16.2.1 Give the definition of each term below in words and, but for the first, with an equation.

a) angular velocity

b) angular acceleration

Answer:
(a) The rate of change of any marked or imaginary line on a rigid object.
(b) The rate of change of the angular velocity.

a) The rate of change of all marked lines on a rigid object is same and is termed as angular velocity

$\omega = \dot{\theta}_1 = \dot{\theta}_2 = \cdots = \dot{\theta}$ where $\underset{\sim}{\omega} = \omega \hat{\underset{\sim}{k}}$

b) Angular acceleration of a particle w.r.t. fixed frame is the rate of change of angular velocity

16.2.2 Assume that in the reference configuration, when $\theta = 0$, that the $x'y'$ axes attached to an object are aligned with the xy axes. Consider a point P attached to the moving frame (object) that has coordinates (x', y'). Find each of the quantities below in terms of some or all of $x', y', \hat{\imath}, \hat{\jmath}, \theta, \dot{\theta}$ and $\ddot{\theta}$.

a) $\vec{r}_P$

b) $\vec{v}_P$

c) $\vec{a}_P$

Answer:

(a) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$

(b) $\vec{v}_P = \vec{\omega} \times \vec{r}_P$

(c) $\vec{a}_P = \dot{\vec{\omega}} \times \vec{r}_P + \vec{\omega} \times (\vec{\omega} \times \vec{r}_P)$

$$\underset{\sim}{r}_P = x\underset{\sim}{\hat{\imath}} + y\underset{\sim}{\hat{\jmath}} = x'\underset{\sim}{\hat{\imath}}' + y'\underset{\sim}{\hat{\jmath}}'$$

$$\underset{\sim}{r}_P\Big|_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\underset{\sim}{v}_P = \frac{d}{dt}\left[x'\hat{\imath}' + y'\hat{\jmath}'\right] = x'\left[\underset{\sim}{\omega} \times \underset{\sim}{\hat{\imath}}'\right] + y'\left[\underset{\sim}{\omega} \times \underset{\sim}{\hat{\jmath}}'\right] = \underset{\sim}{\omega} \times (x'\underset{\sim}{\hat{\imath}}' + y'\underset{\sim}{\hat{\jmath}}') = \underset{\sim}{\omega} \times \underset{\sim}{r}_P$$

$$\underset{\sim}{a}_P = \dot{\underset{\sim}{\omega}} \times \underset{\sim}{r}_P + \underset{\sim}{\omega} \times \underset{\sim}{v}_P = \dot{\underset{\sim}{\omega}} \times \underset{\sim}{r}_P + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}_P)$$

16.2.3 Assume that in the reference configuration, when $\theta = 0$, the $x'y'$ axes are aligned with the xy axes. Consider two points P_1 and P_2 attached to the moving frame. P_1 and P_2 have coordinates (x_1', y_1') (x_2', y_2'). Find each of the quantities below in terms of some or all of $x_1', y_1', x_2', y_2', \hat{\imath}, \hat{\jmath}, \theta, \dot{\theta}$ and $\ddot{\theta}$.

a) $\vec{r}_{P_1/P_2}$

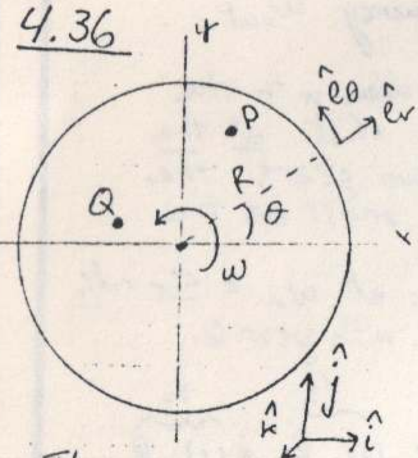
b) $\vec{v}_{P_1/P_2}$

c) $\vec{a}_{P_1/P_2}$

Answer:
(a) $\vec{r}_{P_1/P_2} = (x_2' - x_1')\hat{\imath} + (y_2' - y_1')\hat{\jmath}$
(b) $\vec{v}_{P_1/P_2} = \dot{\theta}\left((x_2' - x_1')\hat{\jmath} - (y_2' - y_1')\hat{\imath}\right)$
(c) $\vec{a}_{P_1/P_2} = \ddot{\theta}\left((x_2' - x_1')\hat{\jmath} - (y_2' - y_1')\hat{\imath}\right) - \dot{\theta}^2\left((x_2' - x_1')\hat{\imath} + (y_2' - y_1')\hat{\jmath}\right)$

SOLUTIONS TO HW 5 • TAM 203 Summer 96
TA: Erich Schneider • July 10, 1996.

4.36



$\underset{\sim}{r}_P = c\hat{\imath} + d\hat{\jmath}$

$\underset{\sim}{r}_Q = 0.5(-d\hat{\imath} + c\hat{\jmath})$

Find $\underset{\sim}{v}_{P/Q}$ & $\underset{\sim}{a}_{P/Q}$

$\underset{\sim}{r}_{P/Q} = \underset{\sim}{r}_P - \underset{\sim}{r}_Q = (c + 0.5d)\hat{\imath} + (d - c)\hat{\jmath}$

Thus, $\underset{\sim}{v}_{P/Q} = \underset{\sim}{\omega} \times \underset{\sim}{r}_{P/Q}$:

$\underset{\sim}{v}_{P/Q} = \omega\hat{k} \times ((c + 0.5d)\hat{\imath} + (d - c)\hat{\jmath}) =$

$\boxed{\underset{\sim}{v}_{P/Q} = \omega(c + 0.5d)\hat{\jmath} - \omega(d - c)\hat{\imath}}$

$\underset{\sim}{a}_{P/Q} = \underset{\sim}{\omega} \times \underset{\sim}{\omega} \times \underset{\sim}{r}_{P/Q} = \underset{\sim}{\omega} \times \underset{\sim}{v}_{P/Q}$

$= \omega\hat{k} \times \omega((c + 0.5d)\hat{\jmath} - \omega(d - c)\hat{\imath})$

$= \boxed{-\omega^2((c + 0.5d)\hat{\imath} + (d - c)\hat{\jmath}) = \underset{\sim}{a}_{P/Q}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

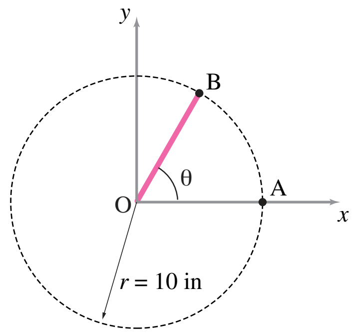
16.2.4 Find $\vec{\boldsymbol{v}} = \vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{r}}$, if $\vec{\boldsymbol{\omega}} = 1.5\,\mathrm{rad/s}\,\hat{\boldsymbol{k}}$ and $\vec{\boldsymbol{r}} = 2\,\mathrm{m}\,\hat{\boldsymbol{\imath}} - 3\,\mathrm{m}\,\hat{\boldsymbol{\jmath}}$.

Answer:
$(5.4\hat{\boldsymbol{\imath}} + 3.0\,\hat{\boldsymbol{\jmath}})\,\mathrm{m/s}$

$\underset{\sim}{v} = \underset{\sim}{\omega} \times \underset{\sim}{r} = 1.5\,\mathrm{rad/s}\,\hat{\underset{\sim}{k}} \times (2\mathrm{m}\,\underset{\sim}{\hat{i}} - 3\mathrm{m}\,\underset{\sim}{\hat{j}}) = [4.5\,\underset{\sim}{\hat{i}} + 3\,\underset{\sim}{\hat{j}}]\,\frac{\mathrm{m}}{\mathrm{s}}$

5.2.6

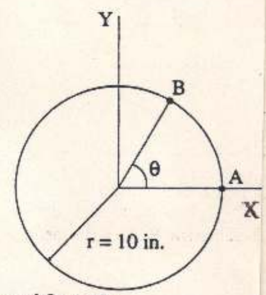
16.2.5 A rod OB rotates with its end O fixed as shown in the figure with angular velocity $\vec{\omega} = 5\,\text{rad/s}\hat{\boldsymbol{k}}$ and angular acceleration $\vec{\alpha} = 2\,\text{rad/s}^2\hat{\boldsymbol{k}}$ at the instant of interest. Find, draw, and label the tangential and normal acceleration of point B at $\theta = 60^\circ$.



Problem 16.5

Answer:
$\vec{a}_t = 20\hat{\boldsymbol{\jmath}}\,\text{in/s}^2$ and $\vec{a}_n = -250\hat{\boldsymbol{\imath}}\,\text{in/s}^2$.

1.46 The tangential acceleration $\underline{\mathbf{a}}_t$ and the normal acceleration $\underline{\mathbf{a}}_n$ of a point in pure circular motion are given by $\underline{\mathbf{a}}_t = \underline{\boldsymbol{\alpha}} \times \underline{\mathbf{r}}$ and $\underline{\mathbf{a}}_n = \underline{\boldsymbol{\omega}} \times (\underline{\boldsymbol{\omega}} \times \underline{\mathbf{r}})$, where $\underline{\boldsymbol{\alpha}}$ is the angular acceleration and $\underline{\mathbf{r}}$ is the position vector of the point. A particle goes around a circle shown in the figure, at the moment of interest, with angular velocity $\underline{\boldsymbol{\omega}} = 5\,\text{rad/s}\,\hat{\mathbf{k}}$ and angular acceleration $\underline{\boldsymbol{\alpha}} = 2\,\text{rad/s}^2\,\hat{\mathbf{k}}$. Find, draw, and label the tangential and normal acceleration of the particle if this occurs at position A.



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Use 2-sides to save paper.

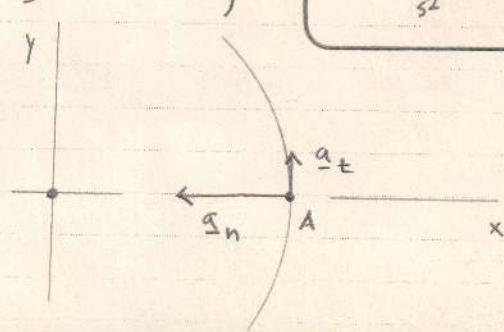
$\underline{a}_t = \underline{\alpha} \times \underline{r}$ $\quad \underline{a}_n = \underline{\omega} \times (\underline{\omega} \times \underline{r})$

at point A, $\underline{r} = 10\,\text{in}\,\hat{\imath}$

$\underline{a}_t = 2\frac{\text{rad}}{s^2}\hat{k} \times 10\,\text{in}\,\hat{\imath} = \boxed{20\,\frac{\text{in}}{s^2}\,\hat{\jmath} = \underline{a}_t}$

Briefly restate problems at start.

$\underline{a}_n = 5\frac{\text{rad}}{s}\hat{k} \times \left(5\frac{\text{rad}}{s}\hat{k} \times 10\,\text{in}\,\hat{\imath}\right) = \boxed{-250\,\frac{\text{in}}{s^2}\,\hat{\imath} = \underline{a}_n}$



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16.2.6 A disc rotates at 15 rpm. How many seconds does it take to rotate by 180 degrees? What is the angular speed of the disc in rad/s?

Answer:
rad/s $= \frac{\pi}{2}$ rad/s.

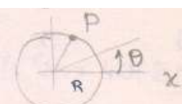
$$N = 15 \text{ rpm} \implies \dot{\theta} = \frac{15 \times 2\pi}{60} \frac{\text{rad}}{\text{s}} = \frac{\pi}{2} \text{ rad/s}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.2.7 A motor turns a uniform disc of radius R counter-clockwise about its mass center at a constant rate ω. The disc lies in the xy-plane and its angular displacement θ is measured (positive counter-clockwise) from the x-axis. What is the angular displacement $\theta(t)$ of the disc if it starts at $\theta(0) = \theta_0$ and $\dot{\theta}(0) = \omega$? What are the velocity and acceleration of a point P at position $\vec{\boldsymbol{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$?

Answer:
$\theta(t) = \theta_0 + \omega t$, $\vec{\boldsymbol{v}} = \omega(x\hat{\boldsymbol{\jmath}} - y\hat{\boldsymbol{\imath}})$, and $\vec{\boldsymbol{a}} = -\omega(x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}})$.

$\theta(0) = \theta_0$ $\quad \dot{\theta}(0) = \omega$ $\quad \underset{\sim}{r} = x\hat{\imath} + y\hat{\jmath}$



$\dot{\theta} = \omega$ $\quad \underset{\sim}{v} = \underset{\sim}{\omega} \times \underset{\sim}{r} = \omega x \hat{\jmath} - \omega y \hat{\imath}$

$\theta\Big|_{\theta_0}^{\theta} = \omega t\Big|_{0}^{t}$ $\quad \underset{\sim}{a} = \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r})$

$\theta(t) = \theta_0 + \omega t$ $\quad \underset{\sim}{a} = -\omega^2 x \hat{\imath} - \omega^2 y \hat{\jmath}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.2.8 Find the angular velocities of the second, minute, and hour hands of a clock.

Answer:
$\omega_{\text{sec}} = \frac{\pi}{30}$ rad/s, $\omega_{\text{min}} = \frac{2\pi}{3600}$ rad/s, and $\omega_{\text{hour}} = \frac{2\pi}{43200}$ rad/s.

15.2.8

$$\omega_{\text{seconds hand}} = 1 \text{ rev/m} = \frac{2\pi}{60} \text{ rad/s} = \frac{\pi}{30} \text{ rad/s}$$

$$\omega_{\text{minutes hand}} = 1 \text{ rev/hr} = \frac{2\pi}{60 \times 60} \text{ rad/s}$$

$$\omega_{\text{hours hand}} = 1 \text{ rev/12 hrs} = \frac{2\pi}{60 \times 60 \times 12} \text{ rad/s}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.2.9 A disc $\mathcal{C}$ spins at a constant rate of two revolutions per second counter-clockwise about its geometric center, G, which is fixed. A point P is marked on the disk at a radius of one meter. At the instant of interest, point P is on the x-axis of an xy-coordinate system centered at point G.

a) Draw a neat diagram showing the disk, the particle, and the coordinate axes.

b) What is the angular velocity of the disk, $\vec{\omega}_{\mathcal{C}}$?

c) What is the angular acceleration of the disk, $\dot{\vec{\omega}}_C$?

d) What is the velocity $\vec{v}_P$ of point P?

e) What is the acceleration $\vec{a}_P$ of point P?

Answer:
(b) $\vec{\omega}_{\mathcal{C}} = 4\pi$ rad/s.
(c) $\dot{\vec{\omega}}_C = 0\,\text{rad/s}^2$.
(d) $\vec{v}_p = 4\pi\,\hat{\jmath}\,\text{m/s}$.
(e) $\vec{a}_p = -16\pi^2\,\hat{\imath}\,\text{m/s}^2$.

15.2:9

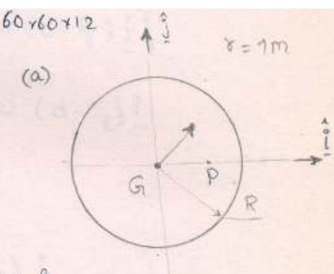
60×60×12

b) $\omega = 2\,\text{rev/s} \Rightarrow \omega_c = 4\pi\,\text{rad/s}$

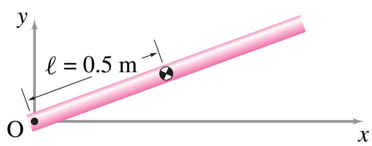
c) $\dot{\underset{\sim}{\omega}}_c = 0$

d) $\underset{\sim}{v}_p = \underset{\sim}{\omega}_c \times \underset{\sim}{r} = 4\pi \frac{\text{rad}}{\text{s}}\hat{\underset{\sim}{k}} \times 1\text{m}\,\hat{\underset{\sim}{\imath}} = 4\pi\,\text{m/s}\,\hat{\underset{\sim}{\jmath}}$

e) $\underset{\sim}{a}_p = \underset{\sim}{\omega}_c \times \underset{\sim}{v}_p = -16\pi^2\,\text{m/s}^2\,\hat{\imath}$



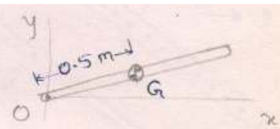
16.2.10 A uniform rigid rod rotates at constant speed in the xy-plane about a peg at point O. The center of mass of the rod may not exceed a specified acceleration $a_{max} = 0.5\,\mathrm{m/s^2}$. Find the maximum angular velocity of the rod.



Problem 16.10

Answer:
$\omega_{\text{max}} = 1\,\text{rad/s}$

15.2.10



$\underset{\sim}{a}_G = \cancel{\dot{\underset{\sim}{\omega}}} \times \underset{\sim}{r}_G + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}_G)$

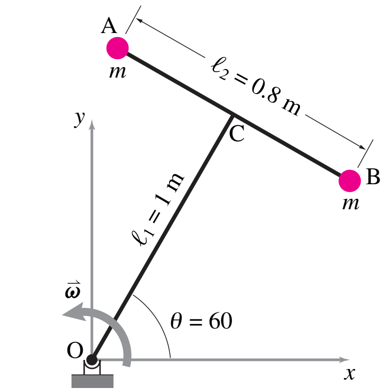
$|\underset{\sim}{a}| = 0.5\,\frac{m}{s^2} \doteq |-\omega^2 l\, \hat{e}_r| \Rightarrow \omega \le \pm 1\,\mathrm{rad/s} \quad \therefore\ \omega_{max} = 1\,\mathrm{rad/s}$

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

16.2.11 A dumbbell AB is welded to a rigid arm OC such that OC is perpendicular to AB. Arm OC rotates about O at a constant angular velocity $\vec{\omega} = 10\,\text{rad/s}\hat{\boldsymbol{k}}$. At the instant when $\theta = 60^\circ$,

a) Find the relative velocity of B with respect to A.

b) Find the acceleration of point B relative to point A.

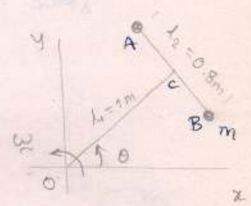


Problem 16.11

Answer:
(a) $\vec{\boldsymbol{v}}_{B/A} = 4\,\text{m/s}(\hat{\boldsymbol{\imath}} + \sqrt{3}\hat{\boldsymbol{\jmath}})$
(b) $\vec{\boldsymbol{a}}_{B/A} = -40\,\text{m/s}^2(\sqrt{3}\hat{\boldsymbol{\imath}} - \hat{\boldsymbol{\jmath}})$

OC ⊥ AB

$\underset{\sim}{\omega} = 10\,\text{rad/s}\,\hat{k}$ & $\theta = 60^\circ$



a) $\underset{\sim}{v}_{B/A} = (\underset{\sim}{\omega} \times \underset{\sim}{r}_{B/A})$

$\underset{\sim}{r}_{B/A} = \underset{\sim}{r}_B - \underset{\sim}{r}_A = (\hat{e}_r - 0.4\hat{e}_\theta) - [1\cdot\hat{e}_r + 0.4\hat{e}_\theta] = [-0.8\hat{e}_\theta]\,\text{m}$

$\underset{\sim}{v}_{B/A} = (\underset{\sim}{\omega} \times \overset{m}{-0.8\hat{e}_\theta}) = 8\,\text{m/s}\,\hat{e}_r = 4(\underset{\sim}{i} + \sqrt{3}\underset{\sim}{j})\,\text{m/s}$

$\underset{\sim}{a}_{B/A} = \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}_{B/A}) = \underset{\sim}{\omega} \times \underset{\sim}{v}_{B/A} = 10\,\text{rad/s}\,\hat{k} \times 8\,\text{m/s}\,\hat{e}_r = 40(-\sqrt{3}\underset{\sim}{i} + \underset{\sim}{j})\ \text{m/s}^2$

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16.2.12 Two discs $\mathcal{A}$ and $\mathcal{B}$ rotate at constant speeds about their centers. Disc $\mathcal{A}$ rotates at 100 rpm and disc $\mathcal{B}$ rotates at 10 rad/s. Which is rotating faster?

Answer:
Disc $\mathcal{A}$ is faster.

15.2.12

∴ Disc A

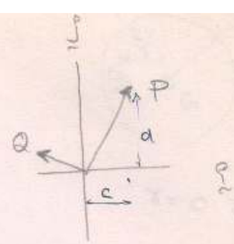
$\omega_A = 100\,\text{rpm} = 100 \times \frac{2\pi}{60} = 10.47\,\text{rad/s}$ is faster than disc B which rotates at $10\,\text{rad/s}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.2.13 A motor turns a uniform disc of radius R counter-clockwise about its mass center at a constant rate ω. The disc lies in the xy-plane and its angular displacement θ is measured (positive counter-clockwise) from the x-axis. What are the velocity and acceleration of a point P at position $\vec{r}_P = c\hat{\imath} + d\hat{\jmath}$ relative to the velocity and acceleration of a point Q at position $\vec{r}_Q = 0.5(-d\hat{\imath} + c\hat{\jmath})$ on the disk? ($c^2 + d^2 < R^2$.)

Answer:
$\vec{v}_{P/Q} = \omega\left((c + \frac{d}{2})\hat{\jmath} - (d - \frac{c}{2})\hat{\imath}\right)$, and
$\vec{a}_{P/Q} = -\omega^2\left((c + \frac{d}{2})\hat{\imath} + (d - \frac{c}{2})\hat{\jmath}\right)$

15.2.13



$$\underset{\sim}{r}_P = c\hat{\underset{\sim}{\imath}} + d\hat{\underset{\sim}{\jmath}}$$

$$\underset{\sim}{r}_Q = 0.5(-d\hat{\underset{\sim}{\imath}} + c\hat{\underset{\sim}{\jmath}})$$

$$\underset{\sim}{r}_{P/Q} = \underset{\sim}{r}_P - \underset{\sim}{r}_Q = \left(c + \frac{d}{2}\right)\hat{\underset{\sim}{\imath}} + \left(d - \frac{c}{2}\right)\hat{\underset{\sim}{\jmath}}$$

$$\underset{\sim}{v}_{P/Q} = \underset{\sim}{\omega} \times \underset{\sim}{r}_{P/Q} = \omega\hat{\underset{\sim}{k}} \times \left[\left(c + \frac{d}{2}\right)\hat{\underset{\sim}{\imath}} + \left(d - \frac{c}{2}\right)\hat{\underset{\sim}{\jmath}}\right]$$

$$= \omega\left(c + \frac{d}{2}\right)\hat{\underset{\sim}{\jmath}} - \omega\left(d - \frac{c}{2}\right)\hat{\underset{\sim}{\imath}}$$

$$\underset{\sim}{a}_{P/Q} = \cancel{\dot{\underset{\sim}{\omega}}}^{\,0} \times \underset{\sim}{r}_{P/Q} + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}) = \underset{\sim}{\omega} \times \underset{\sim}{v}$$

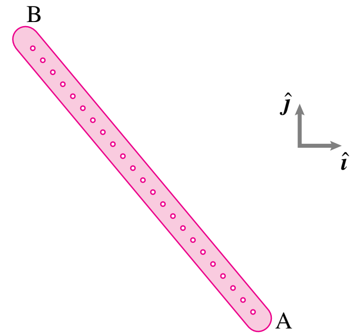
$$= -\omega^2\left(c + \frac{d}{2}\right)\hat{\underset{\sim}{\imath}} - \omega^2\left(d - \frac{c}{2}\right)\hat{\underset{\sim}{\jmath}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.2.14 A 0.4 m long rod AB has many holes along its length such that it can be pegged at any of the various locations. It rotates counter-clockwise at a constant angular speed about a peg whose location is not known. At some instant t, the velocity of end B is $\vec{v}_B = -3\,\mathrm{m/s}\,\hat{j}$. After $\frac{\pi}{20}$ s, the velocity of end B is $\vec{v}_B = -3\,\mathrm{m/s}\hat{\imath}$. If the rod has not completed one revolution during this period,

a) find the angular velocity of the rod, and

b) find the location of the peg along the length of the rod.



Problem 16.14

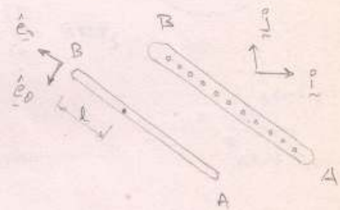
Answer:
(a) $\omega = 30\,\mathrm{rad/s}$.
(b) $\frac{1}{10}$ m from B.

15.2.14

ω = Constant

$V_{B t_1} = -3\,m/s\,\hat{j}$ $\quad \Delta t = \frac{\pi}{20}\,s$

$V_{B t_2} = -3\,m/s\,\hat{i}$



$\Rightarrow \theta = \frac{3\pi}{2}$ rad in $\frac{\pi}{20}\,s \Rightarrow \dot{\theta} = \frac{3\pi}{2}\cdot\frac{20}{\pi}\,\frac{rad}{s} = 30\,rad/s$

$l\omega = 3\,\frac{m}{s}$

$\Rightarrow l = \frac{3\,\frac{m}{s}}{30\,\frac{rad}{s}} \Rightarrow l = 1/10\,m$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

15.2.14

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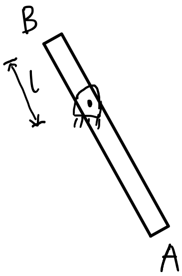
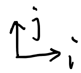




rod AB has length 0.4m
rotates counter-clockwise
about a peg whose location
is unkown.

$\omega = const > 0$ (counter-clockwise)

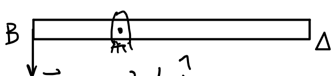
$\vec{V}_{B_{t_1}} = -3 \text{ m/s } \hat{j}$

After $\Delta t = \frac{\pi}{20}$ s

$\vec{V}_{B_{t_2}} = -3 \text{ m/s } \hat{i}$

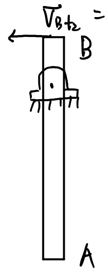
a) Find $\vec{\omega}$

At time t_1



$\vec{V}_{B_{t_1}} = -3 \text{m/s } \hat{j}$

At time t_2 (after $\Delta t = \frac{\pi}{20}$ s
$\vec{V}_{B_{t_2}} = -3\text{m/s } \hat{i}$)



$\Rightarrow \Delta\theta = \frac{3\pi}{2}$ rad in $\Delta t = \frac{\pi}{20}$s

$\omega = \dot{\theta} = \frac{\Delta\theta}{\Delta t} = \frac{3\pi}{2} \text{rad} \Big/ \frac{\pi}{20}\text{s} = 30 \text{ rad/s}.$

b) Find the location of the peg.

$\omega l = V_B$

$\boxed{l = \frac{v_B}{\omega} = \frac{3 \text{ m/s}}{30 \text{ rad/s}} = 0.1 \text{ m}}$ from B.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

HW9-P47-16.2.14

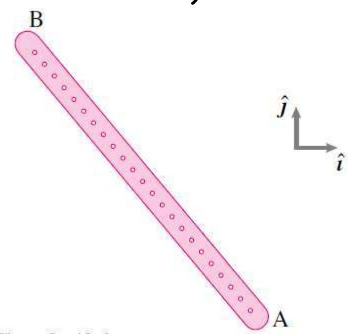
Tuesday, April 6, 2021 3:00 PM

Solution by Michael Zakoworotny

16.2.14 A 0.4 m long rod AB has many holes along its length such that it can be pegged at any of the various locations. It rotates counter-clockwise at a constant angular speed about a peg whose location is not known. At some instant t, the velocity of end B is $\overline{v}_B = -3\,\text{m/s}\,\hat{\jmath}$. After $\frac{\pi}{20}$ s, the velocity of end B is $\overline{v}_B = -3\,\text{m/s}\hat{\imath}$. If the rod has not completed one revolution during this period,

a) find the angular velocity of the rod, and *

b) find the location of the peg along the length of the rod. *



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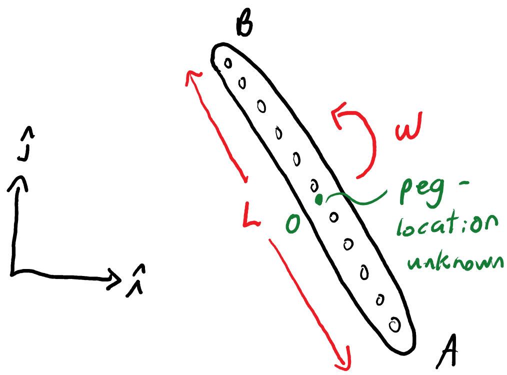
Problem 16.2.14



Given: • $L = 0.4\,m$

• $\vec{v}_B(t = t^*) = -3\,m/s\,\hat{\jmath}$

• $\vec{v}_B(t = t^* + \frac{\pi}{20}) = -3\,m/s\,\hat{\imath}$

• $\vec{\omega} = \omega\hat{k}$, $\omega > 0$ (ccw)

• < 1 full revolution completed

a) Find angular velocity, ω

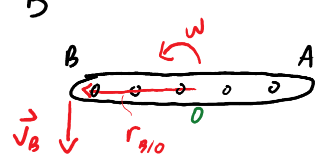
<u>At $t=t^*$</u>: $\vec{v}_B = \vec{\omega} \times \vec{r}_B$

$$-3\hat{\jmath} = \omega\hat{k} \times \vec{r}_B(t^*)$$

need $-\hat{\jmath} = \hat{k} \times -\hat{\imath}$

Must have $\vec{r}_B(t^*) = r_{OB} \cdot -\hat{\imath} = \frac{3}{\omega}(-\hat{\imath})$

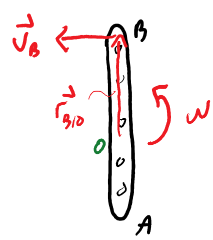
where r_{OB} is distance from the peg to B



<u>At $t=t^*+\frac{\pi}{20}$</u>: $\vec{v}_B = \vec{\omega} \times \vec{r}_B$

$$-3\hat{\imath} = \omega\hat{k} \times \vec{r}_B(t^*+\tfrac{\pi}{20})$$

Must have $\vec{r}_B(t^*+\frac{\pi}{20}) = r_{OB}\,\hat{\jmath} = \frac{3}{\omega}\hat{\jmath}$



It is clear that the rod rotated CCW by $\frac{3\pi}{2}$ from $t=t^*$ to $t^*+\frac{\pi}{20}$ between its two configurations. We can therefore compute the constant ω as:

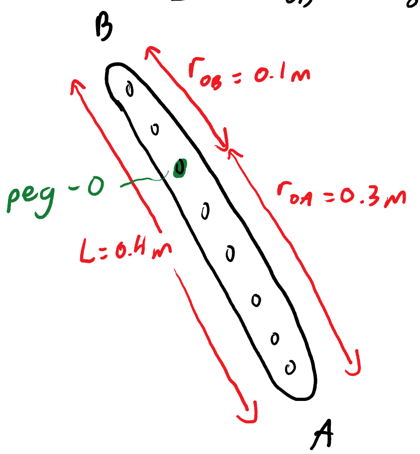
$$\boxed{\omega = \frac{3\pi/2 \text{ rad}}{\pi/20 \text{ s}} = 30 \text{ s}^{-1}}$$

b) Find location of peg

We found in a) that $r_{OB} = \frac{3}{\omega}$. This can also be understood by $|\vec{v}_B| = \omega|\vec{r}_{B/O}|$ for circular motion.

$$\boxed{r_{OB} = \frac{3 \text{ m/s}}{30 \text{ s}^{-1}} = 0.1 \text{ m}}$$

$$L = r_{OB} + r_{OA} \Rightarrow r_{OA} = L - r_{OB}$$

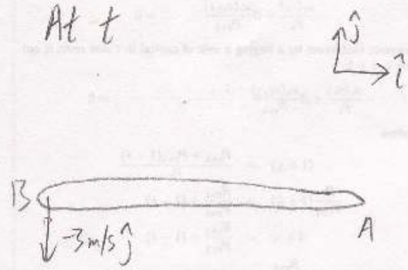


$$r_{OA} = 0.4 \text{m} - 0.1 \text{m}$$

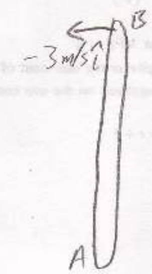
$$\boxed{r_{OA} = 0.3 \text{m}}$$

13.73

At t



At $t + \frac{\pi}{20}$



a)

Assume rod has not completed one rev., it rotates $\frac{3}{4}T$ counter-clockwise where T = period

$$\frac{3}{4}T = \frac{\pi}{20}\ \text{s}$$

$$T = \frac{4\pi}{3(20)} = \frac{\pi}{15}$$

angular velocity $\dot{\theta} = \frac{2\pi}{T} = \frac{30\pi}{\pi} = \boxed{30 \text{ rad/s}}$

b) find location along the length of rod

$$\frac{|v|}{|r|} = \dot{\theta}$$

$$|r| = \frac{|v|}{\dot{\theta}} = \frac{3\text{ m/s}}{30\text{ rad/s}}$$

$$|r| = 0.1\text{ m}$$

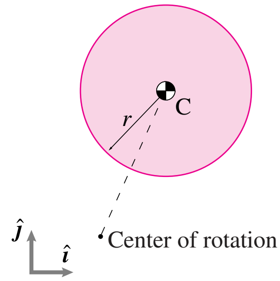
Location the peg = 0.1 m from B

16.2.15 A circular disc of radius $r = 250\,\mathrm{mm}$ rotates in the xy-plane about a point which is at a distance $d = 2r$ away from the center of the disk. At the instant of interest, the linear speed of the center C is $0.60\,\mathrm{m/s}$ and the magnitude of its centripetal acceleration is $0.72\,\mathrm{m/s^2}$.

a) Find the rotational speed of the disk.

b) Is the given information enough to locate the center of rotation of the disk?

c) If the acceleration of the center has no component in the $\hat{\jmath}$ direction at the instant of interest, can you locate the center of rotation? If yes, is the point you locate unique? If not, what other information is required to make the point unique?



Problem 16.15

Answer:
(a) $\dot{\theta} = 1.2\,\mathrm{rad/s}$.
(b) No.
(c) If the acceleration of the center has no component in the $\hat{\jmath}$ direction the center of rotation can lie anywhere on the x-axis.

$r = 250\,\mathrm{mm}$ $\quad V = 0.6\,\mathrm{m/s} = r\omega\,\hat{e}_\theta$

$\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) = -\omega^2 r\,\hat{e}_r$

$\dfrac{|\underline{v}|}{|\underline{a}|} = \dfrac{r\omega}{r\omega^2} \Rightarrow \dfrac{1}{\omega} = \dfrac{0.6\,\frac{m}{s}}{0.72\,\frac{m}{s^2}} \Rightarrow \omega = 1.2\,\mathrm{rad/s}$

b) Not enough to locate center of rotation

c) If the acceleration of center has no $\hat{\underline{i}}$ component

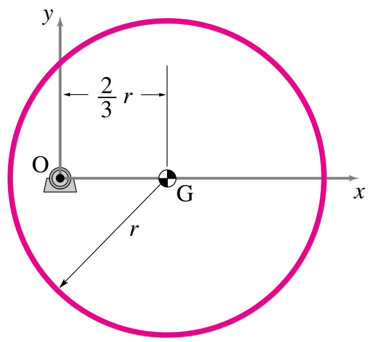
rotation center lies on x-axis, direction $\hat{\underline{i}}$ is required for uniqueness

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.2.16 A uniform disc of radius $r = 200\,\text{mm}$ is mounted eccentrically on a motor shaft at point O. The motor rotates the disc at a constant angular speed. At the instant shown, the velocity of the center of mass is $\vec{v}_G = -1.5\,\text{m/s}\,\hat{\jmath}$.

a) Find the angular velocity of the disc.

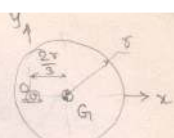
b) Find the point with the highest linear speed on the disc. What is its velocity?



Problem 16.16

Answer:
(a) $\vec{\omega} = -11.25\,\text{rad/s}\hat{\boldsymbol{k}}$
(b) Speed is maximum at $\vec{r} = \frac{5r}{3}\hat{\imath}$; $\vec{v}_{\text{max}} = -3.75\,\text{m/s}\hat{\jmath}$

ω = Constant $\qquad \underset{\sim}{V}_G = -1.5\,\text{m/s}\,\underset{\sim}{\hat{j}}$



$\underset{\sim}{V} = \underset{\sim}{\omega} \times \underset{\sim}{r} = \omega\underset{\sim}{\hat{k}} \times \frac{2}{3} r\,\underset{\sim}{\hat{i}}$

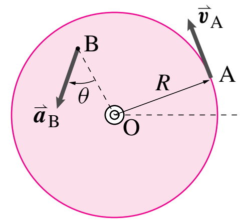
$r = 0.2\,\text{m}$

$\Rightarrow \frac{2\omega r}{3} = -1.5\,\frac{\text{m}}{\text{s}} \Rightarrow \omega = \frac{-4.5\,\frac{\text{m}}{\text{s}}}{2\times 0.2\,\text{m}} \Rightarrow \underset{\sim}{\omega} = -11.25\,\text{rad/s}\,\underset{\sim}{\hat{k}}$

b)
$\underset{\sim}{V}_{max} = \underset{\sim}{\omega} \times \underset{\sim}{r}_{max} = -11.25\,\text{rad/s}\,\underset{\sim}{\hat{k}} \times \left(\frac{5r}{3}\right)\underset{\sim}{\hat{i}} = -3.75\,\text{m/s}\,\hat{j}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

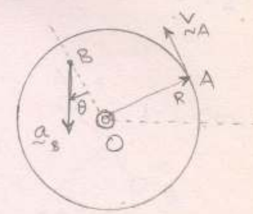
16.2.17 The circular disc of radius $R = 100\,\text{mm}$ rotates about its center O. At a given instant, point A on the disk has a velocity $v_A = 0.8\,\text{m/s}$ in the direction shown. At the same instant, the tangent of the angle θ made by the total acceleration vector of any point B with its radial line to O is 0.6. Compute the angular acceleration α of the disc.



Problem 16.17

Answer:
$\alpha = 38.4\,\text{rad/s}^2$

$R = 0.1\text{m} \qquad |\underset{\sim}{v}_A| = 0.8\,\text{m/s}$



$\tan\theta = 0.6 \Rightarrow \theta = 30.964^\circ$

$\underset{\sim}{v} = r\omega\,\hat{e}_\theta \Rightarrow \omega = \dfrac{|v|}{r} = \dfrac{0.8\,\frac{\text{m}}{\text{s}}}{0.1\,\text{m}} = 8\,\text{rad/s}$

$\underset{\sim}{a}_B = \dot{\omega} \times \underset{\sim}{r} + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r})$

$= \ddot{\theta}\, \underset{\sim}{r}_{B/O}\, \hat{e}_\theta + - r_{B/O}\,\omega^2\, \hat{e}_r$

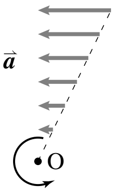
$a\cos\theta = -r_B\omega^2 \quad$ and $\quad a\sin\theta = \ddot{\theta}\, r_{B/O}$

$\tan\theta = \dfrac{|r_{B/O}\ddot{\theta}|}{|-r_{B/O}\omega^2|} \Rightarrow \ddot{\theta} = 0.6 \times \omega^2$

$\ddot{\theta} = 38.4\,\text{rad/s}^2$

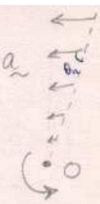
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.2.18 Show that, for non-constant rate circular motion, the acceleration of all points in a given radial line are parallel.



Problem 16.18

$\underset{\sim}{\omega} \neq$ Constant



$$\underset{\sim}{a}_{P_1} = \dot{\omega} \times \underset{\sim}{r}_{P_1/O} + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}_{P_1/O})$$

$$= \dot{\omega}\, r_{P_1/O}\, \hat{\underset{\sim}{e}}_\theta - r_{P_1/O}\, \omega^2\, \hat{\underset{\sim}{e}}_r$$

$$\underset{\sim}{a}_{P_2} = \dot{\omega}\, r_{P_2}\, \hat{\underset{\sim}{e}}_\theta - r_{P_2}\, \omega^2\, \hat{\underset{\sim}{e}}_r$$

$$(\tan\theta_1)^{-1} = \left|\frac{-r_{P_1}\omega^2}{\dot{\omega}\, r_{P_1}}\right|, \overbrace{\left|\frac{-r_{P_2}\omega^2}{\dot{\omega}\, r_{P_2}}\right|}^{\cot\theta_2}, \left|\frac{-r_{P_3}\omega^2}{\dot{\omega}\, r_{P_3}}\right| = \frac{1}{\tan\theta_3}, \ldots$$

As $r_{P_n} = n\, r_{P_1}$ $\quad \theta =$ Constant $\Rightarrow$ All $\underset{\sim}{a}$'s are Parallel

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.2.19 A motor turns a uniform disc of radius R about its mass center at a variable angular rate ω with rate of change $\dot{\omega}$, counter-clockwise. The disc lies in the xy-plane and its angular displacement θ is measured from the x-axis, positive counter-clockwise. What are the velocity and acceleration of a point P at position $\vec{\boldsymbol{r}}_P = c\hat{\boldsymbol{\imath}} + d\hat{\boldsymbol{\jmath}}$ relative to the velocity and acceleration of a point Q at position $\vec{\boldsymbol{r}}_Q = 0.5(-d\hat{\boldsymbol{\imath}} + c\hat{\boldsymbol{\jmath}})$ on the disk? ($c^2 + d^2 < R^2$.)

Answer:
$\vec{\boldsymbol{v}}_{P/Q} = \omega\left((c + \frac{d}{2})\hat{\boldsymbol{\jmath}} - (d - \frac{c}{2})\hat{\boldsymbol{\imath}}\right)$, and
$\vec{\boldsymbol{a}}_{P/Q} = -\left(\dot{\omega}(d - \frac{c}{2}) + \omega^2(c + \frac{d}{2})\right)\hat{\boldsymbol{\imath}} + \left(\dot{\omega}(c + \frac{d}{2}) - \omega^2(d - \frac{c}{2})\right)\hat{\boldsymbol{\jmath}}$

$\dot{\omega}$ = non zero in ccw.

$$\underset{\sim}{r}_{P/Q} = r_P - r_Q = \left(c + \frac{d}{2}\right)\hat{\underset{\sim}{i}} + \left(d - \frac{c}{2}\right)\hat{\underset{\sim}{j}}$$

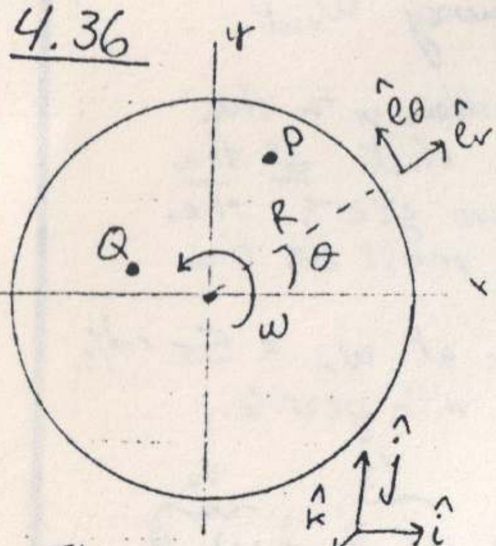
$$\underset{\sim}{v}_{P/Q} = \underset{\sim}{\omega} \times \underset{\sim}{r}_{P/Q} = \omega\left(c + \frac{d}{2}\right)\hat{\underset{\sim}{j}} - \omega\left(d - \frac{c}{2}\right)\hat{\underset{\sim}{i}}$$

$$\underset{\sim}{a}_{P/Q} = \dot{\underset{\sim}{\omega}} \times \underset{\sim}{r}_{P/Q} + \underset{\sim}{\omega} \times \underset{\sim}{v}_{P/Q}$$

$$\underset{\sim}{a}_{P/Q} = \left[-\dot{\omega}\left(d - \frac{c}{2}\right) - \omega^2\left(c + \frac{d}{2}\right)\right]\hat{\underset{\sim}{i}} + \left[\dot{\omega}\left(c + \frac{d}{2}\right) - \omega^2\left(d - \frac{c}{2}\right)\right]\underset{\sim}{j}$$

SOLUTIONS TO HW 5 • TAM 203 Summer 96
TA: Erich Schneider • July 10, 1996.

4.36



$$\underset{\sim}{r}_P = c\hat{i} + d\hat{j}$$

$$\underset{\sim}{r}_Q = 0.5(-d\hat{i} + c\hat{j})$$

Find $\underset{\sim}{v}_{P/Q}$ & $\underset{\sim}{a}_{P/Q}$

$$\underset{\sim}{r}_{P/Q} = \underset{\sim}{r}_P - \underset{\sim}{r}_Q = (c+0.5d)\hat{i} + (d-c)\hat{j}$$

Thus, $\underset{\sim}{v}_{P/Q} = \underset{\sim}{\omega} \times \underset{\sim}{r}_{P/Q}$:

$$\underset{\sim}{v}_{P/Q} = \omega\hat{k} \times ((c+0.5d)\hat{i} + (d-c)\hat{j}) =$$

$$\boxed{\underset{\sim}{v}_{P/Q} = \omega(c+0.5d)\hat{j} - \omega(d-c)\hat{i}}$$

$$\underset{\sim}{a}_{P/Q} = \underset{\sim}{\omega} \times \underset{\sim}{\omega} \times \underset{\sim}{r}_{P/Q} = \underset{\sim}{\omega} \times \underset{\sim}{v}_{P/Q}$$

$$= \omega\hat{k} \times \omega((c+0.5d)\hat{j} - \omega(d-c)\hat{i})$$

$$= \boxed{-\omega^2((c+0.5d)\hat{i} + (d-c)\hat{j}) = \underset{\sim}{a}_{P/Q}}$$

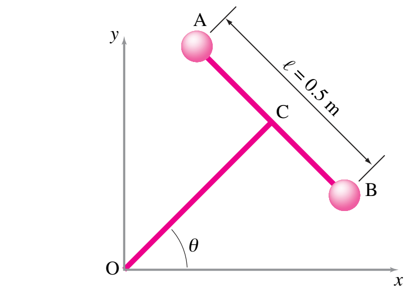
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.2.20 The dumbbell AB shown in the figure rotates counterclockwise about point O with angular acceleration $3\ \mathrm{rad/s^2}$. Bar AB is perpendicular to bar OC. At the instant of interest, $\theta = 45^\circ$ and the angular speed is $2\ \mathrm{rad/s}$.

a) Find the velocity of point B relative to point A. Will this relative velocity be different if the dumbbell were rotating at a *constant* rate of $2\ \mathrm{rad/s}$?

b) Without calculations, draw a vector approximately representing the acceleration of B relative to A.

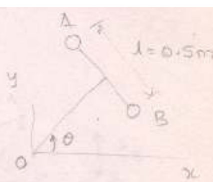
c) Find the acceleration of point B relative to A. What can you say about the direction of this vector as the motion progresses in time?



Problem 16.20

Answer:
(a) $\frac{1}{\sqrt{2}}\ \mathrm{m/s}(\hat{\imath} + \hat{\jmath})$
(c) $\vec{a}_{B/A} = (-0.353\hat{\imath} + 2.474\hat{\jmath})\ \mathrm{m/s^2}$

$\dot{\omega} = 3\ \mathrm{rad/s^2}$ $\quad \omega = 2\ \mathrm{rad/s}$ $\quad \theta = 45^\circ$

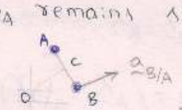


$\underset{\sim}{r}_{B/A} = r_B - r_A = -0.5\,\mathrm{m}\,\hat{\underset{\sim}{e}}_\theta$

$\underset{\sim}{v}_{B/A} = \omega\hat{\underset{\sim}{k}} \times \underset{\sim}{r}_{B/A} = 2\frac{\mathrm{rad}}{\mathrm{s}}\hat{\underset{\sim}{k}} \times -0.5\,\mathrm{m}\,\hat{\underset{\sim}{e}}_\theta = 1\hat{\underset{\sim}{e}}_r \frac{\mathrm{m}}{\mathrm{s}} = \left[\frac{1}{\sqrt{2}}\underset{\sim}{i} + \frac{1}{\sqrt{2}}\underset{\sim}{j}\right]\frac{\mathrm{m}}{\mathrm{s}}$

$\underset{\sim}{v}_{B/A}$ remains same if $\underset{\sim}{\omega}$ = constant

b)



c) $\underset{\sim}{a}_{B/A} = \dot{\underset{\sim}{\omega}} \times \underset{\sim}{r}_{B/A} + \underset{\sim}{\omega} \times \underset{\sim}{v}_{B/A}$

$= 3\hat{\underset{\sim}{k}} \times -0.5\,\hat{\underset{\sim}{e}}_\theta + 2\hat{\underset{\sim}{k}} \times \left(\frac{1}{\sqrt{2}}\underset{\sim}{i} + \frac{1}{\sqrt{2}}\underset{\sim}{j}\right)$

$= \dot{\omega} l\,\hat{\underset{\sim}{e}}_r + \omega l\,\hat{\underset{\sim}{e}}_r = (\dot{\omega} l + \omega l)\,\hat{\underset{\sim}{e}}_r$

$\Rightarrow$ $\underset{\sim}{a}$ is along $\underline{OC}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.2.21 Bit-stream kinematics of a CD. A Compact Disk (CD) has bits of data etched on concentric circular tracks. The data from a track is read by a beam of light from a head that is positioned under the track. The angular speed of the disk remains constant as long as the head is positioned over a particular track. As the head moves to the next track, the angular speed of the disk changes, so that the *linear speed* at any track is always the same. The data stream comes out at a constant rate 4.32×10^6 bits/second. When the head is positioned on the outermost track, for which $r = 56$ mm, the disk rotates at 200 rpm.

a) What is the number of bits of data on the outermost track?
b) find the angular speed of the disk when the head is on the innermost track ($r = 22$ mm), and
c) find the number of bits on the innermost track.

Answer:
(a) 1.296 Megabits.
(b) 509.09 rpm.
(c) 0.509 Megabits

15.2.21

$\underset{\sim}{\omega}$ = Constant ; $r = 0.056$ m : $\omega = 200$ rpm

data stream, $d_s = 4.32e6$ bits/second.

$$t = \frac{2\pi}{\omega} = \frac{2\pi \times 60}{200 \times 2\pi}\,s = 0.3\,s \quad \left(\text{or } t = \frac{2\pi r}{v}\right)$$

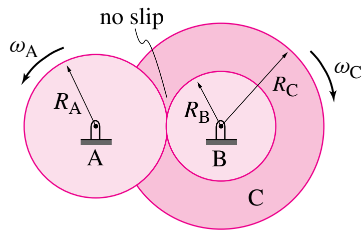
a) No. of bits of data $= d_s \cdot t = 1.296e6$ bits

b) $\omega_1 r_1 = \omega_2 r_2 \Rightarrow \omega_1 = (200 \times 0.056)/0.022 \Rightarrow \omega_{in} = 509.09$ rpm

c) No. of bits of data $= d_s \cdot \frac{2\pi}{\omega_1} = 0.5091e6$ bits

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.2.22 2-D constant rate gear train. The angular velocity of the input shaft (driven by a motor not shown) is a constant, $\omega_{\text{input}} = \omega_A$. What is the angular velocity $\omega_{\text{output}} = \omega_C$ of the output shaft and the speed of a point on the outer edge of disc C, in terms of R_A, R_B, R_C, and ω_A?



Problem 16.22: Gear B is welded to C and engages with A.

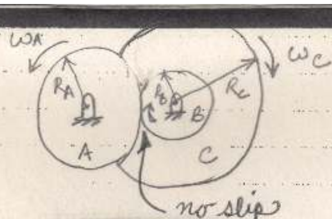
Answer:
$\omega_C = \frac{\omega_A R_A}{R_B}$ and $v_C = \frac{\omega_A R_A R_C}{R_B}$

4.15 Since there is no slip:

$$\omega_A R_A = \omega_B R_B$$

$$\Rightarrow \omega_B = \frac{\omega_A R_A}{R_B}$$

$v_C = ?$



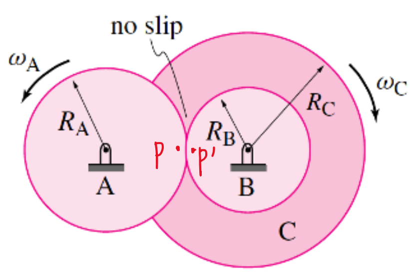
$$\omega_C = \omega_B = \frac{\omega_A R_A}{R_B} \Rightarrow \boxed{\omega_C = \frac{\omega_A R_A}{R_B}}$$

$\boxed{V_C = R_C \omega_C}$ ← speed of a point on the outer edge of disc C : $\boxed{V_C = \frac{\omega_A R_A R_C}{R_B}}$

15.2.22

Saturday, April 6, 2019 10:06 PM

2D constant rate gear train



Filename:ch4-3

Problem 15.2.22: Gear B is welded to C and engages with A.

given:

$W_{input} = W_A$

$R_A \quad R_B \quad R_C$

Find:

$W_{output} = W_C = ?$

$V_C = ?$

(The speed of a point on the outer edge of disc C)

No slip: $\vec{V_P} = \vec{V_{P'}}$

$\omega_A R_A = -\omega_B R_B$

$\Rightarrow \omega_B = -\dfrac{\omega_A R_A}{R_B}$

$$\boxed{W_C = W_B = -\frac{W_A R_A}{R_B}}$$

Note: $\vec{\omega_A} = \omega_A \hat{k}$

$\vec{\omega_B} = \vec{\omega_C} = -\omega_C \hat{k}$

$V_C = R_C W_C$

is speed of a point on the outer edge of disc C

$$\boxed{V_C = -\frac{W_A R_A R_C}{R_B}}$$

MAE 2030 HW9 Problem 48 Solution

16.2.22

Depei Yu

Given: R_A, R_B, R_C, ω_A

Find: $\omega_{output} = \omega_C$

speed of point on outer edge of disc C (v_C)

Solution:

Define a point P on disc A and a point P' on disc B, such that they are in contact. At this point, we can apply the no-slip condition.

P • P'

No-slip condition: $\vec{v}_P = \vec{v}_{P'}$, and $\vec{\omega}_A = \omega_A \hat{k}$, $\vec{\omega}_C = -\omega_C \hat{k}$

$$\vec{\omega}_A \times \vec{r}_{P/A} = \vec{\omega}_C \times \vec{r}_{P'/C}$$

gears B and C are glued together

$$\omega_A \hat{k} \times R_A \hat{\imath} = -\omega_C \hat{k} \times (-R_B)\hat{\imath}$$

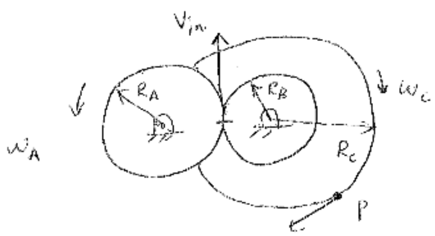
$$\{\omega_A R_A \hat{\jmath} = \omega_C R_B \hat{\jmath}\}$$

$$\{\ \} \cdot \hat{\jmath} \rightarrow \omega_A R_A = \omega_C R_B \rightarrow \omega_C = \frac{\omega_A R_A}{R_B}$$

$$\boxed{\omega_C = \frac{\omega_A R_A}{R_B}} \qquad \boxed{v_C = R_C \omega_C = \frac{\omega_A R_A R_C}{R_B}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

13.81



Given: R_A, R_B, R_C, ω_A

Find ω_C, V_P ?

$\Rightarrow$ For ~~non~~ slip between the gear R_A, R_B

$$V_{in} = \omega_A R_A = \omega_B R_B$$

$$\Rightarrow \omega_B = \omega_A \frac{R_A}{R_B}$$

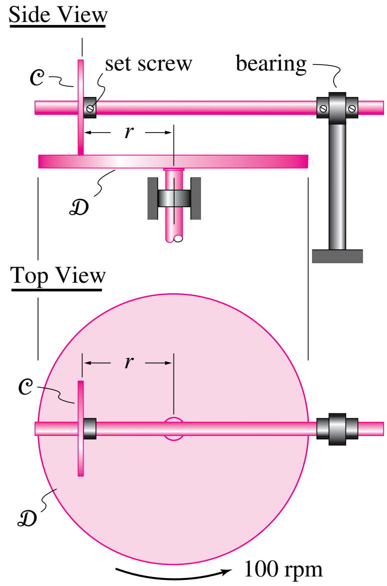
Also $\omega_C = \omega_B$

Thus $\boxed{\omega_C = \omega_A \frac{R_A}{R_B}}$

$$\Rightarrow V_P = \omega_C R_C$$

$$\boxed{V_P = \omega_A \frac{R_A R_C}{R_B}}$$

16.2.23 A horizontal disk $\mathcal{D}$ of diameter $d = 500$ mm is driven at a constant speed of 100 rpm. A small disk $\mathcal{C}$ can be positioned anywhere between $r = 10$ mm and $r = 240$ mm on disk $\mathcal{D}$ by sliding it along the overhead shaft and then fixing it at the desired position with a set screw (see the figure). Disk $\mathcal{C}$ rolls without slip on disk $\mathcal{D}$. The overhead shaft rotates with disk $\mathcal{C}$ and, therefore, its rotational speed can be varied by varying the position of disk $\mathcal{C}$. This gear system is called *brush gearing*. Find the maximum and minimum rotational speeds of the overhead shaft.



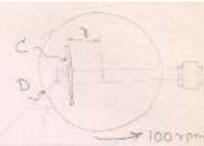
Problem 16.23

Answer:
$\omega_{\text{min}} = 10$ rpm and $\omega_{\text{max}} = 240$ rpm

$d = 0.5$ m $\quad \omega = 100$ rpm

$r_l = 0.01$, $r_h = 0.24$ m

Let $R_c = 0.1$ m (Radius of Disk 'c')



$V = r_1\omega_1 = r_2\omega_2$

$$\omega_{min} = \frac{r_l\omega_1}{R_c} = \frac{0.01 \times 100}{0.1} = 10 \text{ rpm}$$

$$\omega_{max} = \frac{r_h\omega}{R_c} = \frac{0.24 \times 100}{0.1} = 240 \text{ rpm}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.2.24 Two points A and B are on the same machine part that is hinged at an as yet unknown location C. Assume you are given that points at positions $\vec{r}_A$ and $\vec{r}_B$ are supposed to move in given directions, indicated by unit vectors $\hat{\lambda}_A$ and $\hat{\lambda}_B$. For each of the problem parts below, illustrate your results with two numerical examples (in consistent units): i) $\vec{r}_A = 1\hat{\imath}$, $\vec{r}_B = 1\hat{\jmath}$, $\hat{\lambda}_A = 1\hat{\jmath}$, and $\hat{\lambda}_B = -1\hat{\imath}$ (thus $\vec{r}_C = \vec{0}$), and ii) a more complex example of your choosing.

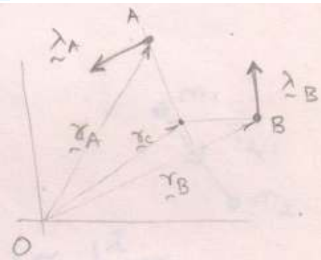
a) Describe in detail what equations must be satisfied by the point $\vec{r}_C$.

b) Write a computer program that takes as input the 4 pairs of numbers $[\vec{r}_A]$, $[\vec{r}_B]$, $[\hat{\lambda}_A]$ and $[\hat{\lambda}_B]$ and gives as output the pair of numbers $[\vec{r}_C]$.

c) Find a formula of the form $\vec{r}_C = \ldots$ that explicitly gives the position vector for point C in terms of the 4 given vectors.

Answer:
(c) $\vec{r}_C = \vec{r}_B + \frac{(\vec{r}_A - \vec{r}_B)\cdot\hat{\lambda}_A}{((\vec{r}_C\times\hat{\lambda}_B)\times\hat{\lambda}_B)\cdot\hat{\lambda}_A}((\vec{r}_C\times\hat{\lambda}_B)\times\hat{\lambda}_B)$

$\underset{\sim}{r}_c = ?$



$\underset{\sim}{r}_A$, $\underset{\sim}{r}_B$ and corresponding

Velocities $\underset{\sim}{\lambda}_A$, $\underset{\sim}{\lambda}_B$ are known.

a) $(\underset{\sim}{r}_{C/A} \cdot \underset{\sim}{\lambda}_A) = 0$ and $(\underset{\sim}{r}_{C/B} \cdot \underset{\sim}{\lambda}_B) = 0$

c) $\underset{\sim}{r}_{C/A} = \{[\underset{\sim}{r}_A \times \underset{\sim}{\lambda}_A] \times \underset{\sim}{\lambda}_A\} \cdot l$ where l is rational no.

$\underset{\sim}{r}_{C/B} = \{[\underset{\sim}{r}_B \times \underset{\sim}{\lambda}_B] \times \underset{\sim}{\lambda}_B\} \cdot m$ " m " "

$\{\underset{\sim}{r}_{C/A} + \underset{\sim}{r}_A = \underset{\sim}{r}_C\} - \{\underset{\sim}{r}_{C/B} + \underset{\sim}{r}_B = \underset{\sim}{r}_C\}$

$\Rightarrow \{l[(\underset{\sim}{r}_A \times \underset{\sim}{\lambda}_A) \times \underset{\sim}{\lambda}_A] - m[(\underset{\sim}{r}_B \times \underset{\sim}{\lambda}_B) \times \underset{\sim}{\lambda}_B] + (\underset{\sim}{r}_A - \underset{\sim}{r}_B) = 0\}$

$\{\ \} \cdot \underset{\sim}{\lambda}_A \Rightarrow 0 - m[(\underset{\sim}{r}_B \times \underset{\sim}{\lambda}_B) \times \underset{\sim}{\lambda}_B] \cdot \underset{\sim}{\lambda}_A + (\underset{\sim}{r}_A - \underset{\sim}{r}_B) \cdot \lambda_A = 0$

$$\Rightarrow \underset{\sim}{r}_C = \underset{\sim}{r}_B + \frac{(\underset{\sim}{r}_A - \underset{\sim}{r}_B) \cdot \underset{\sim}{\lambda}_A}{[(\underset{\sim}{r}_B \times \underset{\sim}{\lambda}_B) \times \underset{\sim}{\lambda}_B] \cdot \underset{\sim}{\lambda}_A} [(\underset{\sim}{r}_B \times \underset{\sim}{\lambda}_B) \times \underset{\sim}{\lambda}_B] \quad //$$

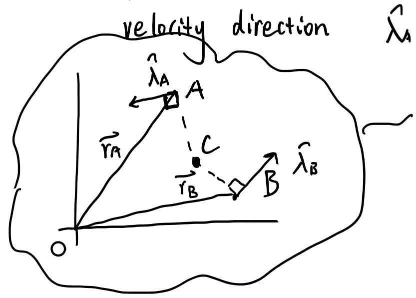
15.2.24

Saturday, April 6, 2019 10:14 PM

Two points A and B are on the same object hinged on an unkown location C.

Given: position $\vec{r}_A$ $\vec{r}_B$

velocity direction $\hat{\lambda}_A$ $\hat{\lambda}_B$ (unit vectors)



object

$\vec{r}_{A/C} \perp \hat{\lambda}_A$

$\vec{r}_{B/C} \perp \hat{\lambda}_B$

$\boxed{\text{Find } \vec{r}_C}$

a) Equations must be satisfied by point $\vec{r}_C$?

$(\vec{r}_A - \vec{r}_C) \cdot \hat{\lambda}_A = 0 \Rightarrow \vec{r}_C \cdot \hat{\lambda}_A = \vec{r}_A \cdot \hat{\lambda}_A$

$(\vec{r}_B - \vec{r}_C) \cdot \hat{\lambda}_B = 0 \qquad \vec{r}_C \cdot \hat{\lambda}_B = \vec{r}_B \cdot \hat{\lambda}_B$

Or: Works for code / pencil

$\hat{\lambda}_A^T * \vec{r}_C = \hat{\lambda}_A \cdot \vec{r}_A$

$\hat{\lambda}_B^T * \vec{r}_C = \hat{\lambda}_B \cdot \vec{r}_B$

$\Downarrow$ Matrix form

$$\begin{bmatrix} \hat{\lambda}_A^T \\ \hat{\lambda}_B^T \end{bmatrix} \begin{bmatrix} \vec{r}_C \end{bmatrix} = \begin{bmatrix} \hat{\lambda}_A \cdot \vec{r}_A \\ \hat{\lambda}_B \cdot \vec{r}_B \end{bmatrix}$$

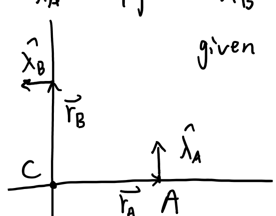
A * x = b;

x = A \ b;

b) Write a program with two examples

i) $\vec{r}_A = 1\hat{i} \qquad \vec{r}_B = 1\hat{j}$

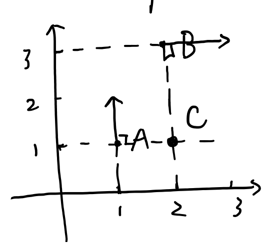
$\hat{\lambda}_A = 1\hat{j} \qquad \hat{\lambda}_B = -1\hat{i}$



given $\vec{r}_A$ $\vec{r}_B$ $\hat{\lambda}_A$ $\hat{\lambda}_B$

C = [0, 0]

ii) more complex one of your choosing



$\vec{r}_A = 1\hat{i} + 1\hat{j}$

$\vec{r}_B = 2\hat{i} + 3\hat{j}$

$\hat{\lambda}_A = 1\hat{j}$

$\hat{\lambda}_B = 1\hat{i}$

Input : 4 pairs of numbers

$[\vec{r}_A]$ $[\vec{r}_B]$ $[\hat{\lambda}_A]$ $[\hat{\lambda}_B]$

Output: 1 pair of number : $[\vec{r}_C]$

See attached matlab

c) Find a formula explicitly of $\vec{r}_C = ?$

given $\vec{r}_A$ $\vec{r}_B$ $\hat{\lambda}_A$ $\hat{\lambda}_B$

$\hat{k}$ is the unit vector perpendicular to the plane (right hand coordinates)

$\vec{r}_{A/C} \perp \hat{k}$, $\vec{r}_{A/C} \perp \hat{\lambda}_A$

$\vec{r}_{A/C}$ is in the direction of $\hat{k} \times \hat{\lambda}_A$

Thus $\vec{r}_{A/C} \parallel \hat{k} \times \hat{\lambda}_A \Rightarrow \vec{r}_{A/C} = m\,\hat{k} \times \hat{\lambda}_A$, m is const (1)

Similarly $\vec{r}_{B/C} = l\,\hat{k} \times \hat{\lambda}_B$, l is const (2)

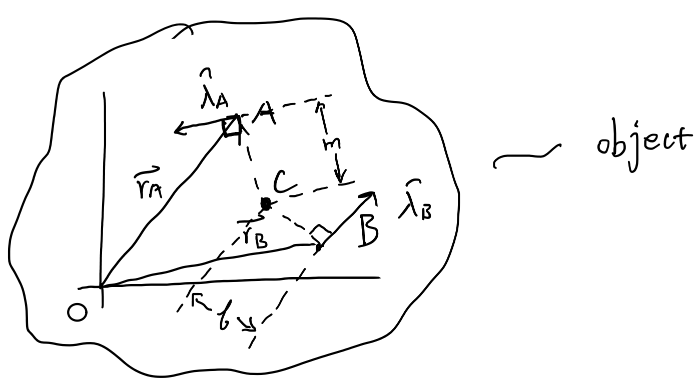
$\vec{r}_C = \vec{r}_C \Rightarrow \vec{r}_A - \vec{r}_{A/C} = \vec{r}_B - \vec{r}_{B/C}$

$\Rightarrow 0 = \{\vec{r}_A - \vec{r}_B - m\hat{k} \times \hat{\lambda}_A + l\,\hat{k} \times \hat{\lambda}_B\}$

$\{\ \} \cdot \hat{\lambda}_A \Rightarrow 0 = (\vec{r}_A - \vec{r}_B) \cdot \hat{\lambda}_A + l(\hat{k} \times \hat{\lambda}_B) \cdot \hat{\lambda}_A$

$$\Rightarrow l = -\frac{(\vec{r}_A - \vec{r}_B) \cdot \hat{\lambda}_A}{(\hat{k} \times \hat{\lambda}_B) \cdot \hat{\lambda}_A}$$

$$\therefore \boxed{\vec{r}_C = \vec{r}_B + \left(\frac{(\vec{r}_A - \vec{r}_B) \cdot \hat{\lambda}_A}{(\hat{k} \times \hat{\lambda}_B) \cdot \hat{\lambda}_A}\right) \hat{k} \times \hat{\lambda}_B}$$



```
% Find the location of the hinge point rC given rA, rB, lambdaA,
 lambdaB
% location of A is given rA = 1i
rA = [1 0];
% location of B is given rB = 1j
rB = [0 1];
% unit vector of velocity of A is given lambdaA = 1j
lambdaA = [0 1];
% unit vector of velocity of B is given lambdaB = -1i;
lambdaB = [-1 0];
rC = FindHinge(rA,rB,lambdaA,lambdaB);
disp('The location vector rC is');
disp(rC);

% Another example
rA = [1 1];
rB = [2 3];
lambdaA = [0 1];
lambdaB = [1 0];
rC = FindHinge(rA,rB,lambdaA,lambdaB);
disp('The location vector rC is');
disp(rC);

function rC = FindHinge(rA,rB,lambdaA,lambdaB)
    % Find the location vector of the hinge rC
    % Equation that rC must satisfy:
    % rC dot lambdaA = rA dot lambdaA
    % rC dot lambdaB = rB dot lambdaB
    % solve for these two equations:
    syms x y
    eqn1 = dot([x y],lambdaA)-dot(rA,lambdaA);
    eqn2 = dot([x y],lambdaB)-dot(rB,lambdaB);
    [x,y] = solve([eqn1,eqn2],[x,y]);
    rC = [x y];
end
```

```
The location vector rC is
[ 0, 0]

The location vector rC is
[ 2, 1]
```

Published with MATLAB® R2018a

#49 Solution Michelle deSouza

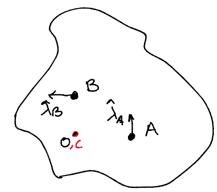
16.2.24 Two points A and B are on the same machine part that is hinged at an as yet unknown location C. Assume you are given that points at positions $\vec{r}_A$ and $\vec{r}_B$ are supposed to move in given directions, indicated by unit vectors $\hat{\lambda}_A$ and $\hat{\lambda}_B$. For each of the problem parts below, illustrate your results with two numerical examples (in consistent units): i) $\vec{r}_A = 1\hat{\imath}$, $\vec{r}_B = 1\hat{\jmath}$, $\hat{\lambda}_A = 1\hat{\jmath}$, and $\hat{\lambda}_B = -1\hat{\imath}$ (thus $\vec{r}_C = \vec{0}$), and ii) a more complex example of your choosing.

a) Describe in detail what equations must be satisfied by the point $\vec{r}_C$.

b) Write a computer program that takes as input the 4 pairs of numbers $[\vec{r}_A]$, $[\vec{r}_B]$, $[\hat{\lambda}_A]$ and $[\hat{\lambda}_B]$ and gives as output the pair of numbers $[\vec{r}_C]$.

c) Find a formula of the form $\vec{r}_C = \ldots$ that explicitly gives the position vector for point C in terms of the 4 given vectors. *

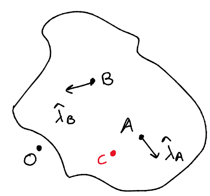
i)



$\vec{r}_A = 1\hat{\imath}$ $\quad \hat{\lambda}_A = 1\hat{\jmath}$

$\vec{r}_B = 1\hat{\jmath}$ $\quad \hat{\lambda}_B = -1\hat{\imath}$

ii)

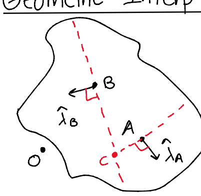


(arbitrary)

$\vec{r}_A = 1.8\hat{\imath} + 0.2\hat{\jmath}$ $\quad \hat{\lambda}_A = 0.64\hat{\imath} - 0.77\hat{\jmath}$

$\vec{r}_B = 0.5\hat{\imath} + 2.3\hat{\jmath}$ $\quad \hat{\lambda}_B = -0.94\hat{\imath} - 0.34\hat{\jmath}$

Geometric Interpretation



$\hat{\lambda}_A \perp \vec{r}_{A/C}$

$\hat{\lambda}_B \perp \vec{r}_{B/C}$

a) Equations that $\vec{r}_c$ must satisfy

For a rigid object, $\dot{\vec{r}}_{A/C} = \omega\hat{k} \times \vec{r}_{A/C}$

$\dot{\vec{r}}_{A/C} = \vec{V}_{A/C} = \vec{V}_A - \cancel{\vec{V}_C}^{\,0}$ C is hinged

$= \vec{V}_A$ → substitute in known direction vector $\hat{\lambda}_A$

$= V_A\hat{\lambda}_A$

$V_A\hat{\lambda}_A = \underline{\omega\hat{k} \times \vec{r}_{A/C}}$ → cross product means that $V_A\hat{\lambda}_A$ is $\perp$ to $\vec{r}_{A/C}$, therefore

$V_A\hat{\lambda}_A \cdot \vec{r}_{A/C} = 0$

$\cancel{V_A}\hat{\lambda}_A \cdot \vec{r}_{A/C} = 0$ → since V_A is a scalar and doesn't affect direction, we can cancel it out

$\hat{\lambda}_A \cdot (\vec{r}_A - \vec{r}_c) = 0$ → distribute the dot product

$$\boxed{\begin{aligned}\hat{\lambda}_A \cdot \vec{r}_A &= \hat{\lambda}_A \cdot \vec{r}_c \\ \hat{\lambda}_B \cdot \vec{r}_B &= \hat{\lambda}_B \cdot \vec{r}_c\end{aligned}}$$

→ can repeat this process with point B

b) Matlab script to output $\vec{r}_c$ given $\vec{r}_A, \vec{r}_B, \hat{\lambda}_A, \hat{\lambda}_B$

[1] Rewrite as a matrix problem $Ax = b$, where $x = \vec{r}_c$, and invert

$\hat{\lambda}_A \cdot \vec{r}_c = \hat{\lambda}_A \cdot \vec{r}_A$

$\hat{\lambda}_B \cdot \vec{r}_c = \hat{\lambda}_B \cdot \vec{r}_B$

$$b = \begin{bmatrix} \hat{\lambda}_A \cdot \vec{r}_A \\ \hat{\lambda}_B \cdot \vec{r}_B \end{bmatrix}$$

$$A = \begin{bmatrix} \hat{\lambda}_A \cdot \hat{\imath} & \hat{\lambda}_A \cdot \hat{\jmath} \\ \hat{\lambda}_B \cdot \hat{\imath} & \hat{\lambda}_B \cdot \hat{\jmath} \end{bmatrix} \quad \text{OR} \quad A = \begin{bmatrix} \hat{\lambda}_A^T \\ \hat{\lambda}_B^T \end{bmatrix}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$\boxed{2}$ Alternatively, solve symbolically as a system of equations

$$\hat{\lambda}_A \cdot (\vec{r}_A - \vec{r}_C) = 0$$
$$\hat{\lambda}_B \cdot (\vec{r}_B - \vec{r}_C) = 0$$

See Methods $\boxed{1}$ and $\boxed{2}$ in Matlab — $\vec{r}_c$ for chosen values in part ii) = $\begin{bmatrix} 1.38 \\ -0.15 \end{bmatrix}$

$\vec{r}_c$ in part i) = $\vec{0}$, confirmed in problem statement

c) Explicit formula for $\vec{r}_c$

$v_A \hat{\lambda}_A = \omega \hat{k} \times \vec{r}_{A/C}$ ← We know that $\hat{k}$ is $\perp$ to $\vec{r}_{A/C}$, and $\hat{\lambda}_A$ is $\perp$ to $\vec{r}_{A/C}$ because of cross product

$\downarrow$

$C_1 (\hat{k} \times \hat{\lambda}_A) = \vec{r}_{A/C}$ ← $C_1 = \frac{v_A}{\omega}$, TBD to get $\vec{r}_c$

$C_1 \hat{k} \times \hat{\lambda}_A = \vec{r}_A - \vec{r}_c$ (A) $\vec{r}_c = -C_1 \hat{k} \times \hat{\lambda}_A + \vec{r}_A$

$\frac{v_B}{\omega}$ → $C_2 \hat{k} \times \hat{\lambda}_B = \vec{r}_B - \vec{r}_c$ (B) $\vec{r}_c = -C_2 \hat{k} \times \hat{\lambda}_B + \vec{r}_B$

} set RHS equal

$$-C_1 \hat{k} \times \hat{\lambda}_A + \vec{r}_A = -C_2 \hat{k} \times \hat{\lambda}_B + \vec{r}_B$$

$C_1 \hat{k} \times \hat{\lambda}_A - C_2 \hat{k} \times \hat{\lambda}_B = \vec{r}_A - \vec{r}_B$ ⟶ in Matlab, $\boxed{3}$ $\underbrace{\begin{bmatrix} \hat{k} \times \hat{\lambda}_A & -\hat{k} \times \hat{\lambda}_B \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \vec{r}_B - \vec{r}_A \end{bmatrix}}_{b}$

$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = A \backslash b$

$C_1 \hat{k} \times \hat{\lambda}_A - C_2 \hat{k} \times \hat{\lambda}_B = \vec{r}_{A/B}$ ⟶ Since $(\hat{k} \times \hat{\lambda}_B)$ is $\perp$ to $\hat{\lambda}_B$, $\{\} \cdot \hat{\lambda}_B$ to get rid of C_2

$\{\} \cdot \hat{\lambda}_B$ $C_1 \hat{k} \times \hat{\lambda}_A \cdot \hat{\lambda}_B = \vec{r}_{A/B} \cdot \hat{\lambda}_B$

$C_1 = \dfrac{\vec{r}_{A/B} \cdot \hat{\lambda}_B}{\hat{k} \times \hat{\lambda}_A \cdot \hat{\lambda}_B}$ → plug into (A)

$$\boxed{\vec{r}_C = \frac{-\vec{r}_{A/B} \cdot \hat{\lambda}_B}{\hat{k} \times \hat{\lambda}_A \cdot \hat{\lambda}_B} (\hat{k} \times \hat{\lambda}_A) + \vec{r}_A}$$

see Method $\boxed{3}$ in Matlab for verification

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

```
%MAE 2030 Solutions
%HW9 #49 (16.2.24)
%Michelle deSouza
close all; clc;

i = [1;0;0];
j=[0;1;0];
k = [0;0;1];
%sample values
rA = 1.8*i + 0.2*j + 0*k; lambdaA = 0.64*i - 0.77*j + 0*k;
rB = 0.5*i + 2.3*j + 0*k; lambdaB = -0.94*i - 0.34*j + 0*k;

%Matrix inversion rC
rC_1 = findRC_1(rA,rB,lambdaA,lambdaB)

%Symbolic solution rC
rC_2 = findRC_2(rA,rB,lambdaA,lambdaB)

%Direct calculation rC (from part c)
rC_3 = findRC_3(rA,rB,lambdaA,lambdaB)



%%%%%%%%%%%METHOD FUNCTIONS BELOW%%%%%%%%%%%
%Method 1: Ax+b form, where rC = x
function rC = findRC_1(rA,rB,lambdaA,lambdaB)
A = [lambdaA';...
     lambdaB'];

b = [dot(lambdaA,rA);...
     dot(lambdaB,rB)];

rC = A\b;
end



%Method 2: Solve symbolically - this method is significantly slower
function rC_ans = findRC_2(rA,rB,lambdaA,lambdaB)
syms rCx rCy
rC = [rCx; rCy; 0];

%0 = lambdaA dot rA/C
%0 = lambdaB dot rB/C
eqA = dot(lambdaA,(rA-rC));
eqB = dot(lambdaB,(rB-rC));

%solve for rC components
[rCx_ans, rCy_ans] = solve(eqA==0, eqB==0, [rCx, rCy]);

rC_ans = [double(rCx_ans); double(rCy_ans)];
end
```


Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

```
%Method 3: Direct solution - formula derived in part c)
function rC = findRC_3(rA,rB,lambdaA,lambdaB)
k = [0;0;1];
%calculate C1 = vA/w, C2 = vB/w
A = [cross(k, lambdaA) -cross(k,lambdaB)];
b = rA - rB;
C = A\b;

rC = rA - C(1)*cross(k,lambdaA);
end
```

```
rC_1 =

    1.3845
   -0.1454
         0


rC_2 =

    1.3845
   -0.1454


rC_3 =

    1.3845
   -0.1454
         0
```

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