2L}\sin\theta$

Page 4

$\boxed{5.16}$ Using energy methods (B) from $\boxed{5.15}$

FBD



$$\dot{W} = \dot{E}_K$$

$$\underline{F} \cdot \underline{v} = \frac{d}{dt}[E_K]$$

$$-mg\hat{\jmath} \cdot \frac{L}{2}\dot{\theta}(\cos\theta\hat{\imath} + \sin\theta\hat{\jmath})$$

$$= \frac{d}{dt}\left[\frac{1}{2}m_{tot}v_{cm}^2 + \frac{1}{2}\underline{\omega} \cdot [I^{cm}] \cdot \underline{\omega}\right]$$

$$-mg\frac{L}{2}\sin\theta\,\dot{\theta} = \frac{d}{dt}\left[\frac{1}{2}m\left(\frac{L}{2}\right)^2\dot{\theta}^2 + \cdot\frac{1}{2}\frac{mL^2}{12}\dot{\theta}^2\right]$$

$$-mg\frac{L}{2}\sin\theta\,\dot{\theta} = m\frac{L^2}{4}\dot{\theta}\ddot{\theta} + \frac{mL^2}{12}\dot{\theta}\ddot{\theta}$$

$$-mg\frac{L}{2}\sin\theta = \frac{mL^2}{3}\ddot{\theta}$$

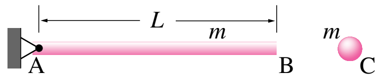
$$\boxed{\ddot{\theta} + \frac{3g}{2L}\sin\theta = 0}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.16 Acceleration of a trap door. A uniform bar AB of mass m and a ball of the same mass are released from rest from the same horizontal position. The bar is hinged at end A. There is gravity.

a) Which point on the rod has the same acceleration as the ball, immediately after release.

b) What is the reaction force on the bar at end A just after release?



Problem 16.16

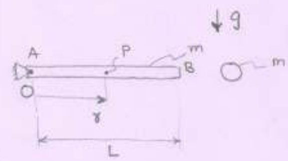
Answer:
(a) Point at $2L/3$ from A
(b) $mg/4$ directed upwards.

15.4.16: Acceleration of a trap door

A bar AB with hinge at 'A' and a ball are released from rest.

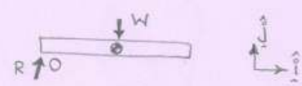
a) $r = ?$ such that acceleration at $P = 1g$.

$(1g = 9.81\,m/s^2)$



$$\underset{\sim}{a}_P = \dot{\underset{\sim}{\omega}} \times \underset{\sim}{r} + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r})$$

As acceleration immediately after release is of interest, $\underset{\sim}{\omega} = \underset{\sim}{0}$



$$\dot{\underset{\sim}{\omega}} = \ddot{\theta}(-\hat{k}) \quad \Rightarrow \quad \underset{\sim}{a}_P = \ddot{\theta}(-\hat{k}) \times r\hat{i} = -\ddot{\theta} r \hat{j} \quad \text{— (1)}$$

To find $\ddot{\theta}$, use AMB about O. $\quad \Sigma \underset{\sim}{M}_{/O} = \dot{\underset{\sim}{H}}_{/O}$

$$\Rightarrow \quad \frac{L}{2}\hat{i} \times W(-\hat{j}) = I\ddot{\theta}(-\hat{k})$$

$$\Rightarrow \quad \left\{ \frac{mgL}{2}(-\hat{k}) = \frac{1}{3} m L^2 \ddot{\theta}(-\hat{k}) \right\}$$

$$\{\ \}\cdot(-\hat{k}) \Rightarrow \quad \ddot{\theta} = \frac{3g}{2L} \quad \text{— (2)}$$

Acceleration of the ball is $g(-\hat{j})$ — (3)

$\therefore$ from (1), (2) and (3) $\quad \underset{\sim}{a}_P = -\frac{3g}{2L} r \hat{j} = -g\hat{j} \quad \Rightarrow \quad \boxed{r = \frac{2L}{3}}$

b) $\underset{\sim}{R} = ?$ immediately after release:

LMB: $\quad \Sigma \underset{\sim}{F} = m \underset{\sim}{a}_{cm}$

$$\underset{\sim}{R} + W(-\hat{j}) = m\left[-\ddot{\theta}\frac{L}{2}\hat{j}\right] \quad (\text{from (1)})$$

$$\Rightarrow \quad \underset{\sim}{R} - mg\hat{j} = -m\frac{3g}{2L}\frac{L}{2}\hat{j}$$

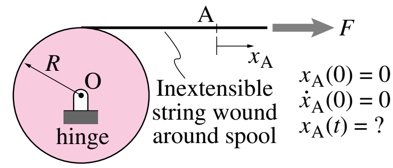
$$\Rightarrow \quad \underset{\sim}{R} = mg\left[1 - \frac{3}{4}\right]\hat{j}$$

$$\Rightarrow \quad \underset{\sim}{R} = \frac{mg}{4}\hat{j}$$

$\therefore$ The reaction at 'O' immediately after release, $\boxed{\underset{\sim}{R} = \frac{mg}{4}\hat{j}}$ (against gravity)

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.17 A disk with radius R has a string wrapped around it which is pulled with a force F. The disk is free to rotate about the axis through O normal to the page. The moment of inertia of the disk about O is I_o. A point A is marked on the string. Given that $x_A(0) = 0$ and that $\dot{x}_A(0) = 0$, what is $x_A(t)$?



Spool has mass moment of inertia, I_O

Problem 16.17

Answer:
$x_A(t) = \frac{Ft^2}{2\left(m+\frac{I}{R^2}\right)}$

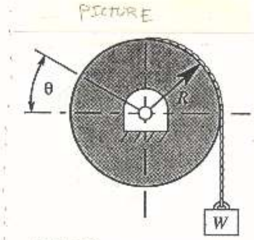
5.12 Conservation of Energy

PICTURE

E_K of the weight $= \frac{1}{2} m \dot{x}^2$

E_K of rotation of wheel $= \frac{1}{2} I \dot{\theta}^2$

E_P of the weight $= -Wx$



probfig 5.12: (Filename:pfigure.blue.136.1)

Suppose at time 0 nothing is moving, and $x = 0$

$0 = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} I \dot{\theta}^2 - Wx$ Cons. of Energy (no energy loss)

$0 = m \dot{x} \ddot{x} + I \dot{\theta} \ddot{\theta} - W\dot{x}$ Differentiating

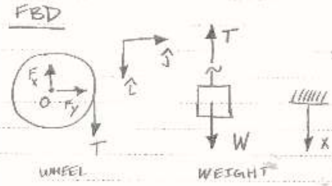
But, for inextensible string we have the constraint that $\dot{x} = R\dot{\theta}$, substituting in for $\dot{\theta}$ above and also $\ddot{x} = R\ddot{\theta}$, by differentiating ∴

$0 = m\dot{x}\ddot{x} + \frac{I}{R^2}\dot{x}\ddot{x} - W\dot{x}$

$0 = \dot{x}\left(m\ddot{x} + \frac{I}{R^2}\ddot{x} - W\right)$ setting term inside parentheses $= 0$

$\boxed{\ddot{x} = \frac{W}{m + \frac{I}{R^2}}}$ or $\boxed{\ddot{\theta} = \frac{W}{mR + \frac{I}{R}}}$ How many degrees of freedom in this system?

FBD



$\underline{\omega} = -\dot{\theta}\hat{k}$

$\dot{\underline{\omega}} = -\ddot{\theta}\hat{k}$

AMB

$\sum \underline{M}_{/O} = \underbrace{\underline{r}_{cm/O}}_{0} \times \underline{a}_{cm} m_{tot} + \underbrace{\cancel{\underline{\omega}}}_{0} \times [I^{cm}] \cdot \underline{\omega} + [I^{cm}] \cdot \dot{\underline{\omega}}$

$\{(R\hat{\jmath} \times T\hat{\imath}) = I(-\ddot{\theta}\hat{k})\} \cdot \hat{k} \Rightarrow TR = I\ddot{\theta}$

LMB $\{-T\hat{\imath} + W\hat{\imath} = m\ddot{x}\hat{\imath}\} \cdot \hat{\imath} \Rightarrow W - T = m\ddot{x}$

we still have $\ddot{\theta} = R\ddot{x} \Rightarrow T = \frac{I}{R^2}\ddot{x}$

putting all together

$\boxed{\ddot{x} = \frac{W}{m + \frac{I}{R^2}}}$ ✓ or $\boxed{\ddot{\theta} = \frac{W}{mR + \frac{I}{R}}}$

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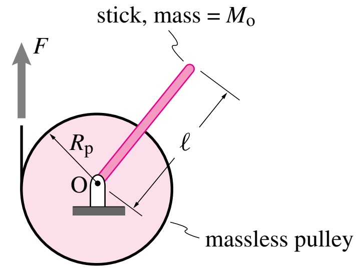
16.4.18 A uniform stick with length ℓ and mass M_o is welded to a pulley hinged at the center O. The pulley has negligible mass and radius R_p. A string is wrapped many times around the pulley. At time $t = 0$, the pulley, stick, and string are at rest and a force F is suddenly applied to the string. How long does it take for the pulley to make one full revolution? Ignore gravity.

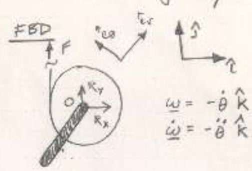


Problem 16.18: String wraps around a pulley with a stick glued to it.

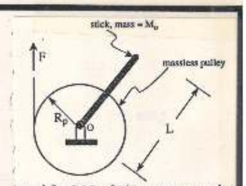
Answer:
$T_{rev} = \sqrt{\dfrac{2M_O\ell^2\pi}{3FR_p}}.$

5.15 no gravity

FBD



$\omega = -\dot{\theta}\,\hat{k}$
$\dot{\omega} = -\ddot{\theta}\,\hat{k}$



probfig 5.15: String wraps around a pulley with a stick glued to it. (Filename:p2.3)

LMB $F\hat{\jmath} + R_y\hat{\jmath} + R_x\hat{\imath} = 0$ doesn't give us equation for $\theta, \dot{\theta}$

AMB $\sum M_{/o} = \underline{r}_{cm/o} \times \underline{a}_{cm}\, m_{tot} + \underline{\omega} \times [I^{cm}]\cdot\underline{\omega} + [I^{cm}]\cdot\dot{\underline{\omega}}$

Could do this way, using $\underline{r}_{cm/o}$ as a vector from O to the cm of the stick and $[I^{cm}]$ as the moment of inertia of the stick about its center of mass. Note that $\underline{\omega} \times [I^{cm}]\cdot\underline{\omega}$ still is zero ($[I^{cm}]\cdot\underline{\omega}$ is parallel to $\underline{\omega}$).

$(-R_p\hat{\imath} \times F\hat{\jmath}) = m\left(\frac{L}{2}\hat{e}_r \times \left[-\dot{\theta}^2\frac{L}{2}\hat{e}_r - \frac{L}{2}\ddot{\theta}\,\hat{e}_\theta\right]\right) + \frac{1}{12}mL^2(-\ddot{\theta}\hat{k})$

$-R_pF\hat{k} = \left(-\frac{mL^2}{4}\ddot{\theta} - \frac{mL^2}{12}\ddot{\theta}\right)\hat{k}$ see pg 228 need to know $\boxed{\underline{a} = \underline{\omega}\times(\underline{\omega}\times\underline{r}) + \dot{\underline{\omega}}\times\underline{r}} \leftarrow \underline{a}_{cm}$

$R_pF = \frac{mL^2}{3}\ddot{\theta}$

Could also use the simplification in this case that

$\sum M_{/o} = \underbrace{\underline{\omega} \times \underline{H}_{/o}}_{0,\ \text{parallel}} + [I^o]\cdot\dot{\underline{\omega}}$ note: $I^o \neq I^{cm}$, $I_o = \frac{mL^2}{3}$

$\{(-R_p\hat{\imath} \times F\hat{\jmath}) = [I^o]\cdot(-\ddot{\theta}\hat{k})\}\cdot\hat{k}$

$R_pF = \frac{mL^2}{3}\ddot{\theta}$

All we need to do now is solve this ODE for $\theta(t)$

$\dot{\theta}(t) = \frac{3R_pF}{mL^2}t + \underbrace{\dot{\theta}(t=0)}_{0,\ \text{given}}$

$\frac{d\theta}{dt} = \frac{3R_pF}{mL^2}t$

$\frac{mL^2}{3R_pF}\int_0^{2\pi} d\theta = \int_0^{T_{rev}} t\,dt$ T_{rev} is time of one revolution

$\frac{2\pi mL^2}{3R_pF} = \frac{1}{2}T_{rev}^2$

$\boxed{T_{rev} = \sqrt{\frac{4\pi mL^2}{3R_pF}}}$ ✓

can also do this with uniformly accelerated motion

$d = d_0 + V_0t + \frac{1}{2}at^2$

$t = \sqrt{\frac{2d}{a}}$ $d_0 = 0$, $V_0 = 0$

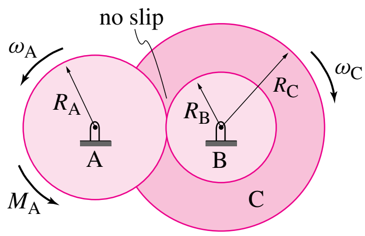
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.19 Constant speed gear train. Gear A is connected to a motor (not shown) and gear B, which is welded to gear C, is connected to a taffy-pulling mechanism. Assume you know the torque $M_{\text{input}} = M_A$ and angular velocity $\omega_{\text{input}} = \omega_A$ of the input shaft. Assume the bearings and contacts are frictionless.

a) What is the input power?

b) What is the output power?

c) What is the output torque $M_{\text{output}} = M_C$, the torque that gear C applies to its surroundings in the clockwise direction?



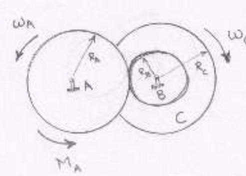
Problem 16.19

Answer:
(a) $P_{\text{in}} = M_A\omega_A$.
(b) $P_{\text{out}} = M_A\omega_A$.
(c) $M_C = M_A\frac{R_B}{R_A}$

15.4.19

a) $P_{in} = M_{in}\,\omega_{in} = M_A\omega_A$

b) If $\eta = 100\%$ => $P_{out} = P_{in} = M_A\omega_A$



c) $V_p = \omega_A R_A = \omega_B R_B$

$\omega_B = \omega_A \frac{R_A}{R_B}$

$M_c = \frac{P_{out}}{\omega_c} = \frac{M_A\omega_A}{\omega_A \frac{R_A}{R_B}} = M_A\frac{R_B}{R_A}$

16.4.20 At the input to a gear box a 100 lbf force is applied to gear A. At the output, the machinery (not shown) applies a force of F_B to the output gear. Gear A rotates at constant angular rate $\omega = 2\,\text{rad/s}$, clockwise.

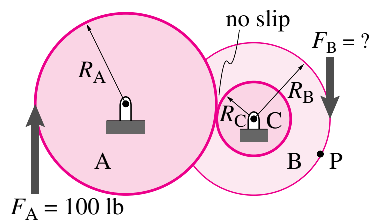
a) What is the angular speed of the right gear?

b) What is the velocity of point P?

c) What is F_B?

d) If the gear bearings had friction, would F_B have to be larger or smaller in order to achieve the same constant velocity?

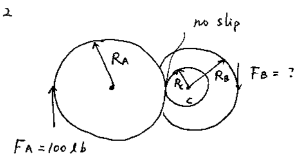
e) If instead of applying a 100 lbf to the left gear it is driven by a motor (not shown) at constant angular speed ω, what is the angular speed of the right gear?



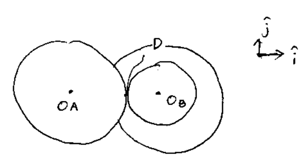
Problem 16.20: Two gears with end loads.

Answer:
(a) $\omega_B = \omega \frac{R_A}{R_C}$
(b) $V_P = \frac{\omega R_A R_B}{R_C}$
(c) $F_B = \frac{F_A R_C}{R_B}$
(d) F_B will have to be smaller.
(e) The left gear is constrained to rotate at $\omega_B = \frac{\omega R_A}{R_C}$.

13.122



(a)



$\hat{i}$ is along the line connecting O_A and O_B. D is the point of contact

$\therefore \quad \vec{r}_{D/O_A} = R_A \hat{i}, \quad \vec{r}_{D/O_B} = -R_c \hat{i}$

No slip condition:

$$\vec{V}_D \big|_{\text{on gear } A} = \vec{V}_D \big|_{\text{on gear } B}$$

$$\cancel{\vec{V}_{O_A}}^{\,0} + \vec{\omega}_A \times \vec{r}_{D/O_A} = \cancel{\vec{V}_{O_B}}^{\,0} + \vec{\omega}_B \times \vec{r}_{D/O_B}$$

Gear A rotates clockwise with $\omega = 2\,rad/s$.

$\therefore \quad \vec{\omega}_A = -\omega \hat{k}$

Assume $\vec{\omega}_B = \omega_B \hat{k}$

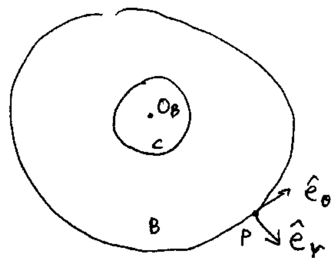
$\Rightarrow \quad -\omega \hat{k} \times R_A \hat{i} = \omega_B \hat{k} \times (-R_c \hat{i})$

$\Rightarrow \quad \boxed{\omega_B = \omega \frac{R_A}{R_c}}$, $\boxed{\omega_B > 0}$ means the right gear rotates counterclockwise

$\therefore$ gear B ~~C~~ rotates counterclockwise with ~~spee~~ angular velocity $\omega_B = \omega \frac{R_A}{R_c}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

b.).



From (a), we know the angular velocity of B, $\vec{\omega}_B = \omega \frac{R_A}{R_C} \hat{k}$

$\therefore \vec{V}_P = \vec{V}_{OB} + \vec{\omega}_B \times \vec{r}_{P/OB} = \omega \frac{R_A}{R_C} R_B \hat{e}_\theta$

$\therefore$ The velocity of P is $\boxed{\frac{\omega R_A R_B}{R_c}}$ and is in $\hat{e}_\theta$ direction.

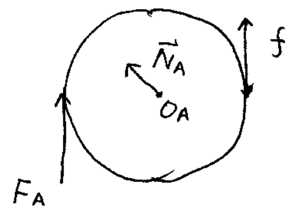
c). Assume friction less bearing.

FBD $\hat{j}$ $\hat{i}$

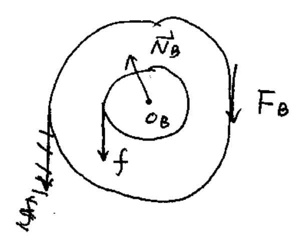
f: friction force at contact point

$\vec{N}_A$: reaction force at gear bearing

Left gear:



Right gear:



AMB of left gear about OA

$\Rightarrow \quad \Sigma \vec{M}_{/OA} = \dot{\vec{H}}_{/OA} = \frac{d}{dt}(I_{OA} \underbrace{\vec{\omega}_A}) = 0$ constant angular velocity

$\Rightarrow -F_A R_A + f R_A = 0$

$\Rightarrow F_A = f$

AMB of right gear about O_B

$\Rightarrow \sum \vec{M}_{/O_B} = \dot{\vec{H}}_{/O_B} = \frac{d}{dt}(I_{O_B}\vec{\omega}_B) = 0$

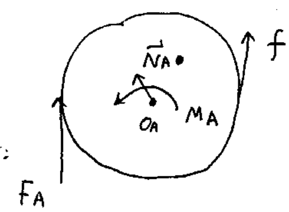
$\Rightarrow -F_B R_B + f R_C = 0 \qquad \Rightarrow F_B = \frac{f R_C}{R_B}$

$\therefore \boxed{F_B = \frac{F_A R_C}{R_B}}$

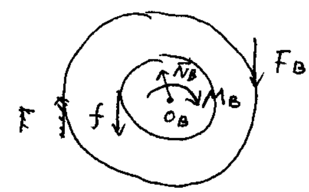
d). If gear bearing had friction,

FBD

Left gear:



right gear :



then there will be moment from the bearing resisting rotation of the gears.

$\because$ A is rotating clockwise, so $\vec{M}_A = M_A\hat{k}$, $M_A > 0$,
i.e., the moment is counterclockwise

B, C is rotating counterclockwise, so $\vec{M}_B = -M_B\hat{k}$, $M_B > 0$
i.e., the moment on B, C. is clockwise.

Use the same argument as in (c).,

Left gear: $\sum \vec{M}_{/O_A} = 0$

$\Rightarrow -F_A R_A + f R_A + M_A = 0$

$\Rightarrow f = F_A - \frac{M_A}{R_A}$

right gear: $\sum \vec{M}_{/O_B} = 0$

$\Rightarrow -F_B R_B + f R_C - M_B = 0 \quad \Rightarrow F_B = f\frac{R_C}{R_B} - \frac{M_B}{R_B}$

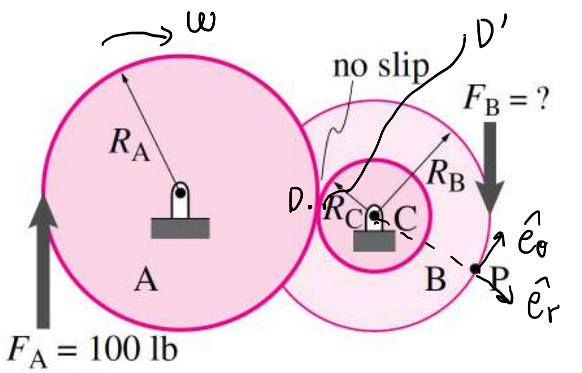
$\therefore \boxed{F_B = \frac{F_A R_C}{R_B} - \frac{M_A R_C}{R_A R_B} - \frac{M_B}{R_B}} < \frac{F_A R_C}{R_B}$ since $M_A, M_B > 0$

∴ F_B is smaller because of friction

e). If the left gear is driven by a motor, angular speed of the right gear is $\boxed{\text{still } \omega_B = \omega \frac{R_A}{R_C}}$, counter clockwise.
Because this result comes from the kinematic constraint that there is no slip between left gear and right gear. It doesn't depend on how the left gear is driven.

15.4.20

Tuesday, April 9, 2019 3:30 PM



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Problem 15.4.20: Two gears with end loads.

Given:

Input force applied to gear A:

$F_A = 100$ lb

gear A rotates clockwise

$\omega = 2$ rad/s

$\vec{\omega} = -\omega \hat{k}$

Note: Just like 15.2.22

a) $\omega_B = ?$

D is the point of contact as shown

No slip $\Rightarrow$ $\vec{V}_D \big|_{\text{on gear } A} = \vec{V}_{D'}$ on gear B

$\{ \vec{\omega}_A \times \vec{r}_{D/0A} = \vec{\omega}_B \times \vec{r}_{D'/0B} \}$

$\{ \} \cdot \hat{j} \Rightarrow -\omega \cdot R_A = \omega_B (-R_c)$

$\Rightarrow \boxed{\omega_B = \omega \frac{R_A}{R_C} > 0}$ meaning ω_B rotates ccw

b) $\vec{V}_P = ?$

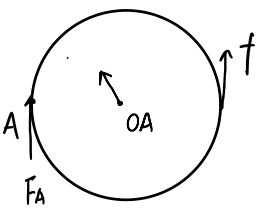
From (a) $\vec{\omega}_B = \omega \frac{R_A}{R_C} \hat{k}$

$\vec{V}_P = \cancel{\vec{V}_{0B}}^{\,0} + \vec{\omega}_B \times \vec{r}_{P/0B} = \omega \frac{R_A}{R_C} R_B \hat{e}_\theta$ $\boxed{F_B = \omega \frac{R_A}{R_C} R_B \hat{e}_\theta}$

c) $\vec{F}_B = ?$ (Assume frictionless bearing)

FBD

Left gear A

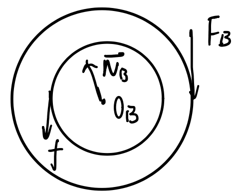


f: friction force at contact point

$\vec{N}_A$: reaction force at gear bearing

$\vec{N}_B$: Similarly

Right gear B



Assume $\vec{F}_B = -F_B \hat{j}$

AMB of $A/_{OA}$ $\quad \Sigma \vec{M}_{/OA} = \dot{\vec{H}}_{/OA} = 0$

$\Rightarrow -F_A R_A + f R_A = 0 \quad \Rightarrow F_A = f$

AMB of $B/_{OB}$ $\quad \Sigma \vec{M}_{/OB} = \dot{\vec{H}}_{/OB} = 0$

$\Rightarrow -F_B R_B + f R_C = 0 \quad \Rightarrow \boxed{F_B = \frac{f R_C}{R_B} = \frac{F_A R_C}{R_B}}$

(magnitude)

Also, Power balance :

$\vec{\omega}_A \cdot \vec{M}_{/OA} + \vec{\omega}_B \cdot \vec{M}_{/OB} = 0 \qquad \vec{\omega}_C = \vec{\omega}_B$

where A, B are contacting point of force with disk

$\vec{\omega}_A \cdot (\vec{R}_{OA} \times \vec{F}_A) + \vec{\omega}_B \cdot (\vec{R}_{OB} \times \vec{F}_B) = 0$

$\therefore \quad \omega_A R_{OA} F_A - \omega_B R_{OB} F_B = 0 \quad ①$

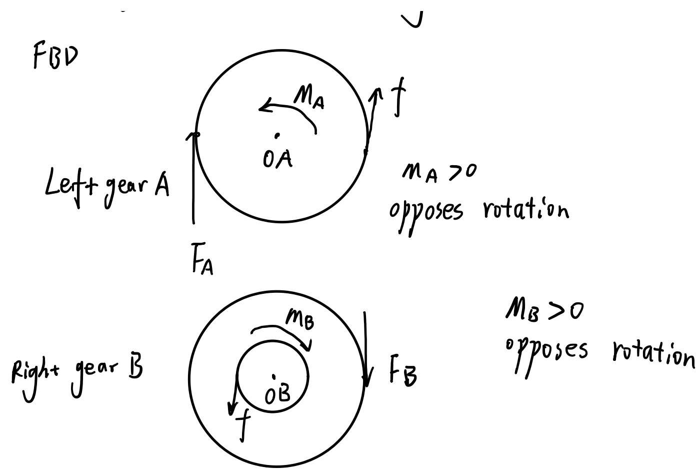
$\omega_B = \omega \frac{R_A}{R_C} \qquad \omega_A = \omega \qquad ②$

① ② $\Rightarrow \boxed{F_B = \frac{F_A R_C}{R_B}}$

d) $\vec{F}_B$ larger / smaller ? (bearing has friction)

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

FBD



$$\vec{M}_A = M_A \hat{k} \qquad \vec{M}_B = -M_B \hat{k}$$

A : AMB $\sum \vec{M}_{/O_A} = \vec{0}$

$$\Rightarrow -F_A R_A + f R_A + M_A = 0$$

$$\Rightarrow f = F_A - \frac{M_A}{R_A}$$

B : AMB $\sum \vec{M}_{/O_B} = \vec{0}$

$$\Rightarrow -F_B R_B + f R_C - M_B = 0$$

$$\Rightarrow F_B = f \frac{R_C}{R_B} - \frac{M_B}{R_B}$$

$$\therefore F_B = \frac{F_A R_C}{R_B} - \frac{M_A R_C}{R_A R_B} - \frac{M_B}{R_B} < \frac{F_A R_C}{R_B}$$

since $M_A > 0$ $M_B > 0$

$\boxed{\therefore F_B \text{ is smaller because of friction}}$

e) $\vec{\omega}_B = ?$ If the left gear is driven by a motor?

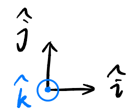
$\boxed{\text{Still } \vec{\omega}_B = \omega \frac{R_A}{R_C} \hat{k}}$

The result comes from the kinemic constraint that there is no slip between left gear A and right gear B. It does not depend on how the left gear is driven.

MAE2030 HW P51 (16.4.20) Solution

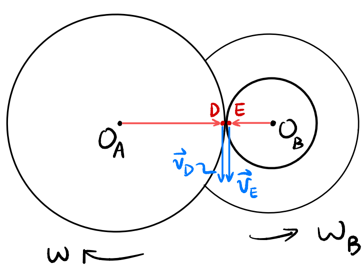
Duan Li

Given :



$\omega = 2$ rad/s

a) Find ω_B



Angular velocity of gear A : $\vec{\omega}_A = -\omega \hat{k}$

Angular velocity of gear B : $\vec{\omega}_B = \omega_B \hat{k}$

D and E are the contacting points of the 2 gears

D right on the edge of gear A

E right on the edge of gear B

No slip $\rightarrow$ $\vec{v}_D = \vec{v}_E$

$\vec{v}_D = \vec{\omega}_A \times \vec{r}_{D/O_A}$

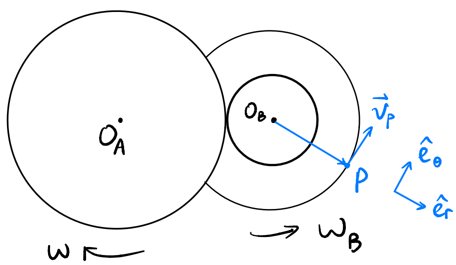
$\vec{v}_E = \vec{\omega}_B \times r_{E/O_B}$

$$-\omega\hat{k}\times R_A\,\hat{\imath} = \omega_B\hat{k}\times(-R_c)\,\hat{\imath}$$

$$\left\{-\omega R_A\,\hat{\jmath} = -\omega_B R_c\,\hat{\jmath}\right\}\cdot\hat{\jmath}$$

$$\boxed{\omega_B = \frac{R_A}{R_c}\,\omega}$$

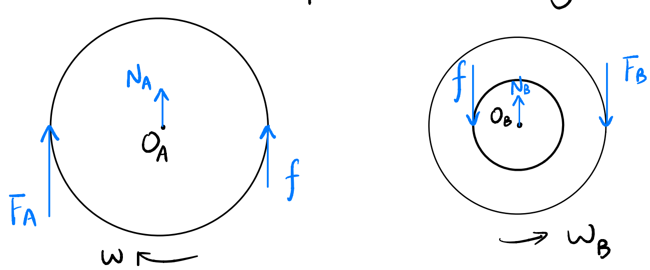
b) Find $\vec{v}_P$



$$\vec{v}_P = \vec{\omega}_B\times r_{P/O_B} = \frac{R_A}{R_c}\,\omega\hat{k}\times R_B\,\hat{e}_r = \boxed{\frac{R_A R_B}{R_c}\,\omega\,\hat{e}_\theta}$$

c) Find F_B

FBDs (Assume frictionless bearings)



AMB of gear A

constant $\vec{\omega}_A$

$$\sum\vec{M}_{A/O_A} = \dot{\vec{H}}_{A/O_A} = \vec{0}$$

$$F_A \hat{j} \times (-R_A)\hat{i} + f\hat{j} \times R_A \hat{i} = \vec{0}$$

$$f = F_A$$

AMB of gear B

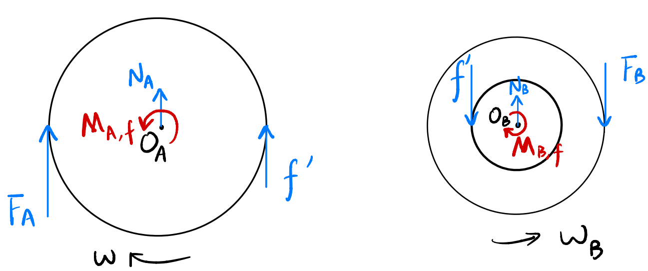
$$\sum \vec{M}_{B/O_B} = \dot{\vec{H}}_{B/O_B} \overset{\text{constant } \vec{\omega}_B}{=} \vec{0}$$

$$-f\hat{j} \times (-R_c)\hat{i} + (-F_B)\hat{j} \times R_B \hat{i} = \vec{0}$$

$$F_B = \frac{R_c}{R_B} f = \boxed{\frac{R_c}{R_B} F_A}$$

d) Compare F_B for bearings with or without friction

FBDs (Bearing w. frictional moments $\vec{M}_f^A$, $\vec{M}_f^B$)



$M_f^A > 0$, $M_f^B > 0$

AMB of gear A

$$\sum \vec{M}_{A/O_A} = \dot{\vec{H}}_{A/O_A} \overset{\text{constant } \vec{\omega}_A}{=} \vec{0}$$

$$F_A \hat{j} \times (-R_A)\hat{i} + f'\hat{j} \times R_A \hat{i} + M_{A,f}\hat{k} = \vec{0}$$

$$f' = F_A - \frac{M_{A,f}}{R_A}$$

AMB of gear B

constant $\vec{\omega}_B$

$$\sum \vec{M}_{B/O_B} = \dot{\vec{H}}_{B/O_B} = \vec{0}$$

$$-f'\hat{j} \times (-R_c)\hat{i} + (-F_B')\hat{j} \times R_B \hat{i} - M_{B,f}\hat{k} = \vec{0}$$

$$F_B' = \frac{R_c}{R_B} f' - \frac{M_{B,f}}{R_B} = \frac{R_c}{R_B} F_A - \frac{R_c}{R_B}\frac{M_A}{R_A} - \frac{M_{B,f}}{R_B} < F_B$$

F_B has to be $\boxed{\text{smaller}}$ to achieve the same equilibrium

Qualitative sanity check:

$\because \omega_B \propto \omega \quad \therefore \omega = \text{const.}$ means $\omega_B = \text{const.}$

$\because M_{A,f} \uparrow$ (increases) $\Rightarrow \omega \downarrow$ (decreases)

$\therefore$ Need $f' \downarrow \Rightarrow \omega \uparrow$

So that ω stays constant

$\because f' \downarrow \Rightarrow \omega_B \downarrow$

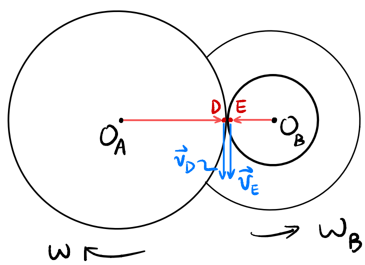
Also $M_{B,f} \uparrow \Rightarrow \omega_B \downarrow$

$\therefore$ Need $\boxed{F_B \downarrow} \Rightarrow \omega_B \uparrow$

So that ω_B stays constant

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

e) Find ω_B

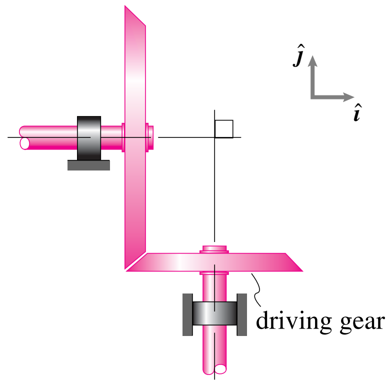


As we can see from the diagram from (a),

$\vec{v}_E$ does NOT depend on how gear A is driven, and neither does $\vec{\omega}_B$

$$\boxed{\omega_B = \frac{R_A}{R_C}\,\omega}$$

16.4.21 A bevel gear system. A bevel-type gear system, shown in the figure, is used to transmit power between two shafts that are perpendicular to each other. The driving gear has a mean radius of 50 mm and rotates at a constant speed $\omega = 150$ rpm. The mean radius of the driven gear is 80 mm and the driven shaft is expected to deliver a torque of $M_{out} = 25$ N·m. Assuming no power loss, find the input torque supplied by the driving shaft.



Problem 16.21: A bevel gear

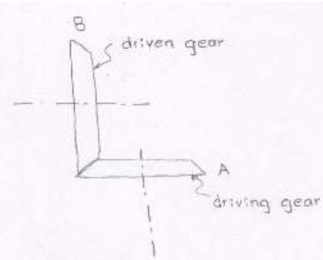
Answer:
$M_{\text{in}} = 15.625\,\text{N}\cdot\text{m}$.

$R_A = 50$ mm.

$\omega_A = 150$ rpm

$R_B = 80$ mm

$M_{out} = 25$ Nm



$V_p = \omega_A R_A = \omega_B R_B$

$$\Rightarrow \quad \omega_B = \frac{\omega_A R_A}{R_B} = 150 \times \frac{50}{80} = 93.75 \text{ rpm}$$

$$P_{out} = M_{out} \cdot \omega_{out} = 25 \times 93.75 \times \frac{2\pi}{60} = 245.4369 \text{ W}$$

$$M_{in} = \frac{P_{in}}{\omega_{in}} = \frac{245.43}{150 \times 2\pi} \times 60 = 15.625 \text{ Nm}$$

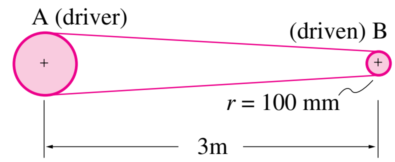
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.22 *Belt drives* are used to transmit power between parallel shafts. Two parallel shafts, 3 m apart, are connected by a belt passing over the pulleys A and B fixed to the two shafts. The driver pulley A rotates at a constant 200 rpm. The speed ratio between the pulleys A and B is 1:2.5. The input torque is 350 N·m. Assume no loss of power between the two shafts.

a) Find the input power.

b) Find the rotational speed of the driven pulley B.

c) Find the output torque at B.



Problem 16.22

Answer:
(a) P_{in} = 7.33kilo-watts
(b) 500 rpm
(c) M_{out} = 140 N·m

$M_{in} = 350.\ N\cdot m$

$P_{in} = M_{in}\omega_{in} = 350 \times 200 \times \frac{2\pi}{60}$

$P_{in} = 7330.38\ W$

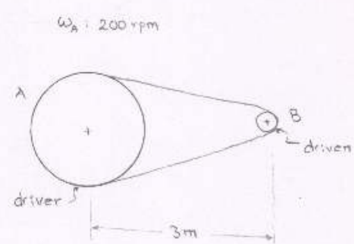
$\frac{\omega_B}{\omega_A} = 2.5$

$\omega_B = 2.5 \times 200 = 500\ rpm$

$= 52.359$ cycles /sec

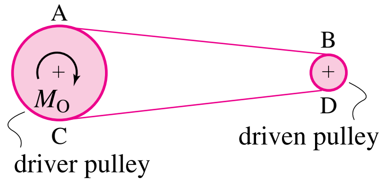
$M_{out} = \frac{P_{out}}{W_{out}} = 139.99\ N\cdot m$

$\frac{\omega_B}{\omega_A} = 2.5$



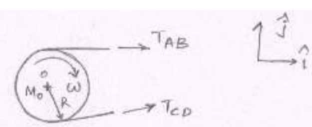
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.23 In the belt drive system shown, assume that the driver pulley rotates at a constant angular speed ω. If the motor applies a constant torque M_O on the driver pulley, show that the tensions in the two parts, AB and CD, of the belt must be different. Which part has a greater tension? Does your conclusion about unequal tension depend on whether the pulley is massless or not? Assume any dimensions you need.



Problem 16.23

Answer:
$M_{out} = 140\,\text{N}\cdot\text{m}$



$\sum M_{/O} = \dot{H}_{/O}$.

$-M_O - T_{AB} R + T_{CD} R = \cancel{\dot{\omega}}^{0} \frac{MR^2}{2} \hat{k}$

$T_{CD} = T_{AB} + \frac{M_O}{R} \Rightarrow T_{CD} > T_{AB}$

NO. It doesnot depend on the mass of the wheel

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

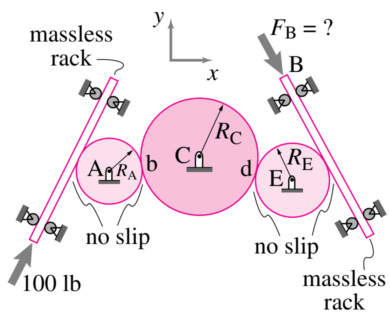
16.4.24 Two racks connected by three constant rate gears. A 100 lbf force is applied to one rack. At the output, the machinery (not shown) applies a force of F_B to the other rack.

a) Assume the gear-train is spinning at constant rate and is frictionless. What is F_B?

b) If the gear bearings had friction would that increase or decrease F_B to achieve the same constant rate?

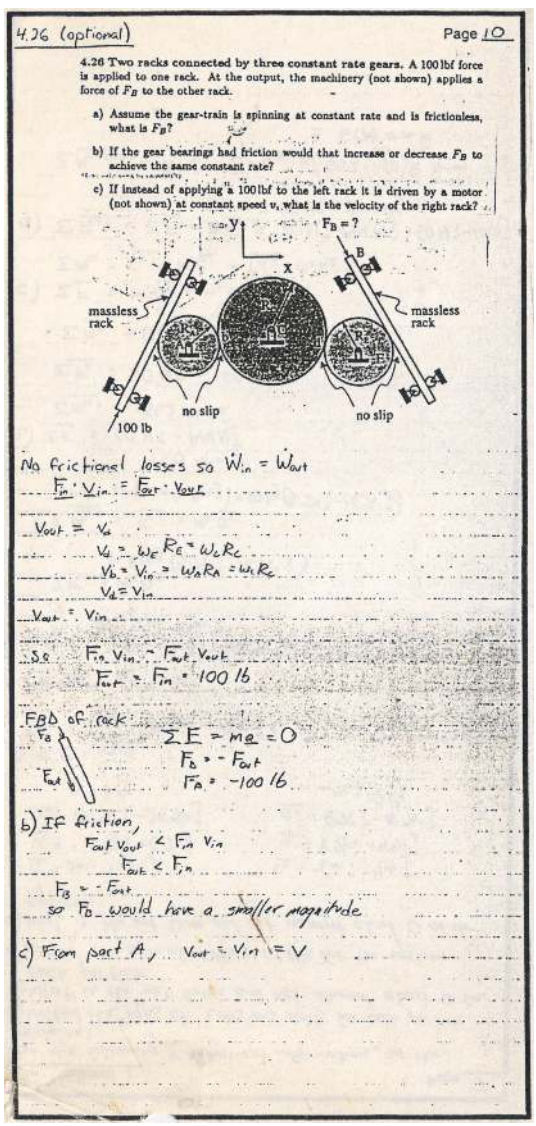
c) If instead of applying a 100 lbf to the left rack it is driven by a motor (not shown) at constant speed v, what is the speed of the right rack?

d) If the angular velocity of the gear is increasing at rate α does this increase or decrease F_B at the given ω.



Problem 16.24: Two racks connected by three gears.

Answer:
(a) $F_B = 100\,\text{lbf}$.
(c) $v_{right} = v$.



4.26 (optional) Page 10

4.26 Two racks connected by three constant rate gears. A 100 lbf force is applied to one rack. At the output, the machinery (not shown) applies a force of F_B to the other rack.

a) Assume the gear-train is spinning at constant rate and is frictionless, what is F_B?

b) If the gear bearings had friction would that increase or decrease F_B to achieve the same constant rate?

c) If instead of applying a 100 lbf to the left rack it is driven by a motor (not shown) at constant speed v, what is the velocity of the right rack?

No frictional losses so $\dot{W}_{in} = \dot{W}_{out}$

$\underline{F_{in}} \cdot \underline{V_{in}} = \underline{F_{out}} \cdot \underline{V_{out}}$

$V_{out} = V_d$

$V_d = \omega_E R_E = \omega_C R_C$

$V_b = V_{in} = \omega_A R_A = \omega_C R_C$

$V_d = V_{in}$

$V_{out} = V_{in}$

So $F_{in} V_{in} = F_{out} V_{out}$

$F_{out} = F_{in} = 100\ lb$

FBD of rack:

F_B

F_{out}

$\sum \underline{F} = m\underline{a} = 0$

$F_B = -F_{out}$

$F_B = -100\ lb$

b) If friction,

$F_{out} V_{out} < F_{in} V_{in}$

$F_{out} < F_{in}$

$F_B = -F_{out}$

so F_B would have a smaller magnitude

c) From part A, $V_{out} = V_{in} = V$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

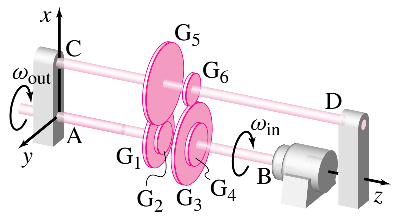
16.4.25 -3-D accelerating gear train. This is really a 2-D problem; each gear turns in a different parallel plane. Shaft B is rigidly connected to gears G_4 and G_5. G_3 meshes with gear G_6. Gears G_6 and G_5 are both rigidly attached to shaft AD. Gear G_5 meshes with G_2 which is welded to shaft A. Shaft A and shaft B spin independently. Assume you know the torque M_{input}, angular velocity ω_{input} and the angular acceleration α_{input} of the input shaft. Assume the bearings and contacts are frictionless.

a) What is the input power?

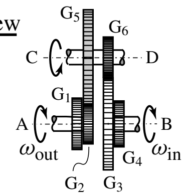
b) What is the output power?

c) What is the angular velocity ω_{output} of the output shaft?

d) What is the output torque M_{output}?



Front view



$R_{G_5} = 3R_{G_2}$
$3R_{G_3} = 5R_{G_6}$

Problem 16.25: A 3-D gear train.

Answer:
(a) $P_{\text{in}} = 2500\pi$ N · m/s.
(b) $P_{\text{out}} = 2500\pi$ N · m/s.
(c) $\omega_{\text{output}} = 25\pi$ rad/s.
(d) $M_{\text{output}} = 100$ N · m.

4.70 A gear train that I won't even try to draw.

a) Find the input power.

$\dot{W}_{in}\left[\frac{N\cdot m}{s}\right] = \underbrace{M}_{\text{N·m}}\ \underbrace{\omega}_{1/s}$ where $\omega = 150$ rev/min $= 5\pi$ rad/s

$\therefore \dot{W}_{in} = (500\ N\cdot m)(5\pi\ \frac{1}{s}) = \boxed{2500\pi\ \frac{N\cdot m}{s} = \dot{W}_{in}}$

b) Find the output power.

Since there is no internal dissipation (efficiency - 100%), all power is transmitted:

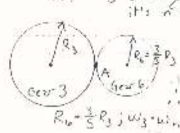
$\boxed{\dot{W}_{out} = \dot{W}_{in} = 2500\pi\ \frac{N\cdot m}{s}}$

4.70 continued. p. 6

c) Find the output frequency ω_{out}.

The key here, like in the problem w/ the rotating cone, is to note that at the contact point between two gears the velocity of the gears must be the same.

Gear 3 is being driven at $\omega_3 = 5\pi$ rad/s. It's in contact with gear 6.

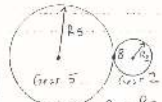


$V_A = \overbrace{\omega_3 R_3}^{V_3} = \overbrace{\omega_6 R_6}^{V_6}$

$R_6 = \frac{3}{5}R_3$; $\omega_3 = \omega_{in}$ $\quad \therefore \omega_6 = \frac{\omega_3 R_3}{R_6} = \frac{5}{3}\omega_{in}$

Gears 5 & 6 are rigidly attached to the same bar $\rightarrow$ $\omega_5 = \omega_6 = \frac{5}{3}\omega_{in}$

Gear 5 is in contact w/ gear 2:



$V_B = \overbrace{\omega_5 R_5}^{V_5} = \overbrace{\omega_2 R_2}^{V_2}$

$\therefore \omega_2 = \frac{\omega_5 R_5}{R_2}$

$R_2 = \frac{R_5}{3}$, $\omega_5 = \frac{5}{3}\omega_{in} \Rightarrow$

$\omega_2 = \frac{5}{3}(\omega_{in})(R_5)\left(\frac{3}{R_5}\right) = 5\omega_{in}$

$\omega_2 = \omega_{out}$, so $\boxed{\omega_{out} = 5\omega_{in} = 25\pi\ \text{rad/s}}$

d) Find M_{out}.

since $\dot{W}_{out} = M_{out}\,\omega_{out}$,

$M_{out} = \frac{2500\pi\ \frac{N\cdot m}{s}}{25\pi\ \frac{1}{s}} = \boxed{100\ N\cdot m = M_{out}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Find ω output.

$\boxed{4{\cdot}23(c)}$ $3R_{G_3} = 5R_{G_6}$, $R_{G_5} = 3R_{G_2}$ } Given

Page 2

Ask yourself how motion is induced to shaft A. G_4 and G_3 rotate at the same rate. Because of G_3, motion is induced to G_6. G_6 and G_5 rotate at the same rate. G_5's motion causes G_2 to rotate. G_2 and G_1 rotate at the same rate.

$$\therefore \quad \omega_3 R_{G_3} = \omega_6 R_{G_6}$$

$$\Rightarrow \omega_3 \tfrac{5}{3} \cancel{R_{G_6}} = \omega_6 \cancel{R_{G_6}} = \boxed{\omega_6 = \tfrac{5}{3}\omega_3 = \omega_5}$$

and $\omega_5 R_{G_5} = \omega_2 R_{G_2}$

$$\Rightarrow \omega_5 3\cancel{R_{G_2}} = \omega_2 \cancel{R_{G_2}} = \boxed{\omega_2 = 3\omega_5}$$

$$\Rightarrow \omega_2 = 3 \times \tfrac{5}{3}\omega_3 = 5\omega_3 = 5 \times 150 = 750 \text{ rev/min}$$

$$\Rightarrow \boxed{\omega_{output} = 750 \text{ rev/min}}$$

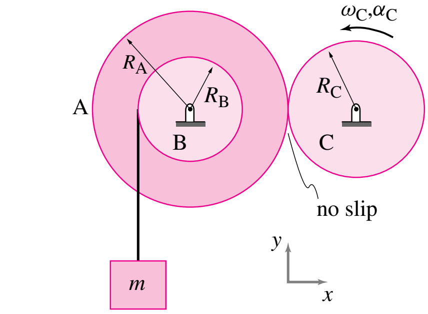
Cut at this border

USE #7B lead and bold print.

Reduce 55% and tape onto 8½" x 11" sheet. (3 per page)

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.26 Gear A with radius $R_A = 400\,\text{mm}$ is rigidly connected to a drum B with radius $R_B = 200\,\text{mm}$. The combined moment of inertia of the gear and the drum about the axis of rotation is $I_{zz} = 0.5\ \text{kg}\cdot\text{m}^2$. Gear A is driven by gear C which has radius $R_C = 300\,\text{mm}$. As the drum rotates, a 5 kg mass m is pulled up by a string wrapped around the drum. At the instant of interest, The angular speed and angular acceleration of the driving gear are 60 rpm and 12 rpm/s, respectively. Find the acceleration of the mass m.



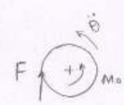
Problem 16.26

Answer:
$a_m = 0.188\,\text{m/s}^2$

$R_A = 0.4\,m$, $R_B = 0.2\,m$, $I_{zz}^{AB/o} = 0.5\ \text{kg m}^2$

$R_C = 0.3\,m$; $m = 5\,\text{kg}$, $\dot\theta_c = 60\,\text{rpm}$, $\ddot\theta_c = 12\,\text{rpm/s}$

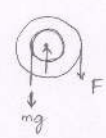
$\sum M_{/o} = \dot{\underline{H}}_{/o}$



$-FR + M_o = \ddot\theta \dfrac{mR^2}{2}$

$V_P = R_C\omega_C = R_A\omega_A$

$\Rightarrow \omega_A = \omega_C \dfrac{R_C}{R_A}$



$a_P\big|_\theta = (\dot{\underline{\omega}} \times \underline{r}) = \alpha_C R_C\big|_\theta = \alpha_A R_A\big|_\theta \Rightarrow \alpha_A = \alpha_C \dfrac{R_C}{R_A}$

$\underline{a}\big|_\theta = \dot{\underline{\omega}} \times \underline{r} = \alpha_c \dfrac{R_c}{R_A}\cdot R_B\big|_\theta = \underbrace{12}_{\text{rpm/s}} \times \dfrac{0.3}{0.4} \times 0.2\,m = 1.8\ \text{rpm/s (m)}$

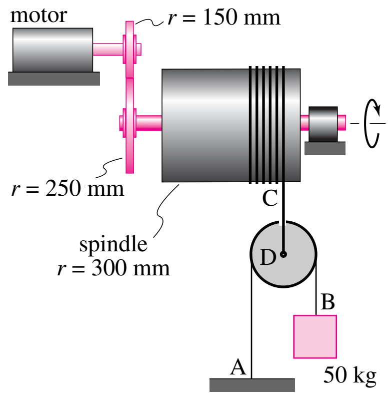
$a\big|_\theta = 1.8 \times \dfrac{2\pi}{60}\dfrac{m}{s^2} = 0.1885\ \text{cycles/sec}^2\ \ m/s^2$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.27 A spindle and pulley arrangement is used to hoist a 50 kg mass as shown in the figure. Assume that the pulley is to be of negligible mass. When the motor is running at a constant 100 rpm,

a) Find the velocity of the mass at B.

b) Find the tension in strings AB and CD.



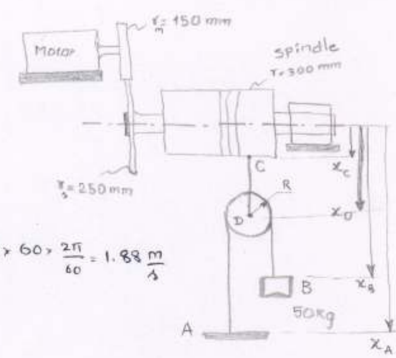
Problem 16.27

Answer:
(a) $V_B = 3.77\,\mathrm{m/s}$.
(b) $T_{AB} = 500\,\mathrm{N}$, $T_{CD} = 1000\,\mathrm{N}$.

a) Find the Velocity of mass at 'B'.

Given motor speed, $\omega_m = 100$ rpm

Assuming no slip, spindle speed $= \omega_s$



$\Rightarrow \omega_s = \dfrac{\omega_m r_m}{r_s} = \dfrac{100 \times 0.15}{0.25} = 60$ rpm

Velocity of point C, $\dot{x}_c = r\omega = 0.3 \times 60 \times \frac{2\pi}{60} = 1.88\ \frac{m}{s}$

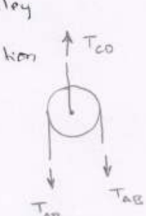
Length of CD $\equiv x_D - x_c$

Length of AB $\equiv (x_A - x_D) + \pi R + (x_B - x_D)$

$\Rightarrow \cancel{\dot{x}_A}^{\,0} + \dot{x}_B - 2\dot{x}_D = 0$

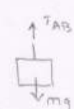
$\Rightarrow \dot{x}_B = 2\dot{x}_D = 2\dot{x}_c = 2 \times 1.88\ \frac{m}{s} = 3.77\ m/s$

b) Consider the free body diagram of pulley with strings. Force balance in vertical direction result in



$T_{CD} - 2T_{AB} = 0$

$\Rightarrow T_{CD} = 2T_{AB}$



As $T_{AB} = mg = 500$ N

$T_{CD} = 1$ KN

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.28 Accelerating rack and pinion. The two gears shown are welded together and spin on a frictionless bearing. The inner gear has radius 0.5 m and negligible mass. The outer disk has 1 m radius and a uniformly distributed mass of 0.2 kg. They are loaded as shown with the force $F = 20$ N on the massless rack which is held in place by massless frictionless rollers. At the time of interest the angular velocity is $\omega = 2$ rad/s (though ω is not constant). The point P is on the disk a distance 1 m from the center. At the time of interest, point P is on the positive y axis.

a) What is the speed of point P?

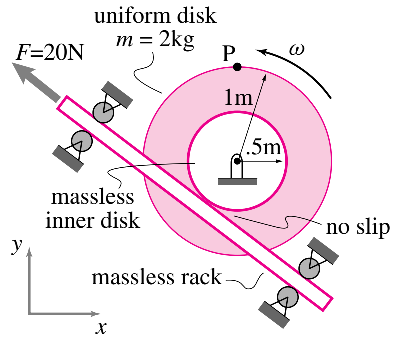
b) What is the velocity of point P?

c) What is the angular acceleration α of the gear?

d) What is the acceleration of point P?

e) What is the magnitude of the acceleration of point P?

f) What is the rate of increase of the speed of point P?



Problem 16.28: Accelerating rack and pinion

Answer:
(a) $|\vec{v}_p| = 2\,\text{m/s}$.
(b) $\vec{v}_p = -2\hat{\imath}\,\text{m/s}$.
(c) $\alpha = 100\,\text{rad/s}^2$.
(d) $\vec{a}_p = (100\hat{\imath} - 4\hat{\jmath})\,\text{m/s}^2$.
(e) $|\vec{a}_p| = 100.08\,\text{m/s}^2$.
(f) $\frac{d|\vec{v}_a|}{dt} = -100\,\text{m/s}^2$.

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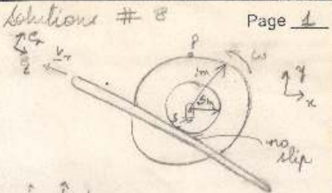
4.14



(a) Speed of point P?

$r_p = 1\hat j$

$\omega = 2\hat k$

$v_p = \omega \times r_p = \begin{vmatrix} \hat i & \hat j & \hat k \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{vmatrix} = \hat i(-2) = -2\hat i$

Speed $= |v_p| = \sqrt{4} = 2$ m/s

(b) Velocity of point P? = $\boxed{v_p = -2\hat i \text{ m/s}}$

(c) Acceleration of point P? $a_p = \frac{d}{dt}(v_p) = \frac{d}{dt}(\omega \times r_p)$

$= \omega \times \frac{d}{dt}(r_p) + \underbrace{\frac{d}{dt}(\omega)}_{0} \times r_p$ ($\because \frac{d}{dt}(\omega) = 0$)

$= \omega \times \frac{d}{dt}(r_p) = \omega \times v_p = \omega \times (\omega \times r) = \omega \times (-2\hat i)$

$= \begin{vmatrix} \hat i & \hat j & \hat k \\ 0 & 0 & 2 \\ -2 & 0 & 0 \end{vmatrix} = \hat i(0) - \hat j(+4) + \hat k(0) = -4\hat j$

$\therefore \boxed{a_p = -4 \text{ m/s}^2\, \hat j}$

(d) Velocity of rack v_r?
the velocity of the rack is the same as the instantaneous velocity of the instantaneous point of contact S.
the direction of motion of the rack is to the right and down.

$v_r = \omega \times r$

$\omega = 2\hat k$; $r = 0.5\hat e_r$

$\omega \times r = \begin{vmatrix} \hat e_r & \hat e_t & \hat k \\ 0 & 0 & 2 \\ 0.5 & 0 & 0 \end{vmatrix} = 1\hat e_t \Rightarrow \boxed{v_r = 1\hat e_t \text{ m/s}}$

(e) While points on the circumference of the lower gear have an acceleration of $(\omega \times \omega \times r)$, the motion induced to the rack is of constant velocity. Hence the rack does not accelerate. So there is no force on the rack

(Cut at this border)

USE #7B lead and bold print.

Reduce 55% and tape onto 8 1/2" x 11" sheet. (3 per page)

Copy onto 3-holed paper for student convenience.

Use 2-sides to save paper.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

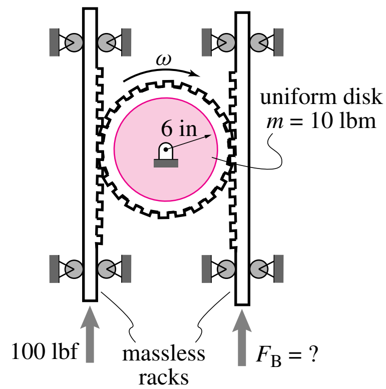
16.4.29 Two racks connected by a gear. A 100 lbf force is applied to one rack. At the output the machinery (not shown) applies a constant force F_B to the other rack.

a) Assume the gear is spinning at constant rate and is frictionless. What is F_B?

b) If the gear bearing had friction, would that increase or decrease F_B to achieve the same constant rate?

c) If the angular velocity of the gear is increasing at rate α, does this increase or decrease F_B at the given ω.

d) If the output load F_B is given then the motion of the machine can be found, assuming friction is negligible. Assume that the machine starts from rest with a given output load. So long as rack B moves down and the output force F_B is positive, the output power is positive. The machine starts from rest and runs for a fixed amount of time T. For what value of F_B is the output work maximum? Once you find an answer, can you find an intuitive explanation for the answer?



Problem 16.29: Two racks connected by a gear.

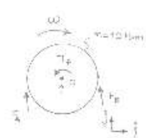
Answer:
(a) $F_B = 100\,\text{lbf}$.
(b) If gear bearing has friction, F_B has to increase.
(c) F_B will decrease if the angular velocity of the gear is increasing.
(d) For work to be maximum, $F_B = 50\,\text{lbf}$. Initially work increases with F_B but too large F_B leads to small ω_0 thus giving smaller net work.

15.4.29 : Two racks connected by a gear

(a) F_B ? (Given F_A)

Assume constant rate and no friction.

∴ Frictional torque, $T_f = 0$ N·m



AMB $\sum \vec{M}_{/O} = \dot{\vec{H}}_{/O}$

Let the radius at which the forces are acting be R.

$\sum \vec{M}_{/O} = R\hat{\imath} \times F_B\hat{\jmath} + R(-\hat{\imath}) \times F_A\hat{\jmath}$

$= F_B R\hat{k} - F_A R\hat{k}$

$\dot{\vec{H}}_{/O} = 0$

∴ $\{F_B R\hat{k} - F_A R\hat{k} = 0\}\cdot\hat{k} \Rightarrow F_B R = F_A R \Rightarrow F_B = F_A = 100$ lbf

(b) F_B = ? if $T_f \neq 0$ N·m to attain the same constant rate.

AMB $\sum \vec{M}_{/O} = \dot{\vec{H}}_{/O}$

$\Rightarrow \{R\hat{\imath} \times F_B\hat{\jmath} + R(-\hat{\imath}) \times F_A\hat{\jmath} + T_f\hat{k} = 0\}$

$\{\ \}\cdot\hat{k} \Rightarrow F_B R - F_A R + T_f = 0 \Rightarrow F_B = \left[F_A - \frac{T_f}{R}\right] < F_A$

∴ F_B decreases.

(c) F_B ? if angular velocity increases at rate α.

AMB $\sum \vec{M}_{/O} = \dot{\vec{H}}_{/O}$

$\Rightarrow R\hat{\imath} \times F_B\hat{\jmath} - R(-\hat{\imath}) \times F_A\hat{\jmath} = I\alpha(-\hat{k})$ (where I = moment of inertia of disk)

$\Rightarrow \{F_B R\hat{k} - F_A R\hat{k} = -I\alpha\hat{k}\}$

$\{\ \}\cdot\hat{k} \Rightarrow F_B R - F_A R = -I\alpha \Rightarrow F_B = \left[F_A - \frac{I\alpha}{R}\right] < F_A$

∴ F_B decreases.

(d) F_B = ? for maximum work.

Assume no friction and the angle of rotation be θ such that θ = ωT

AMB $\sum \underline{M}_{/o} = \dot{\underline{H}}_{/o}$

$\Rightarrow \quad \{ F_B R \hat{\underline{k}} - F_A R \hat{\underline{k}} = -I \dot{\omega} \hat{\underline{k}} \}$

$\{\} \cdot \hat{\underline{k}} \Rightarrow \quad I \dfrac{d\omega}{dt} = (F_A - F_B) R \quad \Rightarrow \quad I\, d\omega = (F_A - F_B) R\, dt$

Let the angular velocity at time 'T' be ω_0

$\Rightarrow \quad \omega_0 = (F_A - F_B) \dfrac{R}{I} T \qquad — ①$

Output work, $W = F_B (R\theta) = F_B R \omega_0 T$

From ①, $W = \dfrac{F_B R^2 (F_A - F_B) T^2}{I}$

Maximizing work w.r.t. F_B:

$$\frac{\partial W}{\partial F_B} = \frac{F_A R^2 T^2 - 2 F_B R^2 T^2}{I} = 0$$

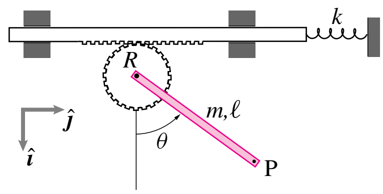
$$\Rightarrow \quad F_B = \frac{F_A}{2}$$

$\therefore$ For work to be maximum, $F_B = \dfrac{F_A}{2} = 50$ lbf.

If we apply too small F_B, initial work is small, Hence, work is small but increases with F_B. But if F_B is too large ω_0 is small, hence, less work.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

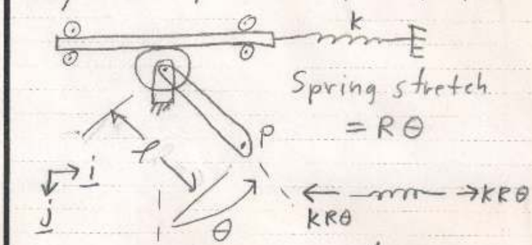
16.4.30 A rack and pinion constrained by a linear spring. Neglect gravity. The spring is relaxed when the angle $\theta = 0$. Assume the system is released from rest at $\theta = \theta_0$. What is the acceleration of the point P at the end of the stick when $\theta = 0$? Answer in terms of any or all of $m, R, \ell, \theta_0, k, \hat{\imath}$, and $\hat{\jmath}$. [Hint: There are several steps of reasoning required. You might want to draw FBD(s), use angular momentum balance, set up a differential equation, solve it, plug values into this solution, and use the result to find the quantities of interest.



Problem 16.30

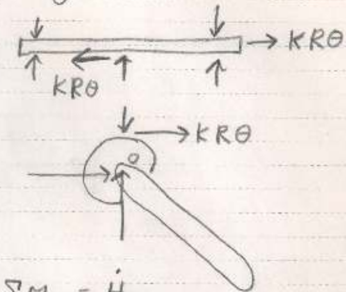
Answer:
$\vec{a}_P = -\frac{3\theta_0^2 R^2 k}{\ell m}\hat{\jmath}$

#2, Prelim 2, TAM 203, 3/28/96



Spring stretch $= R\theta$

FBDs:



AMB$_{/O}$: $\sum \underline{M}_{/O} = \dot{\underline{H}}_{/O}$

$$kR^2\theta\,\underline{k} = -\underbrace{I^O_{zz}}_{m\ell^2/3}\ddot{\theta}\,\underline{k}$$

ODE $\Rightarrow$ $\ddot{\theta} + \frac{3kR^2}{m\ell^2}\theta = 0$

SOLVE $\Rightarrow$ $\theta = A\cos\sqrt{\frac{3kR^2}{m\ell^2}}\,t + B\sin\sqrt{\bullet}\,t$

INIT. COND. $\theta = \theta_0$, $\dot{\theta} = 0$

$\Rightarrow$ $\theta = \theta_0\cos\sqrt{\frac{3kR^2}{m\ell^2}}\,t$

$\dot{\theta} = -\theta_0\sqrt{\bullet}\,\sin\sqrt{\bullet}\,t$

at $\theta = 0$ $\dot{\theta} = \pm\,\theta_0\frac{R}{\ell}\sqrt{\frac{3k}{m}}$, $\ddot{\theta} = 0$

$\underline{a}_P = -\dot{\theta}^2\ell\,\underline{j} + \cancel{\ddot{\theta}\ell\,\underline{i}}^{\,0}$

$\underline{a}_P = -\left(\theta_0^2\frac{R^2}{\ell^2}\frac{3k}{m}\right)\ell\,\underline{j}$

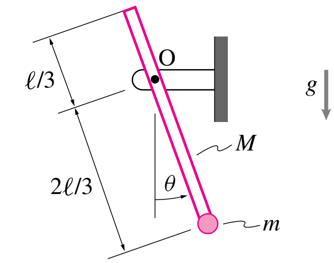
$$\boxed{\underline{a}_P = \frac{-3\theta_0^2R^2k}{\ell m}\underline{j}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.31 A small particle of mass m is attached to the end of a thin rod of mass M (uniformly distributed), which is pinned at hinge O, as depicted in the figure.

a) Obtain the equation of motion governing the rotation θ of the rod.

b) What is the natural frequency of the system for small oscillations θ?



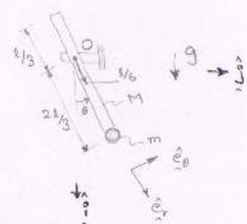
Problem 16.31

Answer:
$\left(\frac{(M+4m)\ell^2}{9}\right)\ddot{\theta} + \left(\frac{(M+4m)g\ell}{6}\right)\sin\theta = 0.$

$f = \frac{1}{2\pi}\sqrt{\frac{3g(M+4m)}{2\ell(M+4m)}}$

a) Obtain the equation of motion:

Using Angular Momentum Balance,



$$\sum \underline{M}_{/O} = \dot{\underline{H}}_{/O}$$

$$\sum \underline{M}_{/O} = \frac{l}{6}\hat{e}_r \times Mg\hat{\imath} + \frac{2l}{3}\hat{e}_r \times mg\hat{\imath}$$

$$= -\frac{l}{6}Mg\sin\theta\hat{k} - \frac{2l}{3}mg\sin\theta\hat{k}$$

$\hat{e}_r = \cos\theta\hat{\imath} + \sin\theta\hat{\jmath}$

$\hat{e}_\theta = -\sin\theta\hat{\imath} + \cos\theta\hat{\jmath}$

$$\dot{\underline{H}}_{/O} = \int \underline{r}_{/O} \times \underline{a}\, dm + (\underline{r}_{m/O} \times \underline{a}_m)\, m$$

$$= \int_{-l/3}^{2l/3} \ddot{\theta} r^2 \frac{M}{l}\hat{k}\, dr + \ddot{\theta}\frac{2l}{3} m \frac{2l}{3}$$

Let $\bar{m} = \frac{M}{L} = \frac{dm}{dr}$

$$= M\left(\frac{l}{3}\right)^2\ddot{\theta} + m\left(\frac{2l}{3}\right)^2\ddot{\theta}$$

$$\Rightarrow \left[\frac{Ml^2}{9} + \frac{4ml^2}{9}\right]\ddot{\theta} + \left[\frac{Mgl}{6} + mg\frac{2l}{3}\right]\sin\theta = 0 \quad //$$

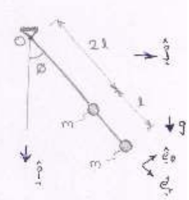
b) For small oscillations, $\sin\theta \approx \theta$,

Frequency of oscillations, $f = \frac{1}{2\pi}\sqrt{\frac{3(Mg + 4mg)}{2(Ml + 4ml)}}$

15.4.32

a) Find $\ddot{\phi}$. $\{\sum \underline{M}_{/O} = \dot{\underline{H}}_{/O}\}$

Using AMB.



$\sum \underline{M}_{/O} = 2l\hat{e}_r \times mg\hat{\imath} + 3l\hat{e}_r \times mg\hat{\imath} = -5mgl\sin\phi\hat{k}$

$\dot{\underline{H}}_{/O} = \underline{r}_{1/O} \times \underline{a}_{m_1} m_1 + \underline{r}_{2/O} \times \underline{a}_{m_2} m_2$

$= (4ml^2\ddot{\phi} + 9ml^2\ddot{\phi})\hat{k}$

$\Rightarrow \{\}\cdot\hat{k} \Rightarrow \ddot{\phi} = \frac{-5g}{13l}\sin\phi$

$\hat{e}_r = \cos\phi\hat{\imath} + \sin\phi\hat{\jmath}$

$\hat{e}_\theta = -\sin\phi\hat{\imath} + \cos\phi\hat{\jmath}$

b) Using LMB $\sum \underline{F} = \dot{\underline{L}}$

$\underline{R} + mg\hat{\imath} + mg\hat{\imath} = m[\ddot{\phi}2l\hat{e}_\theta - \dot{\phi}^2 2l\hat{e}_r] + m[\ddot{\phi}3l\hat{e}_\theta - \dot{\phi}^2 3l\hat{e}_r]$

$\Rightarrow \underline{R} = -2mg\hat{\imath} - 5ml[\ddot{\phi}\sin\phi + \dot{\phi}^2\cos\phi]\hat{\imath} + 5ml[\ddot{\phi}\cos\phi - \dot{\phi}^2\sin\phi]\hat{\jmath}$ //

c) We would not get same answers if $2m$ is put at $2.5l$, as $\dot{H}_{/O}$ reduces. We get $\ddot{\phi} = \frac{-5g}{12.5l}\sin\phi$.

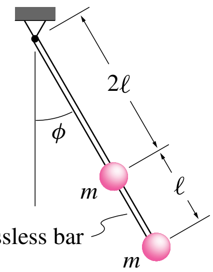
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.32 A rigid massless rod has two equal masses m_B and m_C ($m_B = m_C = m$) attached to it at distances 2ℓ and 3ℓ, respectively, measured along the rod from a frictionless hinge located at a point A. The rod swings freely from the hinge. There is gravity. Let ϕ denote the angle of the rod measured from the vertical. Assume that ϕ and $\dot{\phi}$ are known at the instant of interest.

a) What is $\ddot{\phi}$? Find $\ddot{\phi}$ in terms of m, ℓ, g, ϕ and $\dot{\phi}$.

b) What is the force of the hinge on the rod? Solve in terms of m, ℓ, g, ϕ, $\dot{\phi}$, $\ddot{\phi}$ and any unit vectors you may need to define.

c) Would you get the same answers if you put a mass $2m$ at 2.5ℓ? Why or why not?



Problem 16.32

Answer:
$\ddot{\phi} = -\frac{5g}{13\ell}\sin\phi$.
$\vec{\boldsymbol{F}} = -5m\ell(\dot{\phi}^2 cos\phi + \ddot{\phi}\sin\phi)\hat{\boldsymbol{\imath}} - 2mg\hat{\boldsymbol{\imath}} + 5ml(\ddot{\phi}\cos\phi - \dot{\phi}^2\sin\phi)\hat{\boldsymbol{\jmath}}$.
Answer will change because although total moment of force about the hinge stays same but moment of inertia reduces. In this case we get $\ddot{\phi} = -\frac{5g}{12.5\ell}\sin\phi$.

15.4.32

a) Find $\ddot{\phi}$.

Using AMB. $\left\{\sum \underline{M}_{/O} = \dot{\underline{H}}_{/O}\right\}$

$\sum \underline{M}_{/O} = 2l\hat{e}_r \times mg\hat{\imath} + 3l\hat{e}_r \times mg\hat{\imath} = -5mgl\sin\phi\hat{k}$

$\dot{\underline{H}}_{/O} = \underline{r}_{1/O} \times \underline{a}_{m_1} m_1 + \underline{r}_{2/O} \times \underline{a}_{m_2} m_2$

$= (4ml^2\ddot{\phi} + 9ml^2\ddot{\phi})\hat{k}$

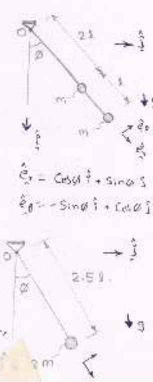
$\Rightarrow \{\}\cdot\hat{k} \Rightarrow \ddot{\phi} = \frac{-5g}{13l}\sin\phi$

b) Using LMB $\sum \underline{F} = \dot{\underline{L}}$

$\underline{R} + mg\hat{\imath} + mg\hat{\imath} = m[\ddot{\phi}2l\hat{e}_\theta - \dot{\phi}^2 2l\hat{e}_r] + m[\ddot{\phi}3l\hat{e}_\theta - \dot{\phi}^2 3l\hat{e}_r]$

$\Rightarrow \underline{R} = -2mg\hat{\imath} - 5ml[\ddot{\phi}\sin\phi + \dot{\phi}^2\cos\phi]\hat{\imath} + 5ml[\ddot{\phi}\cos\phi - \dot{\phi}^2\sin\phi]\hat{\jmath}$

c) We would not get same answers if $2m$ is put at $2.5l$, as $\dot{H}_{/O}$ reduces. We get $\ddot{\phi} = \frac{-5g}{12.5l}\sin\phi$.

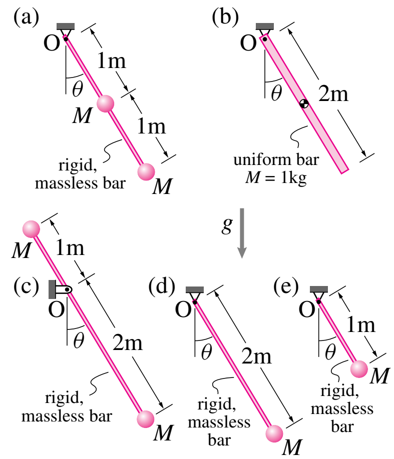


16.4.33 For the pendula in the figure :

a) Without doing any calculations, try to figure out the relative durations of the periods of oscillation for the five pendula (i.e. the order, slowest to fastest) Assume small angles of oscillation.

b) Calculate the period of small oscillations. [Hint: use balance of angular momentum about the point 0].

c) Rank the relative duration of oscillations and compare to your intuitive solution in part (a), and explain in words why things work the way they do.



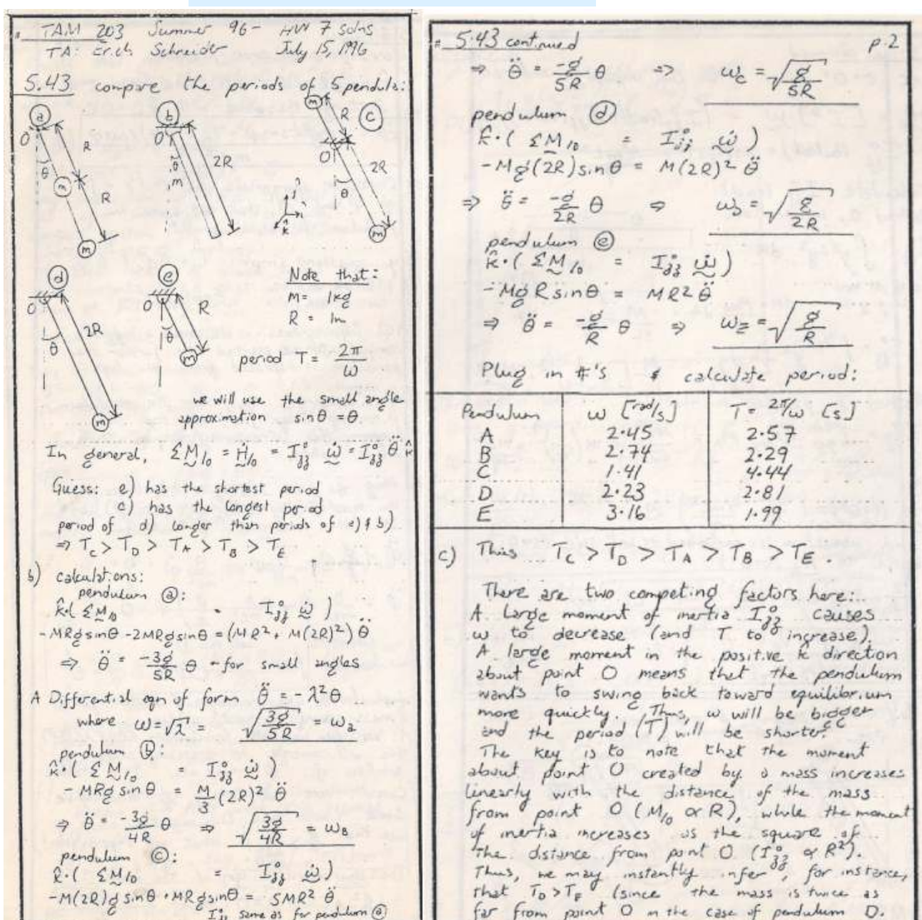
Problem 16.33

Answer:
(b)

(b) $T = 2.29$ s

(e) $T = 1.99$ s

(b) has a longer period than (e) does since in (b) the moment of inertia about the center of mass (located at the same position as the mass in (e)) is non-zero.



TAM 203 Summer 96 – HW 7 Solns
TA: Erich Schreider July 15, 1996

5.43 compare the periods of 5 pendula:

Note that:
M = 1kg
R = 1m

period $T = \frac{2\pi}{\omega}$

we will use the small angle approximation $\sin\theta = \theta$

In general, $\sum \underset{\sim}{M}_{/O} = \dot{\underset{\sim}{H}}_{/O} = I^O_{zz}\, \dot{\underset{\sim}{\omega}} = I^O_{zz}\,\ddot{\theta}\,\hat{k}$

Guess: e) has the shortest period
c) has the longest period
period of d) longer than periods of a) & b)
$\Rightarrow T_C > T_D > T_A > T_B > T_E$

b) calculations:
pendulum a:
$\hat{k}\cdot(\sum \underset{\sim}{M}_{/O} = I^O_{zz}\, \dot{\underset{\sim}{\omega}})$
$-MRg\sin\theta - 2MRg\sin\theta = (MR^2 + M(2R)^2)\,\ddot{\theta}$
$\Rightarrow \ddot{\theta} = \frac{-3g}{5R}\theta$ ← for small angles

A Differential eqn of form $\ddot{\theta} = -\lambda^2\theta$
where $\omega = \sqrt{\lambda^2} = \sqrt{\frac{3g}{5R}} = \omega_a$

pendulum b:
$\hat{k}\cdot(\sum \underset{\sim}{M}_{/O} = I^O_{zz}\, \dot{\underset{\sim}{\omega}})$
$-MRg\sin\theta = \frac{M}{3}(2R)^2\,\ddot{\theta}$
$\Rightarrow \ddot{\theta} = \frac{-3g}{4R}\theta \Rightarrow \sqrt{\frac{3g}{4R}} = \omega_B$

pendulum c:
$\hat{k}\cdot(\sum \underset{\sim}{M}_{/O} = I^O_{zz}\, \dot{\underset{\sim}{\omega}})$
$-M(2R)g\sin\theta + MRg\sin\theta = 5MR^2\,\ddot{\theta}$
I^O_{zz} same as for pendulum a

5.43 continued p.2

$\Rightarrow \ddot{\theta} = \frac{-g}{5R}\theta \Rightarrow \omega_C = \sqrt{\frac{g}{5R}}$

pendulum d
$\hat{k}\cdot(\sum \underset{\sim}{M}_{/O} = I^O_{zz}\, \dot{\underset{\sim}{\omega}})$
$-Mg(2R)\sin\theta = M(2R)^2\,\ddot{\theta}$
$\Rightarrow \ddot{\theta} = \frac{-g}{2R}\theta \Rightarrow \omega_D = \sqrt{\frac{g}{2R}}$

pendulum e
$\hat{k}\cdot(\sum \underset{\sim}{M}_{/O} = I^O_{zz}\, \dot{\underset{\sim}{\omega}})$
$-MgR\sin\theta = MR^2\,\ddot{\theta}$
$\Rightarrow \ddot{\theta} = \frac{-g}{R}\theta \Rightarrow \omega_E = \sqrt{\frac{g}{R}}$

Plug in #'s & calculate period:

Pendulum	ω [rad/s]	$T = 2\pi/\omega$ [s]
A	2.45	2.57
B	2.74	2.29
C	1.41	4.44
D	2.23	2.81
E	3.16	1.99

c) Thus $T_C > T_D > T_A > T_B > T_E$.

There are two competing factors here:
A large moment of inertia I^O_{zz} causes ω to decrease (and T to increase).
A large moment in the positive $\hat{k}$ direction about point O means that the pendulum wants to swing back toward equilibrium more quickly. Thus, ω will be bigger and the period (T) will be shorter.
The key is to note that the moment about point O created by a mass increases linearly with the distance of the mass from point O ($M_{/O} \propto R$), while the moment of inertia increases as the square of the distance from point O ($I^O_{zz} \propto R^2$).
Thus, we may instantly infer, for instance, that $T_D > T_E$ (since the mass is twice as far from point O in the case of pendulum D.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.34 A pegged compound pendulum. A uniform bar of mass m and length ℓ hangs from a peg at point C and swings in the vertical plane about an axis passing through the peg. The distance d from the center of mass of the rod to the peg can be changed by putting the peg at some other point along the length of the rod.

a) Find the angular momentum of the rod about point C.

b) Find the rate of change of angular momentum of the rod about C.

c) How does the period of the pendulum vary with d? Show the variation by plotting the period against $\frac{d}{\ell}$. [Hint, you must first find the equations of motion, linearize for small θ, and then solve.]

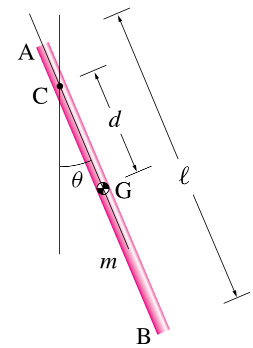
d) Find the total energy of the rod (using point C as a datum for potential energy).

e) Find $\ddot{\theta}$ when $\theta = \pi/6$.

f) Find the reaction force on the rod at C, as a function of m, d, ℓ, θ, and $\dot{\theta}$.

g) For the given rod, what should be the value of d (in terms of ℓ) in order to have the fastest pendulum?

h) **Test of Schuler's pendulum.** The pendulum with the value of d obtained in (g) is called the Schuler's pendulum. It is not only the fastest pendulum but also the "most accurate pendulum". The claim is that even if d changes slightly over time due to wear at the support point, the period of the pendulum does not change much. Verify this claim by calculating the percent error in the time period of a pendulum of length $\ell = 1\,\mathrm{m}$ under the following three conditions: (i) initial $d = 0.15\,\mathrm{m}$ and after some wear $d = 0.16\,\mathrm{m}$, (ii) initial $d = 0.29\,\mathrm{m}$ and after some wear $d = 0.30\,\mathrm{m}$, and (iii) initial $d = 0.45\,\mathrm{m}$ and after some wear $d = 0.46\,\mathrm{m}$. Which pendulum shows the least error in its time period? What is the connection between this result and the plot obtained in (c)?



Problem 16.34

Answer:

(a) $\vec{\boldsymbol{H}}_c = \left(\frac{m\ell^2}{12} + md^2\right)\dot{\theta}\hat{\boldsymbol{k}}$.

(b) $\dot{\vec{\boldsymbol{H}}}_c = \left(\frac{m\ell^2}{12} + md^2\right)\ddot{\theta}\hat{\boldsymbol{k}}$.

(c) $T = \frac{2\pi}{\sqrt{g/\ell}}\sqrt{\frac{1}{12(d/\ell)} + \frac{d}{\ell}}$

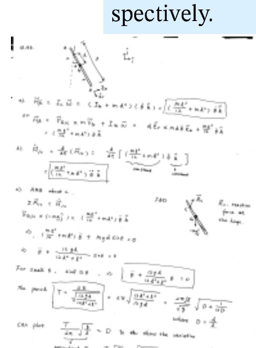
(d) $E_T = \frac{1}{2}\left(\frac{m\ell^2}{12} + md^2\right)\dot{\theta}^2 - mgd\cos\theta$.

(e) $\ddot{\theta} = -\frac{6gd}{12d^2+\ell^2}$.

(f) $\vec{\boldsymbol{R}}_c = -m\left(\frac{12gd}{12d^2+\ell^2}\cos\theta\sin\theta + \dot{\theta}^2 d\sin\theta\right)\hat{\boldsymbol{\imath}} + \left(mg - \frac{12mgd}{12d^2+\ell^2}\sin\theta^2 + m\dot{\theta}^2 d\cos\theta\right)\hat{\boldsymbol{\jmath}}$.

(g) $d = 0.29\,\ell$

(h) The error in the time period of the pendulum for the three cases is (i) 1.767%, (ii) 0.034%, and (iii) 0.470% respectively.



Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

d). Use the height of point c as a datumn for potential energy.

At a position with angle θ.

$$E_p = -mgd\cos\theta$$

$$E_k = \frac{1}{2}mV_G^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}I_c\omega^2 = \frac{1}{2}\left(md^2 + \frac{m\ell^2}{12}\right)\dot\theta^2$$

$$\therefore E_T = E_k + E_p = \boxed{\frac{1}{2}\left(md^2 + \frac{m\ell^2}{12}\right)\dot\theta^2 - mgd\cos\theta}$$

e). $\theta = \frac{\pi}{6} \Rightarrow \sin\theta = \frac{1}{2}$

$\because \ddot\theta + \frac{12gd}{12d^2+\ell^2}\sin\theta = 0$ is satisfied all the time

$$\therefore \ddot\theta = -\frac{12gd}{12d^2+\ell^2}\sin\theta = \boxed{-\frac{6gd}{12d^2+\ell^2}} \quad \text{if } \theta = \frac{\pi}{6}$$

f). Use LMB

$$\Sigma\vec F = m\vec a_G \Rightarrow \vec R_c - mg\hat j = m\vec a_G$$

where $\vec a_G = \ddot\theta d\hat e_\theta - \dot\theta^2 d\hat e_r = \left(-\frac{12gd}{12d^2+\ell^2}\sin\theta\,\hat e_\theta\right) - \dot\theta^2 d\hat e_r$

$\because \hat e_\theta = \cos\theta\hat\imath + \sin\theta\hat\jmath$, $\hat e_r = \sin\theta\hat\imath - \cos\theta\hat\jmath$

$$\Rightarrow \vec a_G = -\left(\frac{12gd}{12d^2+\ell^2}\cos\theta\sin\theta + \dot\theta^2 d\sin\theta\right)\hat\imath - \left(\frac{12gd}{12d^2+\ell^2}\sin^2\theta - \dot\theta^2 d\cos\theta\right)\hat\jmath$$

$\therefore$ The reaction force

$$\boxed{\vec R_c = -m\left(\frac{12gd}{12d^2+\ell^2}\cos\theta\sin\theta + \dot\theta^2 d\sin\theta\right)\hat\imath + \left(mg - \frac{12mgd}{12d^2+\ell^2}\sin^2\theta + m\dot\theta^2 d\cos\theta\right)\hat\jmath}$$

g). $\because T = 2\pi\sqrt{\frac{12d^2+\ell^2}{12gd}}$

To find the minimum T, set $\frac{\partial T}{\partial d} = 0$

$$\because \frac{\partial T}{\partial d} = 2\pi\sqrt{\frac{12gd}{12d^2+\ell^2}}\left(\frac{1}{g} - \frac{\ell^2}{12gd^2}\right) = 0$$

$$\Rightarrow \frac{1}{g} - \frac{\ell^2}{12gd^2} = 0 \Rightarrow \boxed{d_m = \sqrt{\frac{1}{12}}\,\ell} \simeq 0.2887\ell$$

When $d < d_m$, $\frac{\partial T}{\partial d} < 0$; when $d > d_m$, $\frac{\partial T}{\partial d} > 0$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$\therefore\ d_m = \sqrt{\frac{1}{12}}\,\ell$ is the minimum point for T.

h). $\ell = 1\,m$, $g = 9.8\,m/s^2$, $T = 2\pi\sqrt{\frac{12d^2+\ell^2}{12gd}} = \frac{2\pi\sqrt{\ell}}{\sqrt{g}}\sqrt{D+\frac{1}{12D}}$ $\boxed{D=\frac{d}{\ell}}$

i). ~~initid~~ initial $d_o = 0.15\,m$

$T_o = 1.6859\,s$

after some wear, $d = 0.16\,m$, $T = 1.6561\,s$

error $\left|\frac{T-T_o}{T_o}\right| = 1.767\,\%$

ii) initial $d_o = 0.29\,m$, $T_o = 1.5251\,s$

after some wear $d = 0.30\,m$, $T = 1.5256\,s$

error $\left|\frac{T-T_o}{T_o}\right| = 0.0343\,\%$

iii) initial $d_o = 0.45\,m$, $T_o = 1.5996\,s$

after some wear $d = 0.46\,m$, $T = 1.6071\,s$

error $\left|\frac{T-T_o}{T_o}\right| = 0.470\,\%$

The second case where $d_o = 0.29\,m$ shows the least error.

Since $d_o = 0.29\,m$ is close to $d_m = \sqrt{\frac{1}{12}}\,\ell \simeq 0.2887\,\ell$,

and

$$\Delta T = |T - T_o| \approx \left|\left.\frac{\partial T}{\partial d}\right|_{d=d_o}(d-d_o)\right|$$

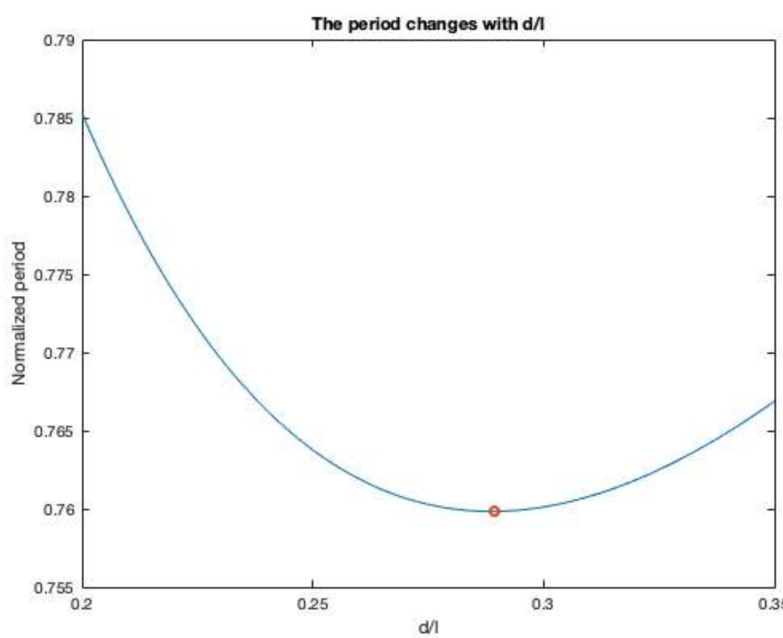
For all the 3 cases, $d - d_o = 0.1\,m$. However, $d_o = 0.29\,m$ is close to $d_m \simeq 0.2887\,\ell$ where $\frac{\partial T}{\partial d} = 0$. So $\left|\left.\frac{\partial T}{\partial d}\right|_{d=d_o}\right|$ is the least when $d_o = 0.29\,m$.

From the graph given in c). one can see the slope near $d_m = \sqrt{\frac{1}{12}}\,\ell$ is very small, that is, the curve is flat near d_m.

```
clc;
clear;
close all;

% Define D = d/l, ranging from 0 to 0.5
% Pick D ranging from 0.15 to 0.35 to clearly see approximate minimum
D = linspace(0.2,0.35,100);
% Plot the normalized period vs D=d/l;
T = sqrt(D+1./(12*D));
plot(D,T)
hold on
title('The period changes with d/l');
xlabel('d/l');
ylabel('Normalized period');
% Find numerical/approximate mim, for analytical/acurate min see
 derivative
Tmin = min(T);
Dmin = D(find(T==Tmin));
plot(Dmin,Tmin,'o');
fprintf('Mimimum normalized time period is %.4f\n',Tmin);
fprintf('When D is equal to %.4f\n',Dmin);

Mimimum normalized time period is 0.7598
When D is equal to 0.2894
```



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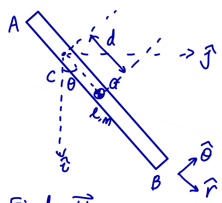
16.4.34 HW9 P52

Teaya Yang



* $\hat{r}$ and $\hat{\theta}$ used here are the same as $\hat{e}_r$ and $\hat{e}_\theta$ used in lecture.

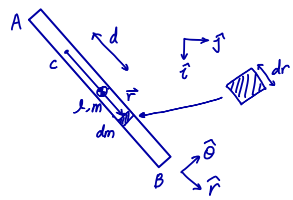
a) Find $\vec{H}_{/C}$.

Approach # 1:

$\vec{H}_{/C} = \int \vec{r}_{/C} \times \vec{v}\, dm$ let $\vec{r}_{/C} = r\hat{r}$, where r is distance defined from C

$\vec{H}_{/C} = \int r\hat{r} \times r\dot{\theta}\hat{\theta}\, dm$ then $\vec{v} = r\dot{\theta}\hat{\theta}$

$\vec{H}_{/C} = \int r^2\dot{\theta}\,\hat{k}\, dm$



$= \int_{d-\frac{\ell}{2}}^{d+\frac{\ell}{2}} r^2\dot{\theta}\,\hat{k}\,\frac{m}{\ell}\,dr$

$dm = \underbrace{\frac{m}{\ell}}_{\text{mass/length}} dr$

$= \int_{d-\frac{\ell}{2}}^{d+\frac{\ell}{2}} \frac{m}{\ell}\dot{\theta}\, r^2\,\hat{k}\, dr$

or can think of as $\frac{dm}{m} = \frac{dr}{\ell}$

mass ratio = length ratio

$= \frac{1}{3}\frac{m}{\ell}\dot{\theta}\, r^3\,\hat{k}\,\Big|_{d-\frac{\ell}{2}}^{d+\frac{\ell}{2}}$ $\vec{r}_{A/C} = -(\frac{\ell}{2}-d)\,\hat{r}$

$\vec{r}_{B/C} = d+\frac{\ell}{2}\,\hat{r}$ $\Rightarrow d-\frac{\ell}{2} \leq r \leq d+\frac{\ell}{2}$

$= \frac{1}{3}\frac{m}{\ell}\dot{\theta}\left[(d+\frac{\ell}{2})^3-(d-\frac{\ell}{2})^3\right]\hat{k}$

$= \frac{1}{3}\frac{m}{\ell}\dot{\theta}\left[\cancel{d^3}+3d^2(\frac{\ell}{2})+\cancel{3d(\frac{\ell}{2})^2}+(\frac{\ell}{2})^3\cancel{-d^3}+3d^2(\frac{\ell}{2})\cancel{-3d(\frac{\ell}{2})^2}+(\frac{\ell}{2})^3\right]\hat{k}$

$= \frac{1}{3}\frac{m}{\ell}\dot{\theta}\left(3d^2\ell+\frac{\ell^3}{4}\right)\hat{k}$

$$\boxed{\vec{H}_{/C} = m\dot{\theta}\left(d^2+\frac{\ell^2}{12}\right)\hat{k}}$$

Approach # 2:

$\vec{H}_{/C} = I^C\vec{\omega}$ $I^C = \int r^2\,dm = \int_{d-\frac{\ell}{2}}^{d+\frac{\ell}{2}} r^2\left(\frac{m}{\ell}\,dr\right)$

$= \underbrace{\dot{\theta}\hat{k}\int_{d-\frac{\ell}{2}}^{d+\frac{\ell}{2}} r^2\left(\frac{m}{\ell}\right)dr}_{\text{same integral as approach \#1.}}$ $\vec{\omega} = \dot{\theta}\,\hat{k}$

b) $\dot{\vec{H}}_{/C} = \frac{d}{dt}\vec{H}_{/C}$

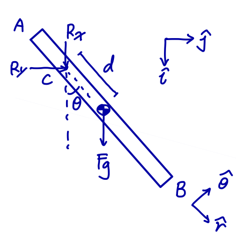
$= \frac{d}{dt}\left[m\dot{\theta}\left(d^2+\frac{\ell^2}{12}\right)\hat{k}\right]$

$$\boxed{\dot{\vec{H}}_{/C} = m\ddot{\theta}\left(d^2+\frac{\ell^2}{12}\right)\hat{k}}$$

c) How does period T depend on d? Plot T vs. $\frac{d}{\ell}$.

Plan: First find EOM, then determine ω and calculate T.

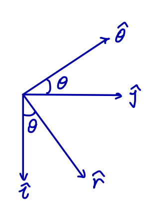
FBD:



$\Sigma\vec{M}_{/C} = \vec{r}_{G/C} \times \vec{F}_g$

$= d\,\hat{r} \times mg\,\hat{\imath}$

$= -\,d\,mg\sin\theta\,\hat{k}$



AMB : $\Sigma \vec{M}_{/c} = \dot{\vec{H}}_{/c}$

$$-d\cancel{m}g\sin\theta\,\cancel{\hat{k}} = \cancel{m}\ddot{\theta}\left(d^2+\frac{\ell^2}{12}\right)\cancel{\hat{k}}$$

$$-dg\sin\theta = \ddot{\theta}\left(d^2+\frac{\ell^2}{12}\right)$$

$$-\sin\theta = \ddot{\theta}\left(\frac{12d^2+\ell^2}{12dg}\right)$$

Small angle approx. : $\sin\theta \approx \theta$

$$\ddot{\theta} = -\left(\frac{12dg}{12d^2+\ell^2}\right)\theta$$

Recall simple harmonic oscillator : $\ddot{x} = -\omega^2 x$

Here : $\omega = \sqrt{\dfrac{12dg}{12d^2+\ell^2}}$

For period : $\boxed{T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{12d^2+\ell^2}{12dg}}}$

To see how T depends on $\frac{d}{\ell}$, set $\frac{d}{\ell} = D$

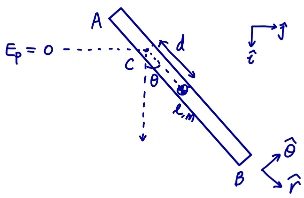
$$T = 2\pi\sqrt{\frac{(12d^2+\ell^2)/\ell^2}{12dg/\ell^2}} = 2\pi\sqrt{\frac{12(\frac{d}{\ell})^2+1}{12(\frac{d}{\ell})g}\,\ell} = 2\pi\sqrt{\frac{12D^2+1}{12D}\,\frac{\ell}{g}}$$

$T = 2\pi\sqrt{\frac{\ell}{g}}\sqrt{\frac{12D^2+1}{12D}} \longrightarrow \frac{T}{2\pi}\sqrt{\frac{g}{\ell}}$ vs. D is plotted on the next page.

S

"Normalized Period" is $\frac{T}{2\pi}\sqrt{\frac{g}{\ell}}$

d) Find total energy of rod. Use C as a datum for E_p.



$E_k = \int \frac{1}{2} v^2 dm$ since it's the sum of KE of all particles.

$\vec{V} = r\dot{\theta}\hat{\theta} \rightarrow |\vec{V}| = v = r\dot{\theta}$

$d - \frac{l}{2} \leq r \leq d + \frac{l}{2}$

$dm = \frac{m}{l} dr$

$$E_k = \int_{d-\frac{l}{2}}^{d+\frac{l}{2}} \frac{1}{2}(r\dot{\theta})^2 \left(\frac{m}{l} dr\right)$$

$$= \frac{1}{2}\dot{\theta}^2 \frac{m}{l} \int_{d-\frac{l}{2}}^{d+\frac{l}{2}} r^2 dr$$

$$= \frac{1}{2}\dot{\theta}^2 \frac{m}{l} \left(\frac{1}{3} r^3 \Big|_{d-\frac{l}{2}}^{d+\frac{l}{2}}\right)$$

$$= \frac{1}{2}\dot{\theta}^2 \frac{m}{l} \frac{1}{3}\left[\left(d+\frac{l}{2}\right)^3 - \left(d-\frac{l}{2}\right)^3\right]$$

$$= \frac{1}{6}\frac{m}{l}\dot{\theta}^2 \left(3d^2 l + \frac{l^3}{4}\right)$$

$$E_k = \frac{1}{2} m\dot{\theta}^2 \left(d^2 + \frac{1}{12} l^2\right)$$

$E_p = -mg\,(d\cos\theta)$ PE of COM.

$$E_{tot} = E_p + E_k$$

$$= \boxed{\frac{1}{2} m\dot{\theta}^2 \left(d^2 + \frac{1}{12} l^2\right) - mgd\cos\theta}$$

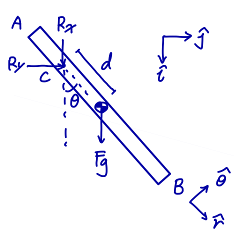
e) Find $\ddot{\theta}$ for $\theta = \frac{\pi}{6}$

$$\ddot{\theta} = -\left(\frac{12dg}{12d^2 + l^2}\right)\sin\theta$$

$$\ddot{\theta} = -\frac{12dg}{12d^2 + l^2}\sin\left(\frac{\pi}{6}\right) \qquad \boxed{\ddot{\theta} = -\frac{6dg}{12d^2 + l^2}}$$

f) Find reaction force at C as function of $m, d, l, \theta, \dot{\theta}$.

FBD:

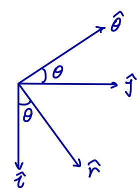


More kinematics:

since $\vec{V}_G = d\dot{\theta}\hat{\theta}$,

$$\vec{a}_G = d\ddot{\theta}\hat{\theta} - d\dot{\theta}^2\hat{r}$$

$$= d\ddot{\theta}(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}) - d\dot{\theta}^2(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$



$$\vec{a}_G = (-d\ddot{\theta}\sin\theta - d\dot{\theta}^2\cos\theta)\hat{\imath} + (d\ddot{\theta}\cos\theta - d\dot{\theta}^2\sin\theta)\hat{\jmath}$$

LMB: $\Sigma\vec{F} = m\vec{a}_G$

$$\{R_x\hat{\imath} + R_y\hat{\jmath} + mg\hat{\imath} = m\vec{a}_G\}$$

$\{\}\cdot\hat{\imath}$: $\quad R_x + mg = m(-d\ddot{\theta}\sin\theta - d\dot{\theta}^2\cos\theta)$

$$R_x = -m(d\ddot{\theta}\sin\theta + d\dot{\theta}^2\cos\theta + g)$$

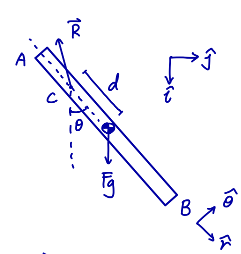
$\{\}\cdot\hat{\jmath}$: $\quad R_y = m(d\ddot{\theta}\cos\theta - d\dot{\theta}^2\sin\theta)$

But we also know that $\ddot{\theta} = -\left(\frac{12dg}{12d^2 + l^2}\right)\sin\theta$

Putting everything together:

$$\vec{R} = R_x\hat{\imath} + R_y\hat{\jmath}$$

$$\boxed{\vec{R} = m\left[\left(\frac{12d^2 g}{12d^2 + l^2}\right)\sin^2\theta - d\dot{\theta}^2\cos\theta - g\right]\hat{\imath} - m\left[\left(\frac{12d^2 g}{12d^2 + l^2}\right)\sin\theta\cos\theta + d\dot{\theta}^2\sin\theta\right]\hat{\jmath}}$$



$\vec{R}$ is always between the vertical and the line paralel to the rod.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

g) What's the value of d (in terms of ℓ) that makes the fastest pendulum?

From b), ratio $D=\frac{d}{\ell} = 0.289$ leads to minimum period T, which means fastest pendulum.
An analytical approach is outlined below.

$$T = 2\pi\sqrt{\frac{\ell}{g}}\sqrt{\frac{12D^2+1}{12D}}$$ to minimize T, $\frac{12D^2+1}{D}$ needs to be minimized.

$$\frac{\partial}{\partial D}\left(\frac{12D^2+1}{D}\right) = 0$$

$$\frac{24D^2 - 12D^2 - 1}{D^2} = 0$$ and the numerator should be 0.

$$12D^2 - 1 = 0 \rightarrow D = \sqrt{\tfrac{1}{12}}$$ since D cannot be negative by definition.

Check if $D = \sqrt{\frac{1}{12}}$ is a minimum:

when $D < \sqrt{\frac{1}{12}}$, $\frac{\partial T}{\partial D} < 0$; when $D > \sqrt{\frac{1}{12}}$, $\frac{\partial T}{\partial D} > 0$ $\Rightarrow$ This is a minimum

So period is minimized when $d = \sqrt{\frac{1}{12}}\,\ell = 0.2887\,\ell$ → matches numerical solution

h) Verify that the Schuler's pendulum is accurate by calculating percent error. Use $\ell = 1m$.

(i) $d_1 = 0.15\,m \longrightarrow d_2 = 0.16m$

$$T_1 = 2\pi\sqrt{\frac{\ell}{g}}\sqrt{\frac{12D^2+1}{12D}} = 2\pi\sqrt{\frac{1m}{9.81m/s^2}}\sqrt{\frac{12(0.15)+1}{12(0.15}} = 1.685\text{ s}$$

$T_2 = 1.655$ s

$$\text{error} = \left|\frac{T_2 - T_1}{T_1}\right| = \boxed{1.77\%}$$

(ii) $d_1 = 0.29m \rightarrow d_2 = 0.30m$

$T_1 = 1.5243$ s

$T_2 = 1.5248$ s

error = $\boxed{0.0365\%}$

Note: the slope of T vs. D is 0 at the minimum, so linearized sensitivity measured at min. is 0. This 0.0365% difference comes from the quadratic dependence.

(iii) $d_1 = 0.45\,m \rightarrow d_2 = 0.46m$

$T_1 = 1.5988$ s

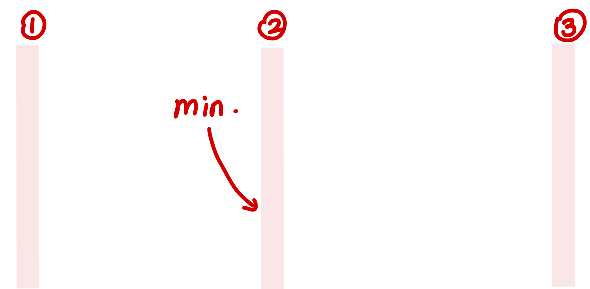
$T_2 = 1.6063$ s

error = $\boxed{0.4692\%}$

The second case has the smallest percent error by far, and that's because the d values are the closest to $d = \sqrt{\frac{1}{12}}$. If we plot the same curve in part b) from 0 to 0.5, the slope of the curve is very small near the minimum, and the slope is 0 at the minimum.

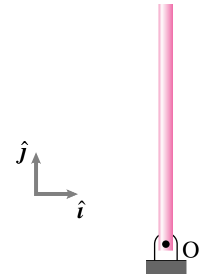
See plot on next page

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*



Also note how the slope at d values for case 1 is steeper than that in case 3. This is why the percent error is larger for case 1.

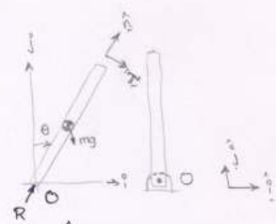
16.4.35 A uniform stick of length ℓ and mass m is a hair away from vertically up position when it is released with no angular velocity (a 'hair' is a technical word that means 'very small amount, zero for some purposes'). It falls to the right. What is the force on the stick at point O when the stick is horizontal. Solve in terms of ℓ, m, g, $\hat{\imath}$, and $\hat{\jmath}$. Carefully define any coordinates, base vectors, or angles that you use.



Problem 16.35

Answer:
$\vec{R} = -\frac{m\ell}{2}\dot{\theta}^2\hat{\imath} + \frac{7mg}{4}\hat{\jmath}$.

Apply AMB about 'O' $\sum \underset{\sim}{M}_{/O} = \dot{\underset{\sim}{H}}_{/O}$



=> $\frac{l}{2}\hat{n} \times mg(-\hat{j}) = \dot{\underset{\sim}{H}}_{/O}$

=> $\left\{-mg\frac{l}{2}\sin\theta\,\hat{k} = \ddot{\theta}\,\frac{1}{3}ml^2\,\hat{k}\right\}$

$\{\ \}\cdot\hat{k}$ => $\ddot{\theta} = -\frac{3g}{2l}\sin\theta$

$\hat{\lambda} = \cos\theta\,\hat{i} - \sin\theta\,\hat{j}$

$\hat{n} = \sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

When the stick is horizontal, $\ddot{\theta} = -\frac{3g}{2l}$; $\hat{n} = \hat{i}$; $\hat{\lambda} = -\hat{j}$

Applying LMB, $\sum F = m\underset{\sim}{a}$

=> $\underset{\sim}{R} - mg\,\hat{j} = m\underset{\sim}{a}$ => $\underset{\sim}{R} = mg\,\hat{j} + m\left[\ddot{\theta}\frac{l}{2}\hat{\lambda} - \dot{\theta}^2\frac{l}{2}\hat{n}\right]$

=> Reaction, $\underset{\sim}{R} = -m\dot{\theta}^2\frac{l}{2}\,\hat{i} + \frac{7mg}{4}\,\hat{j}$

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16.4.36 A massless 10 meter long bar is supported by a frictionless hinge at one end and has a 3.759 kg point mass at the other end. It is released at $t = 0$ from a tip angle of $\phi = .02$ radians measured from vertically upright position (hinge at the bottom). Use $g = 10\text{m s}^{-2}$.

a) Using a small angle approximation and the solution to the resulting linear differential equation, find the angle of tip at $t = 1$s and $t = 7$s. Use a calculator, not a numerical integrator.

b) Using numerical integration of the non-linear differential equation for an inverted pendulum find ϕ at $t = 1$s and $t = 7$s.

c) Make a plot of the angle versus time for your numerical solution. Include on the same plot the angle versus time from the approximate linear solution from part (a).

d) Comment on the similarities and differences in your plots.

Answer:
(a) Angle of tip is 0.031 rad at $t = 1$ s and 10.967 rad at $t = 7$ s.

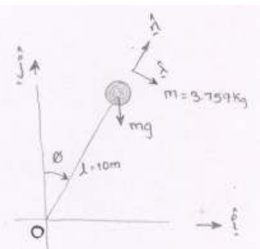
a) $\sum \underline{M}_{/O} = \dot{\underline{H}}_{/O}$ {Applying AMB about 'O'}

$\Rightarrow l\hat{\underline{n}} \times mg(-\hat{\underline{j}}) = l\hat{\underline{n}} \times m\left[\dot{\underline{\omega}} \times l\hat{\underline{n}} + \underline{\omega} \times (\underline{\omega} \times l\hat{\underline{n}})\right]$

$\Rightarrow -l\hat{\underline{n}} \times mg\hat{\underline{j}} = l\hat{\underline{n}} \times m\left[\ddot{\phi} l \hat{\underline{\lambda}} - \dot{\phi}^2 l \hat{\underline{n}}\right]$

$\Rightarrow \left\{-mgl\sin\phi\,\hat{\underline{k}} = -\ddot{\phi} m l^2 \hat{\underline{k}}\right\}$

$\{\ \}\cdot\hat{\underline{k}} \Rightarrow \quad \ddot{\phi} = \frac{g}{l}\sin\phi$



$\hat{\underline{n}} = \sin\phi\,\hat{\underline{i}} + \cos\phi\,\hat{\underline{j}}$

Assuming small angle approximation, $\phi \approx \sin\phi$

$\Rightarrow \quad \ddot{\phi} = \frac{g}{l}\phi$

Solving the second order differential equation $m^2 - \frac{g}{l} = 0 \Rightarrow m = \pm\sqrt{\frac{g}{l}}$

$\Rightarrow \quad \phi = C_1 e^{\sqrt{\frac{g}{l}}t} + C_2 e^{-\sqrt{\frac{g}{l}}t}$

Given at $t=0$, $\phi = 0.02$ rad and $\dot{\phi} = 0$ rad/s, also $g = 10$ m/s and $l = 10$m.

$\Rightarrow \left.\begin{array}{l} C_1 + C_2 = 0.02 \\ C_1 - C_2 = 0 \end{array}\right\} C_1 = 0.01 = C_2$

$\therefore \quad \phi = 0.01(e^{t} + e^{-t})$

At $t = 1$s, $\phi = 0.0309$ rad
and at $t = 7$s, $\phi = 10.9666$ rad

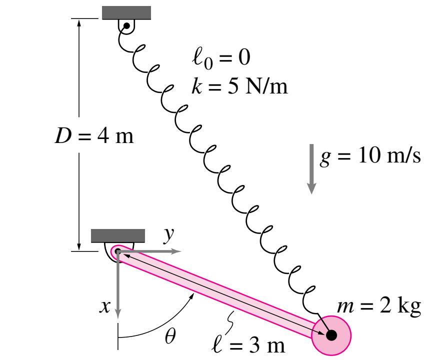
16.4.37 A zero length spring (relaxed length $\ell_0 = 0$) with stiffness $k = 5\,\mathrm{N/m}$ supports the pendulum shown.

a) Find $\ddot{\theta}$ assuming $\dot{\theta} = 2\,\mathrm{rad/s}$, $\theta = \pi/2$.

b) Find $\ddot{\theta}$ as a function of $\dot{\theta}$ and θ (and k, ℓ, m, and g.)

[Hint: use vectors (otherwise it's hard)]

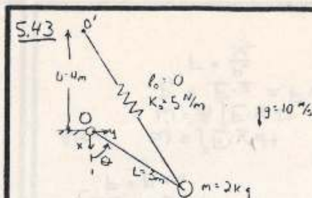
[Hint: For the special case, $kD = mg$, the solution simplifies greatly.]



Problem 16.37

Answer:
(a) $\ddot{\theta} = 0\ \mathrm{rad/s^2}$.
(b) $\ddot{\theta} = \frac{\sin\theta}{m\ell}(Dk - mg)$.

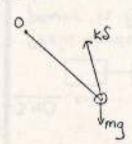
5.43 Page 6



a) find $\ddot{\theta}$ assuming $\dot{\theta} = 2$ rad/s, $\theta = \pi/2$

b) find $\ddot{\theta}$ as a function of $\dot{\theta}$ and θ

FBD



$\Sigma M_{/O} = \dot{H}_{/O}$

$\Sigma M_{/O} = L\sin\theta\,\hat{j} \times mg\,\hat{i} + r_{m/O} \times k_s\, r_{O'/m}$

$= -mgL\sin\theta\,\hat{k} + (L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j}) \times k_s(r_{O'/O} + r_{O/m})$

$= -mgL\sin\theta\,\hat{k} + L(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) \times k_s(-D\,\hat{i} + (-L\cos\theta\,\hat{i} - L\sin\theta\,\hat{j}))$

$\Sigma M_{/O} = -mgL\sin\theta\,\hat{k} + Lk_s(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) \times (-(L\cos\theta + D)\hat{i} - L\sin\theta\,\hat{j})$

$= -mgL\sin\theta\,\hat{k} + Lk_s[\cancel{-L\cos\theta\sin\theta\,\hat{k}} + \cancel{L\cos\theta\sin\theta\,\hat{k}} + D\sin\theta\,\hat{k}]$

$\Sigma M_{/O} = (Dk_s - mg)L\sin\theta\,\hat{k}$

$\dot{H}_{/O} = mL^2\ddot{\theta}\,\hat{k}$

$(\Sigma M_{/O} = \dot{H}_{/O}) \cdot \hat{k}$

$(Dk_s - mg)\cancel{L}\sin\theta = mL^{\cancel{2}}\ddot{\theta}$

$\ddot{\theta} = \frac{\sin\theta}{mL}(Dk_s - mg)$

a) $\ddot{\theta} = \frac{\sin\pi/2}{(2\,\mathrm{kg})(3\,\mathrm{m})}\left[(4\,\mathrm{m})(5\,\tfrac{\mathrm{N}}{\mathrm{m}}) - 20\,\mathrm{kg\,\tfrac{m}{s^2}}\right]$

$\ddot{\theta} = 0$

b) $\ddot{\theta} = \frac{\sin\theta}{mL}(Dk_s - mg)$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

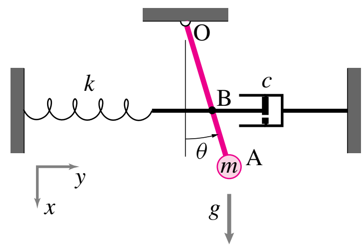
16.4.38 A spring-mass-damper system is depicted in the figure. The horizontal damping force applied at B is given by $F_D = -c\dot{y}_B$
The dimensions and parameters are as follows:

$$\begin{aligned} r_{B/0} &= 2ft \\ r_{A/0} &= \ell = 3ft \\ k &= 2\,\mathrm{lbf/ft} \\ c &= 0.3\,\mathrm{lbf\cdot s/ft} \end{aligned}$$

For small θ, assume that $\sin(\theta) \approx \theta$ and $\cos(\theta) \approx 1$.

a) Determine the natural circular frequency of small oscillations about equilibrium for the pendulum shown. The static equilibrium position is $\theta = 0$ (pendulum hanging vertically), so the spring is at its rest point in this position. Idealize the pendulum as a point mass attached to a rigid massless rod of length 1, so $I^O = m\ell^2$. Also use the "small angle approximation" where appropriate.

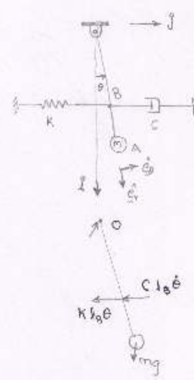
b) Sketch a graph of θ as a function of $t (t \geq 0)$ if the pendulum is released from rest at position $\theta = 0.2$ rad when $t = 0$. Your graph should show the correct qualitative behavior, but calculations are not necessary.



Problem 16.38

Answer:
(a) $\omega_n = \sqrt{\frac{k\ell_B^2 + \ell_A mg}{m\ell_A^2}}$.

Given, $F_D = -c\dot{y}_B$; $r_{B/O} = 2ft$; $r_{A/O} = l = 3ft$; $K = 2\,lbf/ft$, $C = 0.3\,lbf\,s/ft$
$\sin\theta \approx \theta$; $\cos\theta \approx 1$



$\hat{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$
$\hat{e}_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

Apply AMB about 'O': $\sum \underline{M}_{/O} = \dot{\underline{H}}_{/O}$

$\Rightarrow \underline{r}_B \times [K r_B \theta(-\hat{j})] + \underline{r}_B \times F_D(-\hat{j}) + \underline{r}_A \times mg\,\hat{i} = m l_A^2 \ddot{\theta}\,\hat{k}$

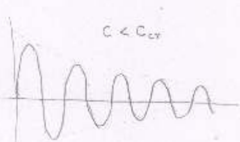
$\Rightarrow \{-K l_B^2 \theta\,\hat{k} - l_A mg\theta\,\hat{k} - c l_B^2 \dot{\theta}\,\hat{k} = m l_A^2 \ddot{\theta}\,\hat{k}\}$

$\{\,\}\cdot\hat{k} \Rightarrow m l_A^2 \ddot{\theta} + c l_B^2 \dot{\theta} + (K l_B^2 + l_A mg)\theta = 0$

Natural circular frequency,

$\omega_n = \sqrt{\frac{K l_B^2 + l_A mg}{m l_A^2}}$

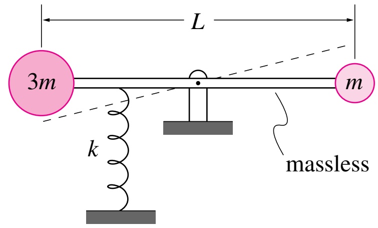
b) $\omega_D = \sqrt{\frac{K l_B^2 + l_A mg}{m l_A^2} - \frac{1}{4}\left[\frac{c l_B^2}{m l_A^2}\right]^2}$



$\left(\frac{c}{2m}\right)^2 < \frac{k}{m} \Rightarrow \frac{c^2 l_B}{4 m l_A^2} < K l_B^2 + l_A mg$

For an under damped system, m has to be selected such that $4 l_A^3 m^2 g + 4 K l_A^2 l_B^2 m - c^2 l_B^4 > 0$
From the given data $m > 0.00147$ lb

16.4.39 The asymmetric dumbbell shown in the figure is pivoted in the center and also attached to a spring at one quarter of its length from the bigger mass. When the bar is horizontal, the compression in the spring is y_s. At the instant of interest, the bar is at an angle θ from the horizontal; θ is small enough so that $y \approx \frac{L}{2}\theta$. If, at this position, the velocity of mass 'm' is $v\hat{\boldsymbol{j}}$ and that of mass $3m$ is $-v\hat{\boldsymbol{j}}$, evaluate the power term $(\sum \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{v}})$ in the energy balance equation.



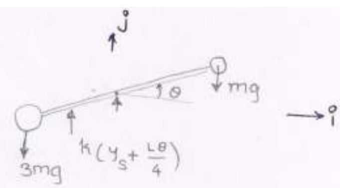
Problem 16.39

Answer:
$P = 2mg - \frac{kv}{2}\left(\frac{L\theta}{4} + y_s\right)$.

16.4.39

Given, mass 'm' has $v\hat{j}$ velocity,

mass '3m' has $-v\hat{j}$ velocity.



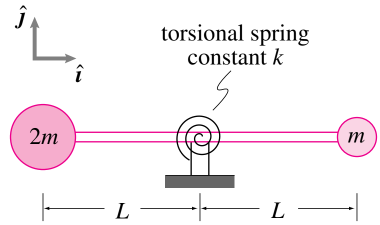
Power, $P = \sum F \cdot v$

$= 3mg(-\hat{j})\cdot(-v\hat{j}) + mg(-\hat{j})\cdot v\hat{j} + k(\frac{L}{4}\theta + y_s)\hat{j} \cdot (-\frac{v}{2}\hat{j})$

$\Rightarrow P = 3mgv - mgv - \frac{kv}{2}(\frac{L\theta}{4} + y_s)$

$\therefore P = 2mg - \frac{kv}{2}(\frac{L\theta}{4} + y_s)$

16.4.40 The dumbbell shown in the figure has a torsional spring with spring constant k (torsional stiffness units are $\frac{\text{kN}\cdot\text{m}}{\text{rad}}$). The dumbbell oscillates about the horizontal position with small amplitude θ. At an instant when the angular velocity of the bar is $\dot{\theta}\hat{\boldsymbol{k}}$, the velocity of the left mass is $-L\dot{\theta}\,\hat{\boldsymbol{\jmath}}$ and that of the right mass is $L\dot{\theta}\,\hat{\boldsymbol{\jmath}}$. Find the expression for the power P of the spring on the dumbbell at the instant of interest.



Problem 16.40

Answer:
$P = mgL\dot{\theta} - k\theta\dot{\theta}$.

15.4.40

Given. Velocity of mass m : $L\dot{\theta}\hat{j}$

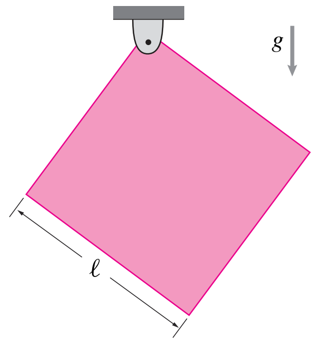
Velocity of mass 2m : $-L\dot{\theta}\hat{j}$

Power P $= 2mg(-\hat{j})\cdot(-L\dot{\theta}\hat{j}) + mg(-\hat{j})\cdot(L\dot{\theta}\hat{j}) + k\theta(-\hat{k})\cdot(\dot{\theta}\hat{k})$

$= 2mgL\dot{\theta} - mgL\dot{\theta} - k\theta\dot{\theta}$

$\Rightarrow P = mgL\dot{\theta} - k\theta\dot{\theta}$ //

16.4.41 A square plate with side ℓ and mass m is hinged at one corner in a gravitational field g. Find the period of small oscillation.



Problem 16.41

Answer:
$T = 2\pi\sqrt{\frac{2\sqrt{2}\ell}{3g}}$.

Apply AMB about 'O' : $\sum \underset{\sim}{M}_{/O} = \dot{\underset{\sim}{H}}_{/O}$

$\Rightarrow \quad a\hat{\underset{\sim}{e}}_r \times mg(\hat{\imath}) = I^O \ddot{\theta}\hat{\underset{\sim}{k}}$

$\hat{\underset{\sim}{e}}_r = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$

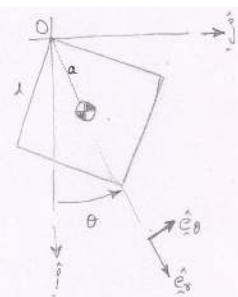
$\hat{\underset{\sim}{e}}_\theta = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$

$\Rightarrow \quad \left\{ mg\frac{l}{\sqrt{2}}\sin\theta(-\hat{\underset{\sim}{k}}) = (I^{cm} + ma^2)\ddot{\theta}\hat{\underset{\sim}{k}} \right\}$

$\{\ \}\cdot\hat{\underset{\sim}{k}} \Rightarrow \quad -mg\frac{l}{\sqrt{2}}\sin\theta = \left(\frac{ml^2}{6} + m\frac{l^2}{2}\right)\ddot{\theta}$

$\Rightarrow \quad \ddot{\theta} + \frac{3g}{2\sqrt{2}l}\sin\theta = 0$

Assuming small oscillations, $\sin\theta = \theta$, $\omega = \sqrt{\frac{3g}{2\sqrt{2}l}}$, $T = 2\pi\sqrt{\frac{2\sqrt{2}l}{3g}}$



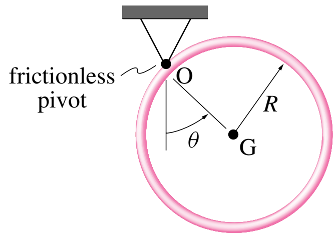
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.42 A thin hoop of radius R and mass M is hung from a point on its edge and swings in its plane. Assuming it swings near to the position where its center of mass G is below the hinge:

a) What is the period of its swinging oscillations?

b) If, instead, the hoop was set to swinging in and out of the plane would the period of oscillations be greater or less?

c) What length ℓ of simple pendulum (a massless rod with a point mass at the end) has the same period of oscillation of this hoop, swinging in its plane?



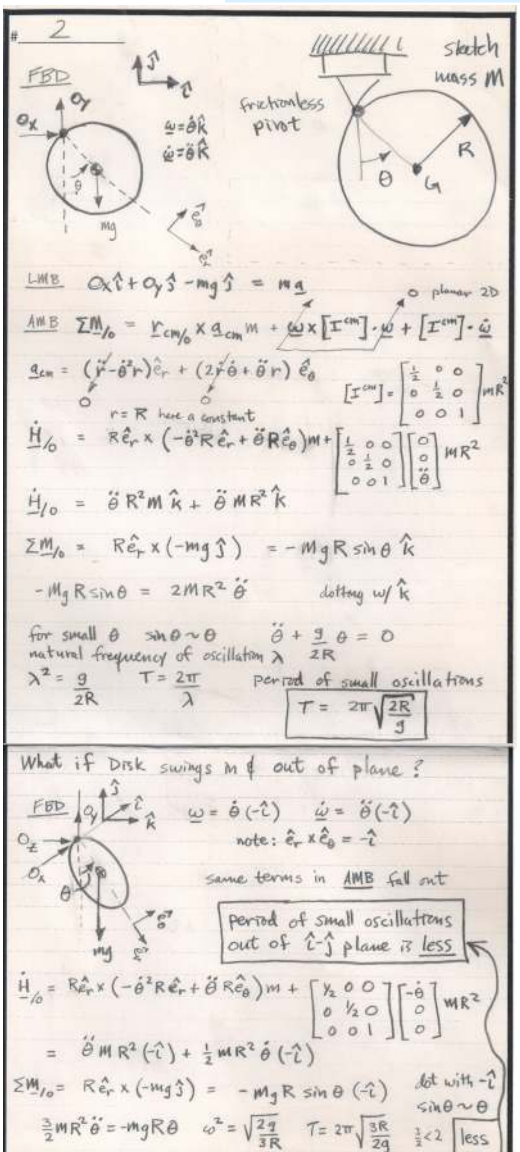
Problem 16.42

Answer:

(a) $T = 2\pi\sqrt{\frac{2R}{g}}$.

(b) The period will reduce to $T = 2\pi\sqrt{\frac{3R}{2g}}$.

$\ell = 2R$



\# 2

FBD

$\underline{\omega} = \dot{\theta}\hat{k}$

$\dot{\underline{\omega}} = \ddot{\theta}\hat{k}$

frictionless pivot

sketch mass M

LMB $O_x\hat{\imath} + O_y\hat{\jmath} - mg\hat{\jmath} = m\underline{a}$

0 planar 2D

AMB $\sum \underline{M}_{/O} = \underline{r}_{cm/O} \times \underline{a}_{cm} m + \underline{\omega} \times [I^{cm}] \cdot \underline{\omega} + [I^{cm}] \cdot \dot{\underline{\omega}}$

$\underline{a}_{cm} = (\ddot{r} - \dot{\theta}^2 r)\hat{e}_r + (2\dot{r}\dot{\theta} + \ddot{\theta} r)\hat{e}_\theta$

$r = R$ here a constant

$[I^{cm}] = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} mR^2$

$\dot{\underline{H}}_{/O} = R\hat{e}_r \times (-\dot{\theta}^2 R\hat{e}_r + \ddot{\theta} R\hat{e}_\theta)m + \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta} \end{bmatrix} mR^2$

$\dot{\underline{H}}_{/O} = \ddot{\theta} R^2 m\hat{k} + \ddot{\theta} mR^2\hat{k}$

$\sum \underline{M}_{/O} = R\hat{e}_r \times (-mg\hat{\jmath}) = -MgR\sin\theta\,\hat{k}$

$-MgR\sin\theta = 2MR^2\ddot{\theta}$ dotting w/ $\hat{k}$

for small θ $\sin\theta \sim \theta$ $\ddot{\theta} + \frac{g}{2R}\theta = 0$

natural frequency of oscillation λ

$\lambda^2 = \frac{g}{2R}$ $T = \frac{2\pi}{\lambda}$ period of small oscillations

$\boxed{T = 2\pi\sqrt{\frac{2R}{g}}}$

What if Disk swings in & out of plane?

FBD

$\underline{\omega} = \dot{\theta}(-\hat{\imath})$ $\dot{\underline{\omega}} = \ddot{\theta}(-\hat{\imath})$

note: $\hat{e}_r \times \hat{e}_\theta = -\hat{\imath}$

same terms in AMB fall out

$\boxed{\text{period of small oscillations out of } \hat{\imath}\text{-}\hat{\jmath} \text{ plane is less}}$

$\dot{\underline{H}}_{/O} = R\hat{e}_r \times (-\dot{\theta}^2 R\hat{e}_r + \ddot{\theta} R\hat{e}_\theta)m + \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\ddot{\theta} \\ 0 \\ 0 \end{bmatrix} mR^2$

$= \ddot{\theta} mR^2(-\hat{\imath}) + \frac{1}{2} mR^2\ddot{\theta}(-\hat{\imath})$

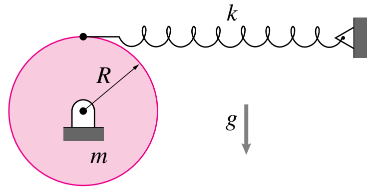
$\sum \underline{M}_{/O} = R\hat{e}_r \times (-mg\hat{\jmath}) = -MgR\sin\theta(-\hat{\imath})$ dot with $-\hat{\imath}$

$\sin\theta \sim \theta$

$\frac{3}{2}mR^2\ddot{\theta} = -mgR\theta$ $\omega^2 = \sqrt{\frac{2g}{3R}}$ $T = 2\pi\sqrt{\frac{3R}{2g}}$ $\frac{3}{2} < 2$ $\boxed{\text{less}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

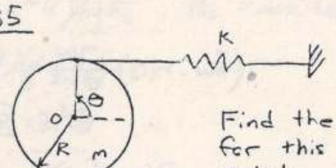
16.4.43 Oscillating disk. A uniform disk with mass m and radius R pivots around a frictionless hinge at its center. It is attached to a massless spring which is horizontal and relaxed when the attachment point is directly above the center of the disk. Assume small rotations and the consequent geometrical simplifications. Assume the spring can carry compression. What is the period of oscillation of the disk if it is disturbed from its equilibrium configuration? [You may use the fact that, for the disk shown, $\dot{\vec{H}}_{/O} = \frac{1}{2}mR^2\ddot{\theta}\hat{\boldsymbol{k}}$, where θ is the angle of rotation of the disk.].



Problem 16.43

Answer:
period $= \pi\sqrt{\frac{2m}{k}}$.

3.35



Find the period of oscillation for this system assuming small rotations.

Small rotations : F_{spring} acts only in $\hat{\imath}$ direction and $\delta_{spring} = R\theta$

FBD:

$$\sum \underline{M}_{/o} = \underline{\dot{H}}_{/o}$$

$$\cancel{R}\hat{j} \times K\cancel{R}\theta\,\hat{\imath} = \tfrac{1}{2}mR^{\cancel{2}}\ddot{\theta}\,\hat{k}$$

$$(-K\theta\hat{k} = \tfrac{1}{2}m\ddot{\theta}\,\hat{k}) \cdot \hat{k}$$

$$\ddot{\theta} + \frac{2k}{m}\theta = 0$$

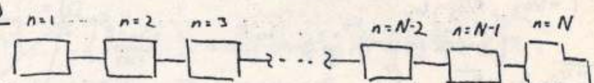
so $\omega^2 = \frac{2k}{m}$

$$\omega = \sqrt{\frac{2k}{m}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}$$

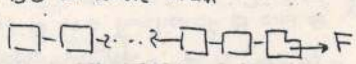
$$T = \pi\sqrt{\frac{2m}{k}}$$

3.40



Train consists of N cars each of mass M. The engine's power is P_t and velocity is V_t. Find the tension between car n and car $n+1$

FBD of whole train:



$$(\sum \underline{F} = m\underline{a}) \cdot \hat{\imath}$$

$$F = Nma$$

Also: $W = \int \underline{F} \cdot \underline{v}\, dt$

$$\dot{W} = \frac{d}{dt}\int \underline{F} \cdot \underline{v}\, dt$$

$$P_t = \underline{F} \cdot \underline{v} = FV_t$$

$$F = \frac{P_t}{V_t}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.44 Robotics problem: Simplest balancing of an inverted pendulum. You are holding a stick upside down, one end is in your hand, the other end sticking up. To simplify things, think of the stick as massless but with a point mass at the upper end. Also, imagine that it is only a two-dimensional problem (either you can ignore one direction of falling for simplicity or imagine wire guides that keep the stick from moving in and out of the plane of the paper on which you draw the problem).

You note that if you model your holding the stick as just having a stationary hinge then you get $\ddot{\phi} = \frac{g}{l}\sin\phi$. Assuming small angles, this hinge leads to exponentially growing solutions. Upside-down sticks fall over. How can you prevent this falling?

One way to do keep the stick from falling over is to firmly grab it with your hand, and if the stick tips, apply a torque in order to right it. This corrective torque is (roughly) how your ankles keep you balanced when you stand upright. Your task in this assignment is to design a robot that keeps an inverted pendulum balanced by applying appropriate torque.

Your model is: Inverted pendulum, length ℓ, point mass m, and a hinge at the bottom with a motor that can apply a torque T_m. The stick might be tipped an angle ϕ from the vertical. A *horizontal* disturbing force $F(t)$ is applied to the mass (representing wind, annoying friends, etc).

a) Draw a picture and a FBD

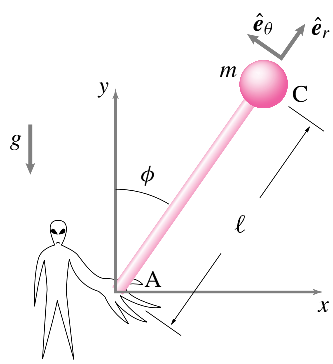
b) Write the equation for angular momentum balance about the hinge point.

c) Imagine that your robot can sense the angle of tip ϕ and its rate of change $\dot{\phi}$ and can apply a torque in response to that sensing. That is you can make T_m any function of ϕ and $\dot{\phi}$ that you want. Can you find a function that will make the pendulum stay upright? Make a guess (you will test it below).

d) Test your guess the following way: plug it into the equation of motion from part (b), linearize the equation, assume the disturbing force is zero, and see if the solution of the differential equation has exponentially growing (i.e. unstable) solutions. Go back to (c) if it does and find a control strategy that works.

e) Pick numbers and model your system on a computer using the full non-linear equations. Use initial conditions both close to and far from the upright position and plot ϕ versus time.

f) If you are ambitious, pick a non-zero forcing function $F(t)$ (say a sine wave of some frequency and amplitude) and see how that affects the stability of the solution in your simulations.



Problem 16.44

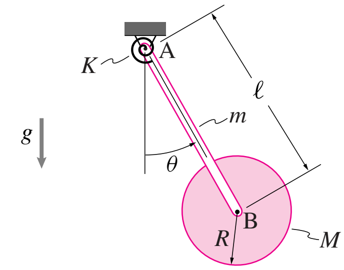
Answer:
(b) $-F(t)\ell\cos\phi - mg\ell\sin\phi + T_m = -m\ell^2\ddot{\phi}$.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.45 A thin rod of mass m and length ℓ is hinged with a torsional spring of stiffness K at A, and is connected to a thin disk of mass M and radius R at B. The spring is uncoiled when $\theta = 0$. Determine the natural frequency ω_n of the system for small oscillations θ, assuming that the disk is:

a) welded to the rod, and

b) pinned frictionlessly to the rod.



Problem 16.45

Answer:

(a) $\omega_n = \sqrt{\frac{gL\left(M+\frac{m}{2}\right)+K}{\left(M+\frac{m}{3}\right)L^2+M\frac{R^2}{2}}}$.

(b) $\omega_n = \sqrt{\frac{gL\left(M+\frac{m}{2}\right)+K}{\left(M+\frac{m}{3}\right)L^2}}$. Frequency higher than in (a)

15.4.45

b). Applying AMB about the hinge point 'O':

$$\sum \underline{M}_{/O} = \dot{\underline{H}}_{/O}$$

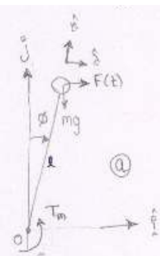
$\Rightarrow T_m \hat{\underline{k}} + \ell \hat{\underline{n}} \times mg(-\hat{\jmath}) + \ell \hat{\underline{n}} \times F(t)\hat{\underline{\imath}} = \underline{r} \times m\underline{a}$

$\Rightarrow \left\{ \left[T_m - mg\ell\sin\phi - F(t)\cos\phi\right]\hat{\underline{k}} = \ell\hat{\underline{n}} \times m\left[\ddot{\phi}\ell\hat{\underline{\lambda}} - \dot{\phi}^2\ell\hat{\underline{n}}\right]\right\}$

$\{\ \}\cdot\hat{\underline{n}} \Rightarrow T_m - mg\ell\sin\phi - F(t)\cos\phi = -m\ell^2\ddot{\phi}$

c) Let $T_m = 2mg\ell\sin\phi$ and $F(t) = 0$

$\Rightarrow m\ell^2\ddot{\phi} + mg\ell\sin\phi = 0$ //



$\hat{\underline{n}} = \sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath}$

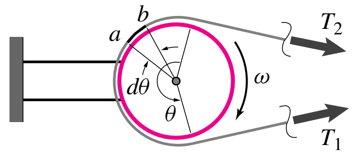
$\hat{\underline{\lambda}} = \cos\phi\,\hat{\imath} - \sin\phi\,\hat{\jmath}$

16.4.46 Assume that the pulley shown in figure(a) rotates at a constant speed ω. Let the angle of contact between the belt and pulley surface be θ. Assume that the belt is massless and that the condition of impending slip exists between the pulley and the belt. The free-body diagram of an infinitesimal section ab of the belt is shown in figure(b).

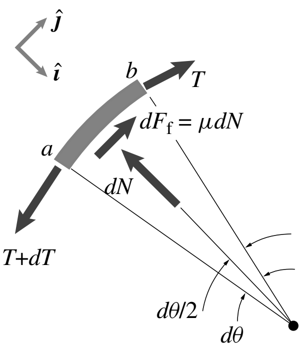
a) Write the equations of linear momentum balance for section ab of the belt in the $\hat{\imath}$ and $\hat{\jmath}$ directions.

b) Eliminate the normal force N from the two equations in part (a) and get a differential equation for the tension T in terms of the coefficient of friction μ and The contact angle θ.

c) Show that the solution to the equation in part (b) satisfies $\frac{T_1}{T_2} = e^{\mu\theta}$, where T_1 and T_2 are the tensions in the lower and the upper segments of the belt, respectively.



(a)



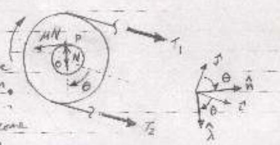
(b)

Problem 16.46

Answer:
(b) $\frac{T_1}{T_2} = \frac{R_o+R_i\sin\theta}{R_o-R_i\sin\theta}$, where $\theta = \tan^{-1}\mu$.

#3.75 Analyze massless pulley, now with friction at the shaft. Find relation between T_1 & T_2

Ⓐ Assume clockwise rotation. $T_1 > T_2$ to overcome friction



Normal contact force N arises at point P friction force μN opposes slip.

LMB $\Sigma \underline{F} = m\underline{a}_{cm}$ note $\underline{a}_{cm} = \underline{0}$, $m=0$

$\{T_1\hat{\imath} + T_2\hat{\imath} - N\hat{\lambda} - \mu N\hat{n} = 0\}\cdot\hat{\jmath}$

$\hat{\lambda}\cdot\hat{\imath} = -\sin\theta$ $\quad \hat{n}\cdot\hat{\jmath} = \cos\theta$

$+N\sin\theta - \mu N\cos\theta = 0$

$\mu = \tan\theta$ so $\boxed{\theta = \phi}$

So, we have found that the location of the contact point (θ) depends on the coefficient of friction. Notice if $\mu = 0 \Rightarrow \theta = 0 \Rightarrow$ N anti-parallel to T_1 & T_2.

AMB $\Sigma \underline{M}_{/P} = \dot{\underline{H}}_{/P}$ pulley has no mass ∴ no inertia

so $\Sigma \underline{M}_{/P} = \underline{0}$ $\quad \dot{H}_{/P} = 0$

$\hat{k}\cdot\{\Sigma\underline{M}_{/P} = -T_1(R_o - R_i\sin\theta)\hat{k} + T_2(R_o + R_i\sin\theta)\hat{k} = 0\}$

$\boxed{\frac{T_1}{T_2} = \frac{R_o + R_i\sin\theta}{R_o - R_i\sin\theta}}$ or $\boxed{\frac{T_1}{T_2} = \frac{1 + {}^{R_i}/_{R_o}\sin\theta}{1 - {}^{R_i}/_{R_o}\sin\theta}}$

write in terms of R_i/R_o

$\mu = .2$

$\frac{R_i}{R_o} = .10$

Ⓑ say $\frac{R_i}{R_o} = 0.10$, $\mu = 0.2$ plugging these into the above equation,

$\frac{T_1}{T_2} = \frac{1+(0.1)(0.2)}{1-(0.1)(0.2)} = 1.04$ note, too, for $\mu\frac{R_i}{R_o}$ small

$T_1/T_2 \cong 1 + 2\mu\,{}^{R_i}/_{R_o} = 1.04$

Ⓒ Beer & Johnston, "Belt Friction", § 8.10 in Statics, 4th Edition derive a formula for the relationship between T_2 & T_1

$\frac{T_1}{T_2} = e^{\mu\beta}$ where β is the total angle of contact between the line and the shaft

try $\mu = 0.2$, in this case $\beta = \pi$

$\frac{T_1}{T_2} = e^{(0.2)(\pi)} = 1.9$ considerably more force required if pulley "locks"

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.47 A belt drive is required to transmit 15 kW power from a 750 mm diameter pulley rotating at a constant 300 rpm to a 500 mm diameter pulley. The centers of the pulleys are located 2.5 m apart. The coefficient of friction between the belt and pulleys is $\mu = 0.2$.

a) (See problem 16.4.46.) Draw a neat diagram of the pulleys and the belt-drive system and find the *angle of lap*, the contact angle θ, of the belt on the driver pulley.

b) Find the rotational speed of the driven pulley.

c) (See the figure in problem 16.4.46.) The power transmitted by the belt is given by *power* = *net tension* × *belt speed*, i.e., $P = (T_1 - T_2)v$, where v is the linear speed of the belt. Find the maximum tension in the belt. [Hint: $\frac{T_1}{T_2} = e^{\mu\theta}$ (see problem 16.4.46).]

d) The belt in use has a 15 mm × 5 mm rectangular cross-section. Find the maximum tensile stress in the belt.

Answer:
(b) $N_2 = 450$ rpm.
(c) $T_{1_{max}} = 2793.8$ N.
(d) $\sigma_{max} = 37.25$ MPa.

Given. Power transmitted : 15kW

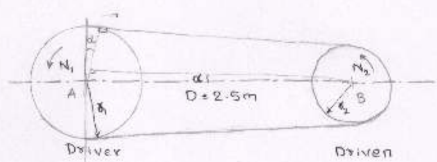
Driving pulley - diameter - 750mm , speed - 300 rpm

Driven pulley . diameter - 500 mm

Center to center distance - 2.5m

Coefficient of friction - belt - pulley , $\mu = 0.2$

a)



Angle of lap on the driver pulley , $\theta = \pi + 2\alpha$

From the figure, $\sin\alpha = \dfrac{r_1 - r_2}{D} \Rightarrow \alpha = \sin^{-1}\left(\dfrac{r_1 - r_2}{D}\right) = 2.87°$

∴ Angle of lap, $\theta = 180° + 2(2.87) = 185.73°$ //

b) Assuming no slip velocity of the belt remains constant.

$\Rightarrow |\underline{\omega}_1 \times \underline{r}_1| = |\underline{\omega}_2 \times \underline{r}_2| \Rightarrow N_1 r_1 = N_2 r_2 \Rightarrow N_2 = \frac{r_1}{r_2} N_1 = 450$ rpm

c) Tension in the belt: Power, $P = (T_1 - T_2)v$

$V = r\omega = 0.375 \cdot 300 \cdot \frac{2\pi}{60}$ m/s

Also $T_1 = T_2 e^{\mu\theta}$

Substituting T_1 in the expression for power.

$P = (T_2 e^{\mu\theta} - T_2) v$

$$\Rightarrow T_2 = \frac{15 \text{ kW}}{\left(e^{[0.2(180°-2\alpha)\frac{\pi}{180°}]} - 1\right)} \cdot \frac{60}{0.375 \cdot 300 \cdot 2\pi \ \frac{m}{s}} = 1520.6 \text{ N}$$

$$T_1 = T_2 e^{\mu(180°-2\alpha)\frac{\pi}{180°}} = 2793.82 \text{ N}$$

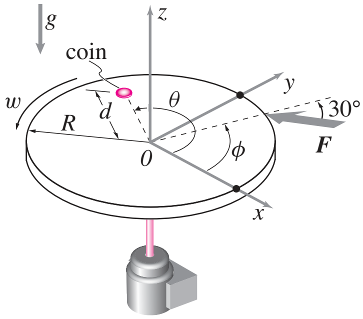
d) Belt cross section, A: 15 mm × 5 mm

Maximum tensile stress, $\sigma_{max} = \frac{T_1}{A} = 37.25$ MPa

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.48 Slippery money A round uniform flat horizontal platform with radius R and mass m is mounted on frictionless bearings with a vertical axis at 0. At the instant of interest it is rotating counter clockwise (looking down) with angular velocity $\vec{\omega} = \omega\hat{\mathbf{k}}$. A force in the xy plane with magnitude F is applied at the perimeter at an angle of 30° from the radial direction. The force is applied at a location that is ϕ from the fixed positive x axis. At the instant of interest a small coin sits on a radial line that is an angle θ from the fixed positive x axis (with mass much much smaller than m). Gravity presses it down, the platform holds it up, and friction (coefficient=μ) keeps it from sliding.

Find the biggest value of d for which the coin does not slide in terms of some or all of $F, m, g, R, \omega, \theta, \phi$, and μ.



Problem 16.48

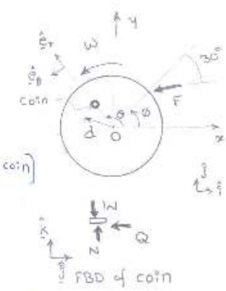
Answer:
$d_{\text{max}} = \dfrac{\mu mgR}{\sqrt{F^2+(mR\omega)^2}}$.

15.4.48: Slippery money

Find 'd' for coin not to slide

From FBD, coin will not slide if $Q < \mu N$ — ①

$Q = ?$



Consider LMB of coin. $\sum \underline{F} = m_c \underline{a}_c$ [m_c = mass of coin]

$\underline{a}_c = (\dot{\underline{\omega}} \times d\hat{e}_r) + \underline{\omega} \times (\underline{\omega} \times d\hat{e}_r)$

$\underline{a}_c = \dot{\omega} d\,\hat{e}_\theta - \omega^2 d\,\hat{e}_r$

$\therefore \sum \underline{F} = m\underline{a}_c \Rightarrow \{N\hat{k} - W\hat{k} + \underline{Q} = m_c[\dot{\omega} d\,\hat{e}_\theta - \omega^2 d\,\hat{e}_r]\}$

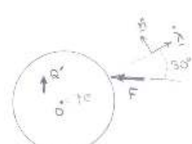
$\{\ \}\cdot\hat{k} \Rightarrow N = W = m_c g$ — ④

$\{\ \}\cdot\hat{e}_r \Rightarrow Q_r = -m_c d\omega^2$

$\{\ \}\cdot\hat{e}_\theta \Rightarrow Q_\theta = m_c d\dot{\omega}$

$\therefore Q = Q_r\hat{e}_r + Q_\theta\hat{e}_\theta$, $|Q| = \left(\sqrt{\dot{\omega}^2 + \omega^4}\right) m_c d$ — ②

$\dot{\omega}$ is determined from AMB



The forces acting on the disk are $\underline{F}$ and $\underline{Q}' = -\underline{Q}$.

AMB: $\sum \underline{M}_{/0} = \dot{\underline{H}}_{/0}$

$\Rightarrow R\hat{\lambda} \times (F\cos 30^\circ(-\hat{\lambda}) + F\sin 30^\circ\,\hat{n}) + d\,\hat{e}_r \times (-Q_r\hat{e}_r - Q_\theta\hat{e}_\theta) = I\dot{\omega}\hat{k}$

$\Rightarrow FR\sin 30^\circ\,\hat{k} - Q_\theta d\,\hat{k} = I\dot{\omega}\hat{k} = \frac{mR^2}{2}\dot{\omega}\hat{k}$

$\Rightarrow \{FR\sin 30^\circ\,\hat{k} = (m_c d^2\dot{\omega} + \frac{mR^2}{2}\dot{\omega})\hat{k}\}$

$\{\ \}\cdot\hat{k} \Rightarrow \dot{\omega} = \dfrac{FR\sin 30^\circ}{\left[m_c d^2 + \frac{mR^2}{2}\right]}$ — ③

Assume $\frac{m_c}{m} \ll 1 \Rightarrow \dot{\omega} \approx \dfrac{2FR\sin 30^\circ}{mR^2}$

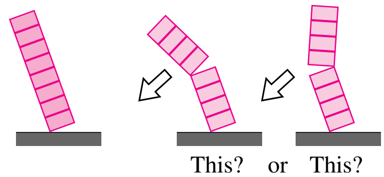
From ①, ②, ③ & ④ $Q < \mu m_c g$ (or) $\frac{m_c d}{mR^2}\sqrt{4F^2R^2\sin^2 30^\circ + m^2R^4\omega^4} \leq \mu m_c g$

$\Rightarrow d < \dfrac{\mu mgR}{\sqrt{4F^2\sin^2 30^\circ + (mR\omega^2)^2}}$ (or) $d < \dfrac{\mu mgR}{\sqrt{F^2 + m^2R^2\omega^4}}$ for coin not to slide.

$\therefore$ The biggest value of 'd' for which coin does not slide is $\dfrac{\mu mgR}{\sqrt{F^2 + (mR\omega^2)^2}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.4.49 Frequently parents will build a tower of blocks for their children. Just as frequently, kids knock them down. In falling (even when they start to topple aligned), these towers invariably break in two (or more) pieces at some point along their length. Why does this breaking occur? What condition is satisfied at the point of the break? Will the stack bend towards or away from the floor after the break?



Problem 16.49

Answer:
The stack will bend away from the floor after the break. This breaking occurs because gravity is not able to provide equal angular momentum to all the falling blocks.