16.4 Dynamics

Preparatory Problems

16.4.1 An object consists of a massless bar with two attached masses m_1 and m_2. The object is hinged at O.

a) What is the moment of inertia of the object about point O (I_{zz}^{O})? *

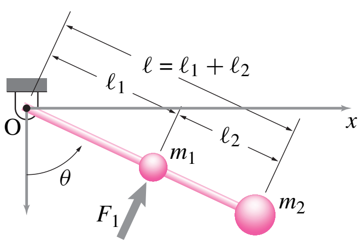
b) Given θ, $\dot{\theta}$, and $\ddot{\theta}$, what is $\vec{\boldsymbol{H}}_{/O}$, the angular momentum about point O? *

c) Given θ, $\dot{\theta}$, and $\ddot{\theta}$, what is $\dot{\vec{\boldsymbol{H}}}_{/O}$, the rate of change of angular momentum about point O? *

d) Given θ, $\dot{\theta}$, and $\ddot{\theta}$, what is T, the total kinetic energy? *

e) Assume that you don't know θ, $\dot{\theta}$ or $\ddot{\theta}$ but you do know that F_1 is applied to the rod, perpendicular to the rod at m_1. What is $\ddot{\theta}$? (Neglect gravity.) *

f) If F_1 were applied to m_2 instead of m_1, would $\ddot{\theta}$ be bigger or smaller? *

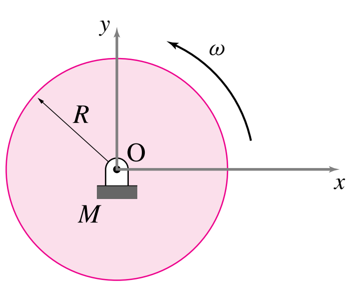


Problem 16.4.1

16.4.2 A uniform circular disc rotates at constant angular speed ω about the origin, which is also the center of the disc. It's radius is R. It's total mass is M.

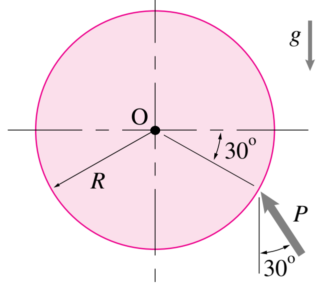
a) What is the total force and moment required to hold it in place (use the origin as the reference point of angular momentum and torque). *

b) What is the total kinetic energy of the disk? *



Problem 16.4.2

16.4.3 The hinged disk of mass m (uniformly distributed) is acted upon by a force P shown in the figure. Determine the *initial* angular acceleration and the reaction forces at the pin O. *



Problem 16.4.3

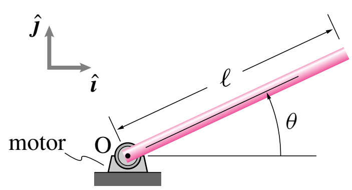
16.4.4 A motor turns a bar. A uniform bar of length ℓ and mass m is turned by a motor whose shaft is attached to the end of the bar at O. The angle that the bar makes (measured counterclockwise) from the positive x axis is $\theta = 2\pi\ t^2/\mathrm{s}^2$. Neglect gravity.

a) Draw a free-body diagram of the bar.

b) Find the force acting on the bar from the motor and hinge at $t = 1\,\mathrm{s}$. *

c) Find the torque applied to the bar from the motor at $t = 1\,\mathrm{s}$. *

d) What is the power produced by the motor at $t = 1\,\mathrm{s}$? *



Problem 16.4.4

16.4.5 A physical pendulum. A swinging stick is sometimes called a 'physical' pendulum. Take the 'body', the system of interest, to be the whole stick.

a) Draw a free-body diagram of the system.

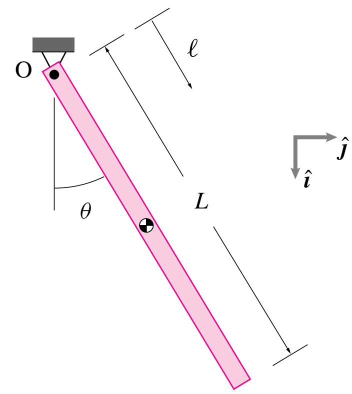
b) Write the equation of angular momentum balance for this system about point O.

c) Evaluate the left-hand-side as explicitly as possible in terms of the forces showing on your Free-Body Diagram.

d) Evaluate the right hand side as completely as possible. You may use the following facts:

$$\begin{aligned}\vec{\boldsymbol{v}} &= l\dot{\theta}\cos\theta\,\hat{\boldsymbol{\jmath}} + -l\dot{\theta}\sin\theta\hat{\boldsymbol{\imath}}\\ \vec{\boldsymbol{a}} &= -l\dot{\theta}^2\left[\cos\theta\hat{\boldsymbol{\imath}} + \sin\theta\,\hat{\boldsymbol{\jmath}}\right]\\ &\qquad +l\ddot{\theta}\left[\cos\theta\,\hat{\boldsymbol{\jmath}} - \sin\theta\hat{\boldsymbol{\imath}}\right]\end{aligned}$$

where ℓ is the distance along the pendulum from the top, θ is the angle by which the pendulum is displaced counter-clockwise from the vertically down position, $\hat{\boldsymbol{\imath}}$ is vertically down, and $\hat{\boldsymbol{\jmath}}$ is to the right. You will have to set up and evaluate an integral. *



Problem 16.4.5

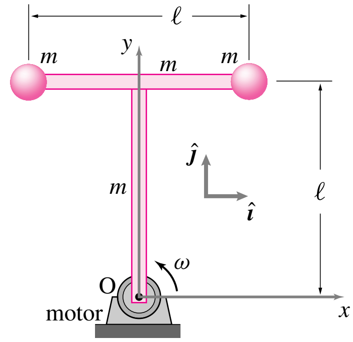
16.4.6 A uniform one meter bar is hung from a hinge that is at the end. It is allowed to swing freely. $g = 10\,\mathrm{m/s^2}$.

a) What is the period of small oscillations for this pendulum? *

b) Suppose the rod is hung 0.4 m from one end. What is the period of small oscillations for this pendulum? Can you explain why it is longer or shorter than when it is hung by its end? *

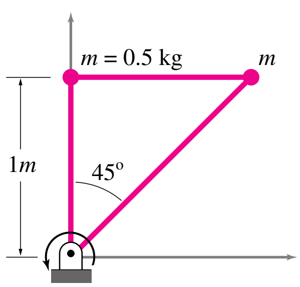
More-Involved Problems

16.4.7 Motor turns a dumbbell. Two uniform bars of length ℓ and mass m are welded at right angles. At the ends of the horizontal bar are two more masses m. The bottom end of the vertical rod is attached to a hinge at O where a motor keeps the structure rotating at constant rate ω (counter-clockwise). What is the net force and moment that the motor and hinge cause on the structure at the instant shown? *



Problem 16.4.7

16.4.8 The structure shown in the figure consists of two point masses connected by three rigid, massless rods such that the whole structure behaves like a rigid body. The structure rotates counterclockwise at a constant rate of 60 rpm. At the instant shown, find the force in each rod. *

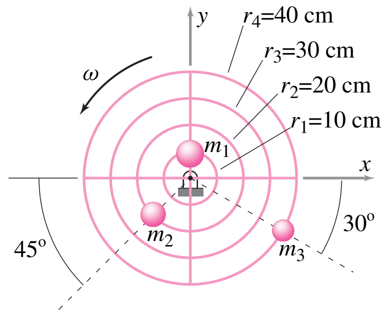


Problem 16.4.8

16.4.9 Balancing a system of rotating particles. A wire frame structure is made of four concentric loops of massless and rigid wires, connected to each other by four rigid wires presently coincident with the x and y axes. Three masses, $m_1 = 200$ grams, $m_2 = 150$ grams and $m_3 = 100$ grams, are glued to the structure as shown in the figure. The structure rotates counterclockwise at a constant rate $\dot{\theta} = 5\,\text{rad/s}$. There is no gravity.

a) Find the net force exerted by the structure on the support at the instant shown. *

b) You are to put a mass m at an appropriate location on the third loop so that the net force on the support is zero. Find the appropriate mass and the location on the loop. *

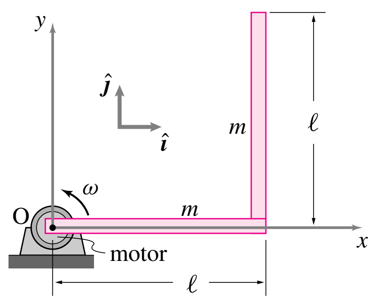


Problem 16.4.9

16.4.10 Motor turns a bent bar. Two uniform bars of length ℓ and uniform mass m are welded at right angles. One end is attached to a hinge at O where a motor keeps the structure rotating at a constant rate ω (counterclockwise). What is the net force and moment that the motor and hinge cause on the structure at the instant shown.

a) neglecting gravity *

b) including gravity. *



Problem 16.4.10: A bent bar is rotated by a motor.

16.4.11 A uniform disk of mass M and radius R rotates about a hinge O in the xy-plane. A point mass m is fixed to the disk at a distance $R/2$ from the hinge. A motor at the hinge drives the disk/point mass assembly with constant angular acceleration α. What torque at the hinge does the motor supply to the system? *

16.4.12 A rigid rod of length ℓ and total mass m is held fixed at one end and whirled around in circular motion at a constant rate ω in the horizontal plane. Ignore gravity.

a) Find the tension in the rod as a function of r, the radial distance from the center of rotation to any desired location on the rod. *

b) Where does the maximum tension occur in the rod? *.

c) At what distance from the center of rotation does the tension drop to half its maximum value? *.

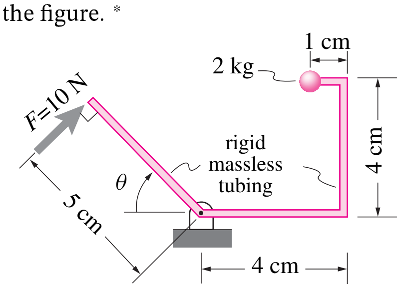
16.4.13 A thin uniform circular disc of mass M and radius R rotates in the xy plane about its center of mass point O. Driven by a motor, it has rate of change of angular speed proportional to angular position, $\alpha = k\theta^{5/2}$. The disc starts from rest at $\theta = 0$.

a) What is the rate of change of angular momentum about the origin at $\theta = \frac{\pi}{3}$ rad? *

b) What is the torque of the motor at $\theta = \frac{\pi}{3}$ rad? *

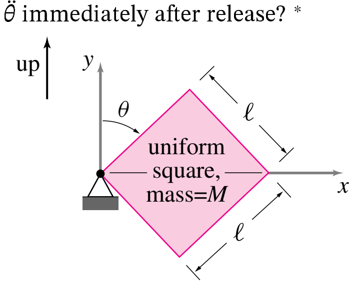
c) What is the total kinetic energy of the disk at $\theta = \frac{\pi}{3}$ rad? *

16.4.14 Neglecting gravity, calculate $\alpha = \dot{\omega} = \ddot{\theta}$ at the instant shown for the system in the figure. *



Problem 16.4.14

16.4.15 The uniform square shown is released from rest at $t = 0$. What is $\alpha = \dot{\omega} = \ddot{\theta}$ immediately after release? *

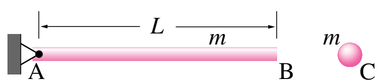


Problem 16.4.15

16.4.16 Acceleration of a trap door. A uniform bar AB of mass m and a ball of the same mass are released from rest from the same horizontal position. The bar is hinged at end A. There is gravity.

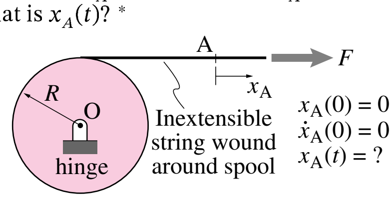
a) Which point on the rod has the same acceleration as the ball, immediately after release. *

b) What is the reaction force on the bar at end A just after release? *



Problem 16.4.16

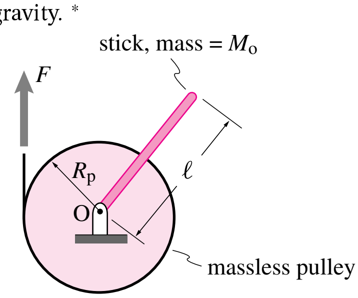
16.4.17 A disk with radius R has a string wrapped around it which is pulled with a force F. The disk is free to rotate about the axis through O normal to the page. The moment of inertia of the disk about O is I_o. A point A is marked on the string. Given that $x_A(0) = 0$ and that $\dot{x}_A(0) = 0$, what is $x_A(t)$? *



Spool has mass moment of inertia, I_O

Problem 16.4.17

16.4.18 A uniform stick with length ℓ and mass M_o is welded to a pulley hinged at the center O. The pulley has negligible mass and radius R_p. A string is wrapped many times around the pulley. At time $t = 0$, the pulley, stick, and string are at rest and a force F is suddenly applied to the string. How long does it take for the pulley to make one full revolution? Ignore gravity. *



Problem 16.4.18: String wraps around a pulley with a stick glued to it.

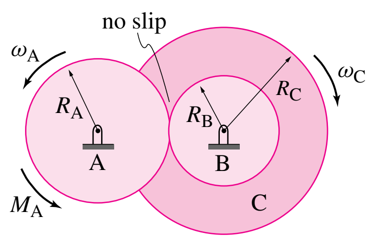
Gears, Belt Drives, and Gear Trains

16.4.19 Constant speed gear train. Gear A is connected to a motor (not shown) and gear B, which is welded to gear C, is connected to a taffy-pulling mechanism. Assume you know the torque $M_{\text{input}} = M_A$ and angular velocity $\omega_{\text{input}} = \omega_A$ of the input shaft. Assume the bearings and contacts are frictionless.

a) What is the input power? *

b) What is the output power? *

c) What is the output torque $M_{\text{output}} = M_C$, the torque that gear C applies to its surroundings in the clockwise direction? *



Problem 16.4.19

16.4.20 At the input to a gear box a 100 lbf force is applied to gear A. At the output, the machinery (not shown) applies a force of F_B to the output gear. Gear A rotates at constant angular rate $\omega = 2\,\text{rad/s}$, clockwise.

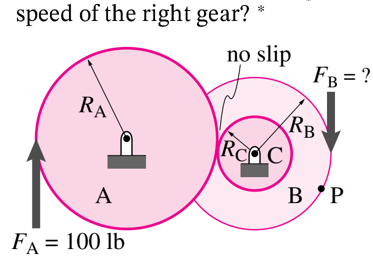
a) What is the angular speed of the right gear? *

b) What is the velocity of point P? *

c) What is F_B? *

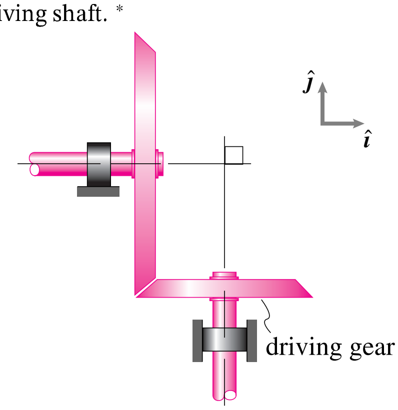
d) If the gear bearings had friction, would F_B have to be larger or smaller in order to achieve the same constant velocity? *

e) If instead of applying a 100 lbf to the left gear it is driven by a motor (not shown) at constant angular speed ω, what is the angular speed of the right gear? *



Problem 16.4.20: Two gears with end loads.

16.4.21 A bevel gear system. A bevel-type gear system, shown in the figure, is used to transmit power between two shafts that are perpendicular to each other. The driving gear has a mean radius of 50 mm and rotates at a constant speed $\omega = 150$ rpm. The mean radius of the driven gear is 80 mm and the driven shaft is expected to deliver a torque of $M_{out} = 25\,\text{N·m}$. Assuming no power loss, find the input torque supplied by the driving shaft. *



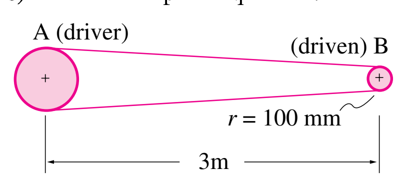
Problem 16.4.21: A bevel gear

16.4.22 *Belt drives* are used to transmit power between parallel shafts. Two parallel shafts, 3 m apart, are connected by a belt passing over the pulleys A and B fixed to the two shafts. The driver pulley A rotates at a constant 200 rpm. The speed ratio between the pulleys A and B is 1:2.5. The input torque is 350 N·m. Assume no loss of power between the two shafts.

a) Find the input power. *

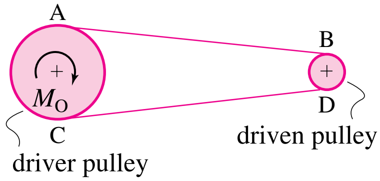
b) Find the rotational speed of the driven pulley B *.

c) Find the output torque at B. *



Problem 16.4.22

16.4.23 In the belt drive system shown, assume that the driver pulley rotates at a constant angular speed ω. If the motor applies a constant torque M_O on the driver pulley, show that the tensions in the two parts, AB and CD, of the belt must be different. Which part has a greater tension? Does your conclusion about unequal tension depend on whether the pulley is massless or not? Assume any dimensions you need. *



Problem 16.4.23

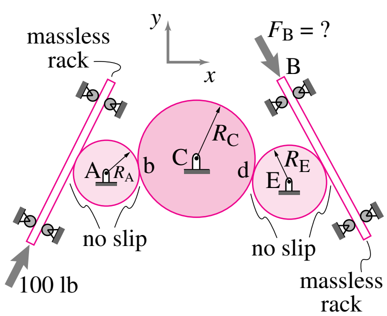
16.4.24 Two racks connected by three constant rate gears. A 100 lbf force is applied to one rack. At the output, the machinery (not shown) applies a force of F_B to the other rack.

a) Assume the gear-train is spinning at constant rate and is frictionless. What is F_B? *

b) If the gear bearings had friction would that increase or decrease F_B to achieve the same constant rate?

c) If instead of applying a 100 lbf to the left rack it is driven by a motor (not shown) at constant speed v, what is the speed of the right rack? *

d) If the angular velocity of the gear is increasing at rate α does this increase or decrease F_B at the given ω.



Problem 16.4.24: Two racks connected by three gears.

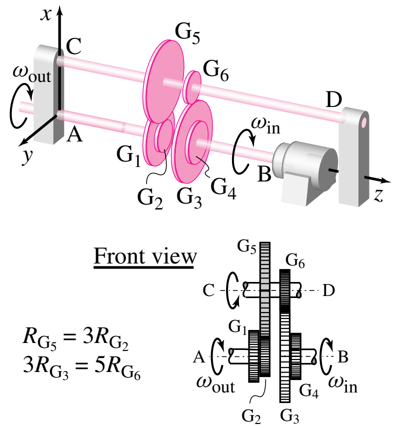
16.4.25 -3-D accelerating gear train. This is really a 2-D problem; each gear turns in a different parallel plane. Shaft B is rigidly connected to gears G_4 and G_5. G_3 meshes with gear G_6. Gears G_6 and G_5 are both rigidly attached to shaft AD. Gear G_5 meshes with G_2 which is welded to shaft A. Shaft A and shaft B spin independently. Assume you know the torque M_{input}, angular velocity ω_{input} and the angular acceleration α_{input} of the input shaft. Assume the bearings and contacts are frictionless.

a) What is the input power? *

b) What is the output power? *

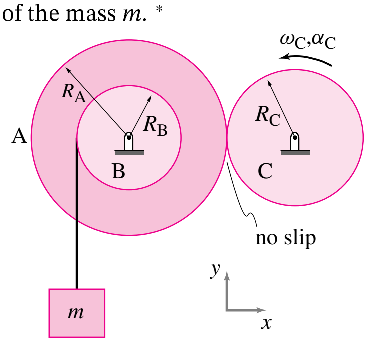
c) What is the angular velocity ω_{output} of the output shaft? *

d) What is the output torque M_{output}? *



Problem 16.4.25: A 3-D gear train.

16.4.26 Gear A with radius R_A = 400 mm is rigidly connected to a drum B with radius R_B = 200 mm. The combined moment of inertia of the gear and the drum about the axis of rotation is I_{zz} = 0.5 kg · m^2. Gear A is driven by gear C which has radius R_C = 300 mm. As the drum rotates, a 5 kg mass m is pulled up by a string wrapped around the drum. At the instant of interest, The angular speed and angular acceleration of the driving gear are 60 rpm and 12 rpm/s, respectively. Find the acceleration of the mass m. *

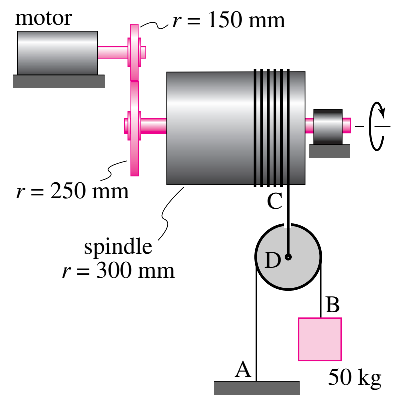


Problem 16.4.26

16.4.27 A spindle and pulley arrangement is used to hoist a 50 kg mass as shown in the figure. Assume that the pulley is to be of negligible mass. When the motor is running at a constant 100 rpm,

a) Find the velocity of the mass at B. *

b) Find the tension in strings AB and CD. *



Problem 16.4.27

16.4.28 Accelerating rack and pinion. The two gears shown are welded together and spin on a frictionless bearing. The inner gear has radius 0.5 m and negligible mass. The outer disk has 1 m radius and a uniformly distributed mass of 0.2 kg. They are loaded as shown with the force F = 20 N on the massless rack which is held in place by massless frictionless rollers. At the time of interest the angular velocity is ω = 2 rad/s (though ω is not constant). The point P is on the disk a distance 1 m from the center. At the time of interest, point P is on the positive y axis.

a) What is the speed of point P? *

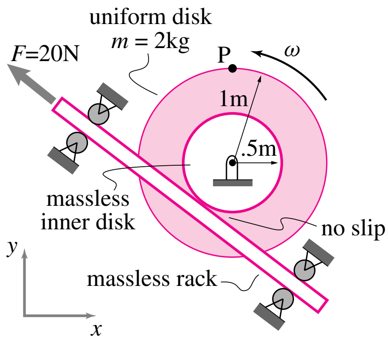
b) What is the velocity of point P? *

c) What is the angular acceleration α of the gear? *

d) What is the acceleration of point P? *

e) What is the magnitude of the acceleration of point P? *

f) What is the rate of increase of the speed of point P? *



Problem 16.4.28: Accelerating rack and pinion

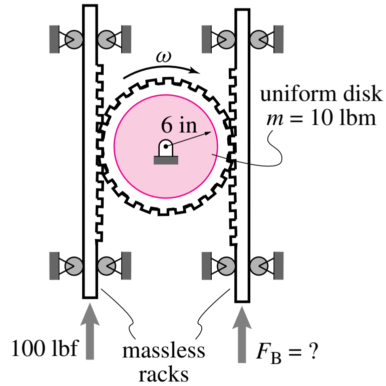
16.4.29 Two racks connected by a gear. A 100 lbf force is applied to one rack. At the output the machinery (not shown) applies a constant force F_B to the other rack.

a) Assume the gear is spinning at constant rate and is frictionless. What is F_B? *

b) If the gear bearing had friction, would that increase or decrease F_B to achieve the same constant rate? *

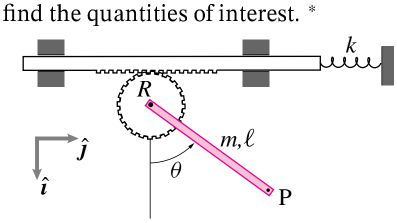
c) If the angular velocity of the gear is increasing at rate α, does this increase or decrease F_B at the given ω. *

d) If the output load F_B is given then the motion of the machine can be found, assuming friction is negligible. Assume that the machine starts from rest with a given output load. So long as rack B moves down and the output force F_B is positive, the output power is positive. The machine starts from rest and runs for a fixed amount of time T. For what value of F_B is the output work maximum? Once you find an answer, can you find an intuitive explanation for the answer? *



Problem 16.4.29: Two racks connected by a gear.

16.4.30 A rack and pinion constrained by a linear spring. Neglect gravity. The spring is relaxed when the angle $\theta = 0$. Assume the system is released from rest at $\theta = \theta_0$. What is the acceleration of the point P at the end of the stick when $\theta = 0$? Answer in terms of any or all of $m, R, \ell, \theta_0, k, \hat{\imath}$, and $\hat{\jmath}$. [Hint: There are several steps of reasoning required. You might want to draw FBD(s), use angular momentum balance, set up a differential equation, solve it, plug values into this solution, and use the result to find the quantities of interest. *



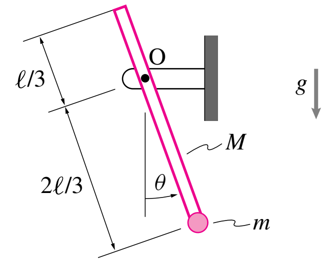
Problem 16.4.30

Pendulum Problems

16.4.31 A small particle of mass m is attached to the end of a thin rod of mass M (uniformly distributed), which is pinned at hinge O, as depicted in the figure.

a) Obtain the equation of motion governing the rotation θ of the rod. *

b) What is the natural frequency of the system for small oscillations θ? *



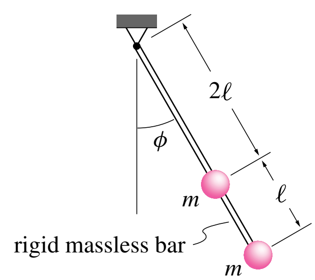
Problem 16.4.31

16.4.32 A rigid massless rod has two equal masses m_B and m_C ($m_B = m_C = m$) attached to it at distances 2ℓ and 3ℓ, respectively, measured along the rod from a frictionless hinge located at a point A. The rod swings freely from the hinge. There is gravity. Let ϕ denote the angle of the rod measured from the vertical. Assume that ϕ and $\dot{\phi}$ are known at the instant of interest.

a) What is $\ddot{\phi}$? Find $\ddot{\phi}$ in terms of m, ℓ, g, ϕ and $\dot{\phi}$. *

b) What is the force of the hinge on the rod? Solve in terms of m, ℓ, g, ϕ, $\dot{\phi}$, $\ddot{\phi}$ and any unit vectors you may need to define. *

c) Would you get the same answers if you put a mass $2m$ at 2.5ℓ? Why or why not? *



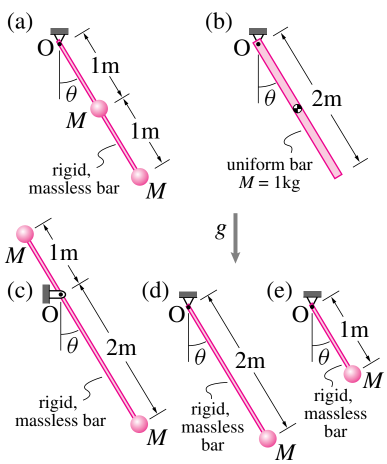
Problem 16.4.32

16.4.33 For the pendula in the figure :

a) Without doing any calculations, try to figure out the relative durations of the periods of oscillation for the five pendula (i.e. the order, slowest to fastest) Assume small angles of oscillation.

b) Calculate the period of small oscillations. [Hint: use balance of angular momentum about the point 0]. *

c) Rank the relative duration of oscillations and compare to your intuitive solution in part (a), and explain in words why things work the way they do.



Problem 16.4.33

16.4.34 A pegged compound pendulum. A uniform bar of mass m and length ℓ hangs from a peg at point C and swings in the vertical plane about an axis passing through the peg. The distance d from the center of mass of the rod to the peg can be changed by putting the peg at some other point along the length of the rod.

a) Find the angular momentum of the rod about point C. *

b) Find the rate of change of angular momentum of the rod about C. *

c) How does the period of the pendulum vary with d? Show the variation by plotting the period against $\frac{d}{\ell}$. [Hint, you must first find the equations of motion, linearize for small θ, and then solve.] *

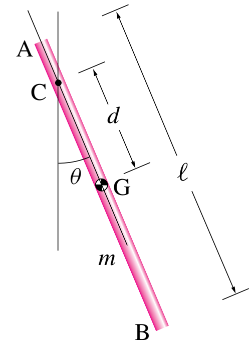
d) Find the total energy of the rod (using point C as a datum for potential energy). *

e) Find $\ddot{\theta}$ when $\theta = \pi/6$. *

f) Find the reaction force on the rod at C, as a function of m, d, ℓ, θ, and $\dot{\theta}$. *

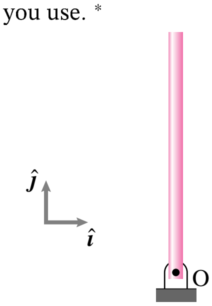
g) For the given rod, what should be the value of d (in terms of ℓ) in order to have the fastest pendulum? *

h) **Test of Schuler's pendulum.** The pendulum with the value of d obtained in (g) is called the Schuler's pendulum. It is not only the fastest pendulum but also the "most accurate pendulum". The claim is that even if d changes slightly over time due to wear at the support point, the period of the pendulum does not change much. Verify this claim by calculating the percent error in the time period of a pendulum of length $\ell = 1\,\text{m}$ under the following three conditions: (i) initial $d = 0.15\,\text{m}$ and after some wear $d = 0.16\,\text{m}$, (ii) initial $d = 0.29\,\text{m}$ and after some wear $d = 0.30\,\text{m}$, and (iii) initial $d = 0.45\,\text{m}$ and after some wear $d = 0.46\,\text{m}$. Which pendulum shows the least error in its time period? What is the connection between this result and the plot obtained in (c)? *



Problem 16.4.34

16.4.35 A uniform stick of length ℓ and mass m is a hair away from vertically up position when it is released with no angular velocity (a 'hair' is a technical word that means 'very small amount, zero for some purposes'). It falls to the right. What is the force on the stick at point O when the stick is horizontal. Solve in terms of ℓ, m, g, $\hat{\imath}$, and $\hat{\jmath}$. Carefully define any coordinates, base vectors, or angles that you use. *



Problem 16.4.35

16.4.36 A massless 10 meter long bar is supported by a frictionless hinge at one end and has a 3.759 kg point mass at the other end. It is released at $t = 0$ from a tip angle of $\phi = .02$ radians measured from vertically upright position (hinge at the bottom). Use $g = 10\,\text{m}\,\text{s}^{-2}$.

a) Using a small angle approximation and the solution to the resulting linear differential equation, find the angle of tip at $t = 1\text{s}$ and $t = 7\text{s}$. Use a calculator, not a numerical integrator. *

b) Using numerical integration of the non-linear differential equation for an inverted pendulum find ϕ at $t = 1\text{s}$ and $t = 7\text{s}$.

c) Make a plot of the angle versus time for your numerical solution. Include on the same plot the angle versus time from the approximate linear solution from part (a).

d) Comment on the similarities and differences in your plots.

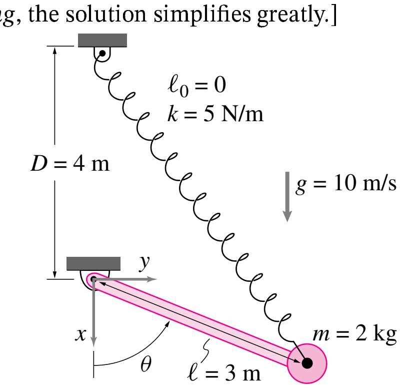
16.4.37 A zero length spring (relaxed length $\ell_0 = 0$) with stiffness $k = 5\,\text{N/m}$ supports the pendulum shown.

a) Find $\ddot{\theta}$ assuming $\dot{\theta} = 2\,\text{rad/s}$, $\theta = \pi/2$. *

b) Find $\ddot{\theta}$ as a function of $\dot{\theta}$ and θ (and k, ℓ, m, and g.) *

[Hint: use vectors (otherwise it's hard)]

[Hint: For the special case, $kD = mg$, the solution simplifies greatly.]



Problem 16.4.37

16.4.38 A spring-mass-damper system is depicted in the figure. The horizontal damping force applied at B is given by $F_D = -c\dot{y}_B$

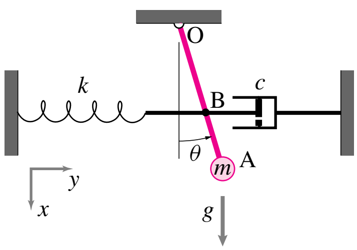
The dimensions and parameters are as follows:

$$\begin{aligned} r_{B/0} &= 2ft \\ r_{A/0} &= \ell = 3ft \\ k &= 2\,\text{lbf}/\text{ft} \\ c &= 0.3\,\text{lbf}\cdot\text{s}/\text{ft} \end{aligned}$$

For small θ, assume that $\sin(\theta) \approx \theta$ and $\cos(\theta) \approx 1$.

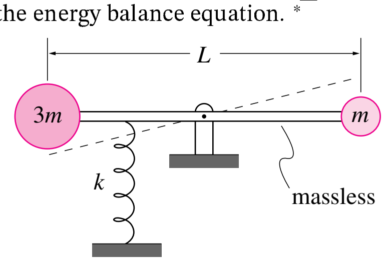
a) Determine the natural circular frequency of small oscillations about equilibrium for the pendulum shown. The static equilibrium position is $\theta = 0$ (pendulum hanging vertically), so the spring is at its rest point in this position. Idealize the pendulum as a point mass attached to a rigid massless rod of length 1, so $I^O = m\ell^2$. Also use the "small angle approximation" where appropriate. *

b) Sketch a graph of θ as a function of $t (t \geq 0)$ if the pendulum is released from rest at position $\theta = 0.2\,\text{rad}$ when $t = 0$. Your graph should show the correct qualitative behavior, but calculations are not necessary.



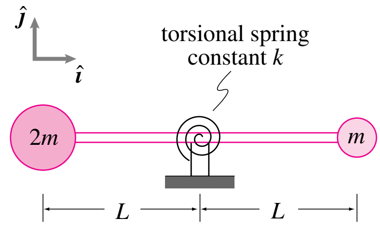
Problem 16.4.38

16.4.39 The asymmetric dumbbell shown in the figure is pivoted in the center and also attached to a spring at one quarter of its length from the bigger mass. When the bar is horizontal, the compression in the spring is y_s. At the instant of interest, the bar is at an angle θ from the horizontal; θ is small enough so that $y \approx \frac{L}{2}\theta$. If, at this position, the velocity of mass 'm' is $v\hat{\boldsymbol{\jmath}}$ and that of mass $3m$ is $-v\hat{\boldsymbol{\jmath}}$, evaluate the power term $(\sum \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{v}})$ in the energy balance equation. *



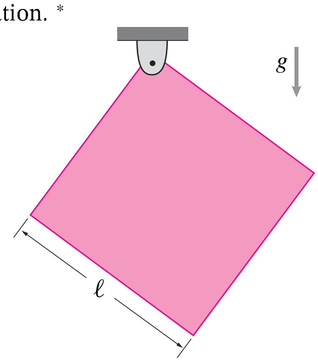
Problem 16.4.39

16.4.40 The dumbbell shown in the figure has a torsional spring with spring constant k (torsional stiffness units are $\frac{\text{kN}\cdot\text{m}}{\text{rad}}$). The dumbbell oscillates about the horizontal position with small amplitude θ. At an instant when the angular velocity of the bar is $\dot{\theta}\hat{\boldsymbol{k}}$, the velocity of the left mass is $-L\dot{\theta}\hat{\boldsymbol{\jmath}}$ and that of the right mass is $L\dot{\theta}\hat{\boldsymbol{\jmath}}$. Find the expression for the power P of the spring on the dumbbell at the instant of interest. *



Problem 16.4.40

16.4.41 A square plate with side ℓ and mass m is hinged at one corner in a gravitational field g. Find the period of small oscillation. *



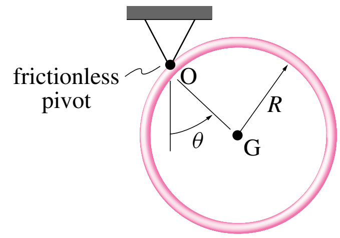
Problem 16.4.41

16.4.42 A thin hoop of radius R and mass M is hung from a point on its edge and swings in its plane. Assuming it swings near to the position where its center of mass G is below the hinge:

a) What is the period of its swinging oscillations? *

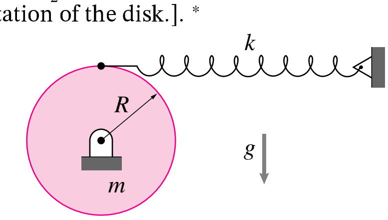
b) If, instead, the hoop was set to swinging in and out of the plane would the period of oscillations be greater or less? *

c) What length ℓ of simple pendulum (a massless rod with a point mass at the end) has the same period of oscillation of this hoop, swinging in its plane? *



Problem 16.4.42

16.4.43 Oscillating disk. A uniform disk with mass m and radius R pivots around a frictionless hinge at its center. It is attached to a massless spring which is horizontal and relaxed when the attachment point is directly above the center of the disk. Assume small rotations and the consequent geometrical simplifications. Assume the spring can carry compression. What is the period of oscillation of the disk if it is disturbed from its equilibrium configuration? [You may use the fact that, for the disk shown, $\dot{\vec{\boldsymbol{H}}}_{/O} = \frac{1}{2}mR^2\ddot{\theta}\hat{\boldsymbol{k}}$, where θ is the angle of rotation of the disk.]. *



Problem 16.4.43

16.4.44 Robotics problem: Simplest balancing of an inverted pendulum. You are holding a stick upside down, one end is in your hand, the other end sticking up. To simplify things, think of the stick as massless but with a point mass at the upper end. Also, imagine that it is only a two-dimensional problem (either you can ignore one direction of falling for simplicity or imagine wire guides that keep the stick from moving in and out of the plane of the paper on which you draw the problem).

You note that if you model your holding the stick as just having a stationary hinge then you get $\ddot{\phi} = \frac{g}{l}\sin\phi$. Assuming small angles, this hinge leads to exponentially growing solutions. Upside-down sticks fall over. How can you prevent this falling?

One way to do keep the stick from falling over is to firmly grab it with your hand, and if the stick tips, apply a torque in order to right it. This corrective torque is (roughly) how your ankles keep you balanced when you stand upright. Your task in this assignment is to design a robot that keeps an inverted pendulum balanced by applying appropriate torque.

Your model is: Inverted pendulum, length ℓ, point mass m, and a hinge at the bottom with a motor that can apply a torque T_m. The stick might be tipped an angle ϕ from the vertical. A *horizontal* disturbing force $F(t)$ is applied to the mass (representing wind, annoying friends, etc).

a) Draw a picture and a FBD

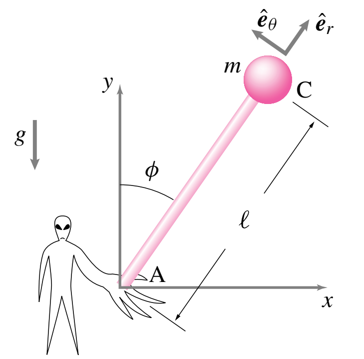
b) Write the equation for angular momentum balance about the hinge point. *

c) Imagine that your robot can sense the angle of tip ϕ and its rate of change $\dot{\phi}$ and can apply a torque in response to that sensing. That is you can make T_m any function of ϕ and $\dot{\phi}$ that you want. Can you find a function that will make the pendulum stay upright? Make a guess (you will test it below).

d) Test your guess the following way: plug it into the equation of motion from part (b), linearize the equation, assume the disturbing force is zero, and see if the solution of the differential equation has exponentially growing (i.e. unstable) solutions. Go back to (c) if it does and find a control strategy that works.

e) Pick numbers and model your system on a computer using the full non-linear equations. Use initial conditions both close to and far from the upright position and plot ϕ versus time.

f) If you are ambitious, pick a nonzero forcing function $F(t)$ (say a sine wave of some frequency and amplitude) and see how that affects the stability of the solution in your simulations.

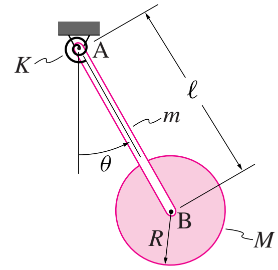


Problem 16.4.44

16.4.45 A thin rod of mass m and length ℓ is hinged with a torsional spring of stiffness K at A, and is connected to a thin disk of mass M and radius R at B. The spring is uncoiled when $\theta = 0$. Determine the natural frequency ω_n of the system for small oscillations θ, assuming that the disk is:

a) welded to the rod, and *

b) pinned frictionlessly to the rod. *



Problem 16.4.45

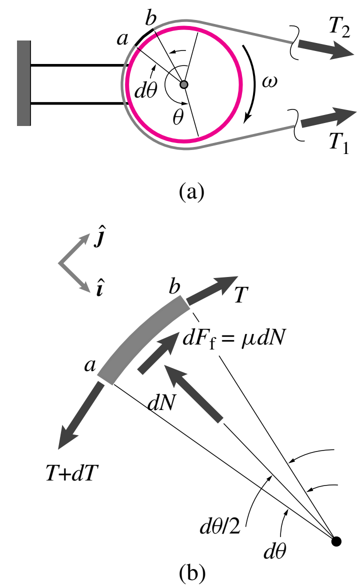
Mixed Problems

16.4.46 Assume that the pulley shown in figure(a) rotates at a constant speed ω. Let the angle of contact between the belt and pulley surface be θ. Assume that the belt is massless and that the condition of impending slip exists between the pulley and the belt. The free-body diagram of an infinitesimal section ab of the belt is shown in figure(b).

a) Write the equations of linear momentum balance for section ab of the belt in the $\hat{\imath}$ and $\hat{\jmath}$ directions.

b) Eliminate the normal force N from the two equations in part (a) and get a differential equation for the tension T in terms of the coefficient of friction μ and The contact angle θ. *

c) Show that the solution to the equation in part (b) satisfies $\frac{T_1}{T_2} = e^{\mu\theta}$, where T_1 and T_2 are the tensions in the lower and the upper segments of the belt, respectively.



Problem 16.4.46

16.4.47 A belt drive is required to transmit 15 kW power from a 750 mm diameter pulley rotating at a constant 300 rpm to a 500 mm diameter pulley. The centers of the pulleys are located 2.5 m apart. The coefficient of friction between the belt and pulleys is $\mu = 0.2$.

a) (See problem 16.4.46.) Draw a neat diagram of the pulleys and the belt-drive system and find the *angle of lap*, the contact angle θ, of the belt on the driver pulley.

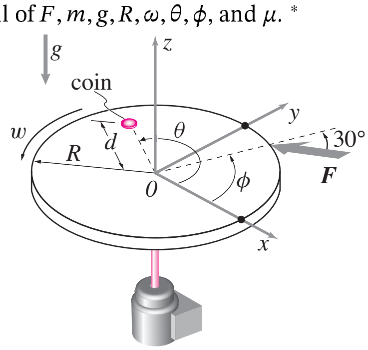
b) Find the rotational speed of the driven pulley. *

c) (See the figure in problem 16.4.46.) The power transmitted by the belt is given by *power = net tension × belt speed*, i.e., $P = (T_1 - T_2)v$, where v is the linear speed of the belt. Find the maximum tension in the belt. [Hint: $\frac{T_1}{T_2} = e^{\mu\theta}$ (see problem 16.4.46).] *

d) The belt in use has a 15 mm × 5 mm rectangular cross-section. Find the maximum tensile stress in the belt. *

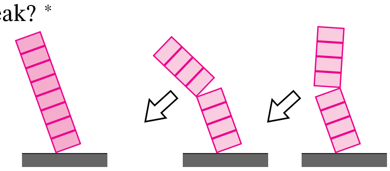
16.4.48 Slippery money A round uniform flat horizontal platform with radius R and mass m is mounted on frictionless bearings with a vertical axis at 0. At the instant of interest it is rotating counter clockwise (looking down) with angular velocity $\vec{\omega} = \omega\hat{\boldsymbol{k}}$. A force in the xy plane with magnitude F is applied at the perimeter at an angle of 30° from the radial direction. The force is applied at a location that is ϕ from the fixed positive x axis. At the instant of interest a small coin sits on a radial line that is an angle θ from the fixed positive x axis (with mass much much smaller than m). Gravity presses it down, the platform holds it up, and friction (coefficient=μ) keeps it from sliding.

Find the biggest value of d for which the coin does not slide in terms of some or all of $F, m, g, R, \omega, \theta, \phi$, and μ. *



Problem 16.4.48

16.4.49 Frequently parents will build a tower of blocks for their children. Just as frequently, kids knock them down. In falling (even when they start to topple aligned), these towers invariably break in two (or more) pieces at some point along their length. Why does this breaking occur? What condition is satisfied at the point of the break? Will the stack bend towards or away from the floor after the break? *



Problem 16.4.49