16.1.1 Give the definition of each term below in words and, if possible, with an equation.

a) rotation angle

b) (x', y')

c) (x, y)

d) rotation matrix

Answer:
(b) Rotating frame coordinates of a point.
(c) Fixed basis coordinates of a point.
(d) A matrix that when operated on a column vector of fixed basis coordinates gives the coordinates in a rotated frame.

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

All real or imagined lines marked on a rotating rigid object rotate by the same angle, the rotation angle θ.

16.1.2 Assume that in the reference configuration, when $\theta = 0$, that the $x'y'$ axes attached to an object are aligned with the xy axes. Consider a point P attached to the moving frame (object) that has coordinates (x', y'). Find each of the quantities below in terms of some or all of $x', y', \hat{\imath}, \hat{\jmath}, \theta, \dot{\theta}$ and $\ddot{\theta}$.

a) $\vec{r}_P$

b) The rotation matrix $[R]$.

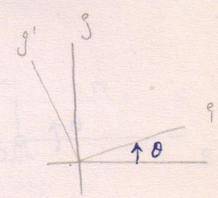
Answer:

(a) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$

(b) $[R] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

15.1.2

(x', y') → Coordinates attached to the moving frame.



$$\imath' = \cos\theta\, \imath + \sin\theta\, \jmath$$

$$\jmath' = -\sin\theta\, \imath + \cos\theta\, \jmath$$

$$\underset{\sim}{r}_P = x \underset{\sim}{\imath} + y \underset{\sim}{\jmath} = x' \underset{\sim}{\imath}' + y' \underset{\sim}{\jmath}'$$

$$\underset{\sim}{r}_P = (x'\cos\theta - y'\sin\theta)\, \underset{\sim}{\imath} + (x'\sin\theta + y'\cos\theta)\, \underset{\sim}{\jmath}$$

$$\underset{\sim}{r}_P = \begin{bmatrix} x'\cos\theta - y'\sin\theta \\ x'\sin\theta + y'\cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.1.3 Assume that in the reference configuration, when $\theta = 0$, that the $x'y'$ axes are aligned with the xy axes. Consider two points P_1 and P_2 attached to the moving frame. P_1 and P_2 have coordinates (x_1', y_1') (x_2', y_2'). Find each of the quantities below in terms of some or all of $x_1', y_1', x_2', y_2', \hat{\imath}, \hat{\jmath}, \theta, \dot{\theta}$ and $\ddot{\theta}$.

a) $\vec{r}_{P_1/P_2}$

b) The rotation matrix $[R]$.

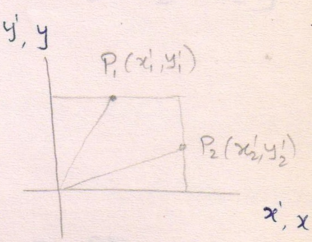
Answer:
(a) $\vec{r}_{P_1/P_2} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} (x_1' - x_2') \\ (y_1' - y_2') \end{bmatrix}$

(b) $[R] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

15.1.3

$$\underset{\sim}{r}_{P_1/P_2} = \left[(x_1'-x_2')\cos\theta - (y_1'-y_2')\sin\theta\right]\underset{\sim}{i} + \left[(x_1'-x_2')\sin\theta + (y_1'-y_2')\cos\theta\right]\underset{\sim}{j}$$

$$[R] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.1.4 The square plate OABC shown in the figure measures $\ell = 1\,\text{m}$ on a side. The coordinate axes $x'y'$ are fixed to the plate and the axes xy are fixed in space. The plate is rotated by an angle $\theta = 60°$ as shown.

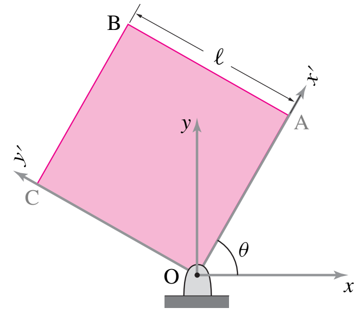
a) Find the coordinates x' and y' of the corner point B in the rotated frame.

b) Find the coordinates x and y of the same point in the fixed frame without using the rotation matrix.

c) Write the rotation matrix R for the given rotation.

d) Find the coordinates of point B in the fixed frame using its (x', y') coordinates and the rotation matrix R.

e) Find the coordinates of point B in the rotated frame using its (x, y) coordinates and the rotation matrix R in its appropriate form.



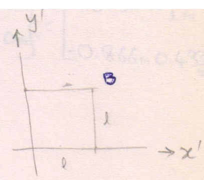
Problem 16.4

Answer:

$[\vec{r}_B]_{x'y'} = \begin{bmatrix} 1\,\text{m} \\ 1\,\text{m} \end{bmatrix}$, $[\vec{r}_B]_{xy} = \begin{bmatrix} -0.37\,\text{m} \\ 1.37\,\text{m} \end{bmatrix}$, $R = \begin{bmatrix} 0.5 & -0.87 \\ 0.87 & 0.5 \end{bmatrix}$

15.1.4

a) $\underset{\sim}{r}_p = l\,\underset{\sim}{i}' + l\,\underset{\sim}{j}'$



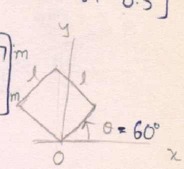
b) $\underset{\sim}{r}_p = x' i' + y' j' = l i' + l j'$

$\underset{\sim}{r}_p = (l\cos\theta - l\sin\theta)\,\underset{\sim}{i} + (l\sin\theta + l\cos\theta)\,\underset{\sim}{j}$

$= [-0.366\,\underset{\sim}{i} + 1.366\,\underset{\sim}{j}]\ m$

c) Rotation Matrix, $R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 0.5 & -0.87 \\ 0.87 & 0.5 \end{bmatrix}$

d) $\begin{bmatrix} x \\ y \end{bmatrix} = [R]\begin{bmatrix} l \\ l \end{bmatrix} = \begin{bmatrix} 0.5 & -0.87 \\ 0.87 & 0.5 \end{bmatrix}\begin{bmatrix} 1m \\ 1m \end{bmatrix} = \begin{bmatrix} -0.37\,m \\ 1.37\,m \end{bmatrix}$



e) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} -0.366\,m \\ 1.366\,m \end{bmatrix} = \begin{bmatrix} 0.9999\,m \\ 0.9999\,m \end{bmatrix} \simeq \begin{bmatrix} 1m \\ 1m \end{bmatrix}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

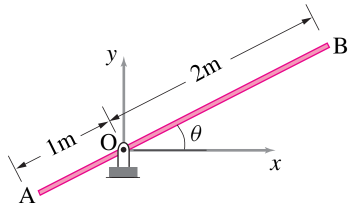
16.1.5 The rod shown in the figure rotates counterclockwise, starting from $\theta = 0$. Find the following quantities.

a) Write the position vector of points A and B as a list of numbers in an array, *e.g.*, $[\vec{\boldsymbol{r}}_A]_{xy} = \begin{bmatrix} x_A \\ y_A \end{bmatrix}$, at $\theta = 0$.

b) Write the rotation matrix R for $\theta = 45°$.

c) Find the coordinates of points A and B using the rotation matrix R for $\theta = 45°$.

d) If the rod rotates at a constant angular speed $\omega = \pi/3\,\text{rad/s}$, find the coordinates of points A and B after 2 seconds, assuming the rod is at $\theta = 0$ when $t = 0$.



Problem 16.5

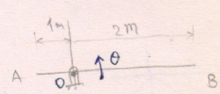
Answer:

(a) $[\vec{\boldsymbol{r}}_A]_{xy} = \begin{bmatrix} -1\,\text{m} \\ 0\,\text{m} \end{bmatrix}$, $[\vec{\boldsymbol{r}}_B]_{xy} = \begin{bmatrix} 2\,\text{m} \\ 0\,\text{m} \end{bmatrix}$

(b) $R = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

(c) $[\vec{\boldsymbol{r}}_A \quad \vec{\boldsymbol{r}}_B]_{xy} = \begin{bmatrix} \frac{-1}{\sqrt{2}} & \sqrt{2} \\ \frac{-1}{\sqrt{2}} & \sqrt{2} \end{bmatrix}$ m

a) $\begin{bmatrix} \underset{\sim}{r}_A & \underset{\sim}{r}_B \end{bmatrix}_{xy} = \begin{bmatrix} -1\text{m} & 2\text{m} \\ 0\text{m} & 0\text{m} \end{bmatrix}$



b) $\theta = 45°$ $\quad R = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

c) $\begin{bmatrix} \underset{\sim}{r}_A & \underset{\sim}{r}_B \end{bmatrix}_{xy} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -1\text{m} & 2\text{m} \\ 0\text{m} & 0\text{m} \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2}\,\text{m} & \sqrt{2}\,\text{m} \\ -1/\sqrt{2}\,\text{m} & \sqrt{2}\,\text{m} \end{bmatrix}$

d) $\omega = \pi/3\ \text{rad/s}$

$\int d\theta = \frac{\pi}{3} \int dt$

$\theta \Big|_0^{\theta_0} = \frac{\pi t}{3} \Big|_0^{t_0} \Rightarrow \theta_0 = \frac{\pi t_0}{3}$; $t_0 = 2\text{s} \Rightarrow \theta_0 = \frac{2\pi}{3}$ rad

$\begin{bmatrix} \underset{\sim}{r}_A & \underset{\sim}{r}_B \end{bmatrix}_{xy}^{t=2s} = \begin{bmatrix} -0.5 & -0.866 \\ 0.866 & -0.5 \end{bmatrix} \begin{bmatrix} -1\text{m} & 2\text{m} \\ 0\text{m} & 0\text{m} \end{bmatrix} = \begin{bmatrix} +0.5\text{m} & -1\text{m} \\ -0.866\text{m} & 0.433\text{m} \end{bmatrix}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.1.6 Find the angle of rotation corresponding to the following rotation matrices:

a) $\begin{bmatrix} 0.7071 & -0.7071 \\ 0.7071 & 0.7071 \end{bmatrix}$.

b) $\begin{bmatrix} -0.7071 & -0.7071 \\ 0.7071 & -0.7071 \end{bmatrix}$.

c) $\begin{bmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{bmatrix}$.

Answer:
(a) $\theta = 45°$
(b) $\theta = 135°$
(c) $\theta = 315°$

15.1.6

a) $\begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix} \Rightarrow \theta = 45°$

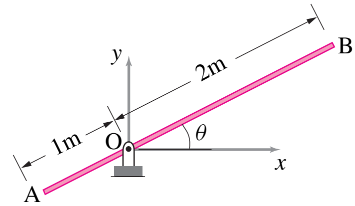
b) $\begin{bmatrix} -0.707 & -0.707 \\ 0.707 & -0.707 \end{bmatrix} \Rightarrow \theta = 135°$

c) $\begin{bmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{bmatrix} \Rightarrow \theta = 315°$

16.1.7 The rod shown in the figure rotates with constant angular speed $\omega = 10\,\text{rad/s}$. $\theta = 0$ at $t = 0\,\text{s}$.

a) Find the position vector $\vec{\boldsymbol{r}}_B|_{t_1}$ and the velocity $\vec{\boldsymbol{v}}_B|_{t_1}$ of point B at $t_1 = 1\,\text{s}$.

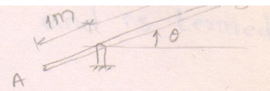
b) Find the position vector $\vec{\boldsymbol{r}}_B|_{t_2}$ of point B at $t_2 = 1.2\,\text{s}$. What is the net displacement of point B during the time interval $\Delta t = t_2 - t1$? Is this net displacement equal to $\vec{\boldsymbol{v}}_B|_{t_1}\Delta t$? Why not?



Problem 16.7

Answer:
(a) $\vec{\boldsymbol{r}}_B|_{t_1} = -1.68\hat{\boldsymbol{\imath}} - 1.09\hat{\boldsymbol{\jmath}}$ and $\vec{\boldsymbol{v}}_B|_{t_1} = 20\,\text{m/s}\,[-0.84 \quad 0.54]$
(b) Net displacement $= (3.366\hat{\boldsymbol{\imath}} + 0.016\hat{\boldsymbol{\jmath}})\,\text{m}$. The net displacement is not equal to $\vec{\boldsymbol{v}}_B|_{t_1}\Delta t$ because the motion is not along a straight line.

$\omega = 10\,\text{rad/s}$

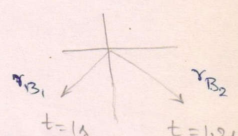


$\theta\Big|_0^\theta = 10t\Big|_0^t \Rightarrow \theta\Big|_{t=1} = 10\,\text{radians}$

$\Rightarrow \vec{r}_B = [R]\begin{bmatrix}2\\0\end{bmatrix} \Rightarrow r_B = \begin{bmatrix}-0.839 & 0.544\\ -0.544 & -0.839\end{bmatrix}\begin{bmatrix}2\\0\end{bmatrix} = \begin{bmatrix}-1.68\\-1.09\end{bmatrix}$

$\underset{\sim}{V}_B = r\dot{\theta}\,\underset{\sim}{\hat{e}_\theta} = 20\,\text{m/s}\,\underset{\sim}{\hat{e}_\theta}$

$t = 1.2\,\text{s} \Rightarrow \theta_{B_2} = 12\,[\text{rad}] \triangleq 687.55\,[\text{rad}]$



$r_{B_2} = \begin{bmatrix}0.844 & 0.536\\ -0.536 & 0.844\end{bmatrix}\begin{bmatrix}2\,\text{m}\\0\,\text{m}\end{bmatrix} = \begin{bmatrix}1.688\,\text{m}\\-1.072\,\text{m}\end{bmatrix}$

$\Delta t = t_2 - t_1 = 0.2\,\text{s}$

$\Delta R = R_2 - R_1 = [3.366\,\underset{\sim}{i} + 0.016\,\underset{\sim}{j}]\,\text{m}$

$V_{B_{t_1}} = 20\,\text{m/s}\cdot\underset{\sim}{\hat{e}_\theta} = [-20\sin 10^\circ\,\underset{\sim}{i} + 20\cos 10^\circ\,\underset{\sim}{j}]\,\frac{\text{m}}{\text{s}} = [10.88\,\underset{\sim}{i} - 16.78\,\underset{\sim}{j}]\,\frac{\text{m}}{\text{s}}$

$V_{B_{t_1}}\cdot\Delta t = [2.176\,\underset{\sim}{i} - 3.356\,\underset{\sim}{j}]\,\text{m} \neq \Delta R$ ∵ The angle between r_{B_1} & r_{B_2} is large.

15.1.8

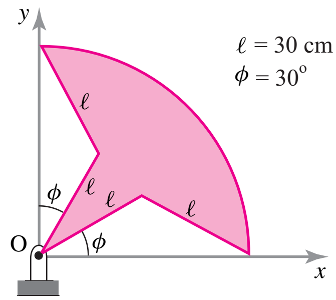
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

16.1.8 Write a computer program to animate the rotation of an object. Your input should be a set of x and y coordinates defining the object (such that `plot y vs x` draws the object on the screen) and the rotation angle θ. The output should be the rotated coordinates of the object.

a) From the geometric information given in the figure, generate coordinates of enough points to define the given object.

b) Using your program, plot the object at $\theta = 20°, 60°, 100°, 160°$, and $270°$.

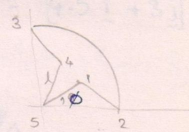
c) Assume that the object rotates with constant angular speed $\omega = 2\,\mathrm{rad/s}$. Find and plot the position of the object at $t = 1\,\mathrm{s},\ 2\,\mathrm{s},$ and $3\,\mathrm{s}$.



Problem 16.8

Answer:
(a) (0, 0); $(15\sqrt{3},\ 15)$; $(30\sqrt{3},\ 0)$; $(0,\ 30\sqrt{3})$; $(15,\ 15\sqrt{3})$.

a) $\ell = 30\text{ cm}$; $\phi = 30°$



① $(30\cos 30,\ 30\sin 30)_{cm} = (15\sqrt{3}, 15)$

② $(60\cos 30,\ 0)_{cm} = (30\sqrt{3}, 0)$

③ $(0,\ 60\cos 30°) = (0, 30\sqrt{3})$

④ $(30\sin 30,\ 30\cos 30) = (15, 15\sqrt{3})$

⑤ $(0, 0)$ $(0, 0)$

c) $\omega = 2\,^c/_s \Rightarrow \theta = 2t \Rightarrow \theta|_{t=1s} = 2^c = 114.59°$; $\theta|_{t=2s} = 229.18°$; $\theta|_{t=3s} = 343.77°$

15.1.8

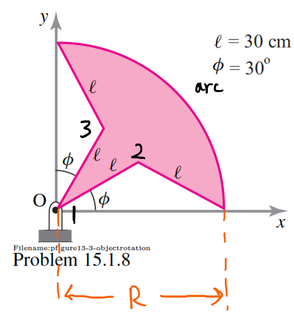
Wednesday, April 3, 2019 8:52 PM

Animation of rotation

Input : x-y coordinates
rotation angle

Output : rotated coordinates

a) Generate coordinates of enough points to define the object. Unit: cm



Problem 15.1.8

```
% Given length and angle as shown in the figure
l = 30;
phi = deg2rad(30);
% The radius of arc part
R = 2*l*cos(phi);
% Coordinates of points labeled on the plot
point1 = [0,0]';
point2 = l*[cos(phi)      , sin(phi)]';
point3 = l*[cos(pi/2-phi), sin(pi/2-phi)]';
% Calculate the x-y coordinates of arc part
arc_angle = linspace(0,pi/2,30);
point_arc = R*[cos(arc_angle);sin(arc_angle)];
```

Need more points to generate the arc

$(x, y) = R(\cos\theta, \sin\theta) \quad 0 \le \theta \le \frac{\pi}{2}$

Matlab can only draw straight line between two points

b) c) See attached

Below are two user-defined functions.
RotCoor is to calculate coordinates
Animate is to do animation of rotation

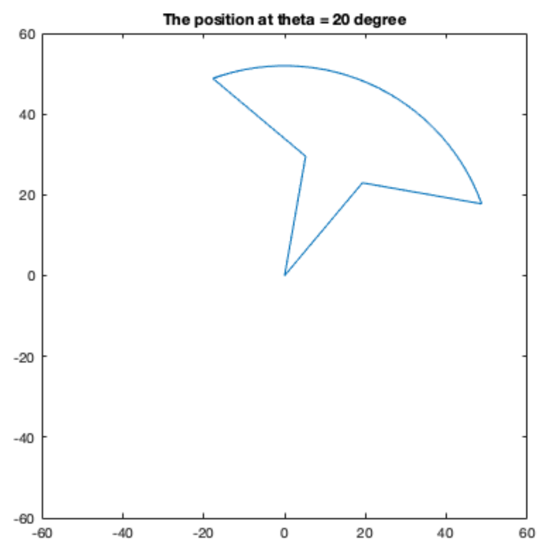
```
function p = RotCoor(p0,angle)
% This function calculates the coordinates after rotation of angle radians
% Input p0 are coordinates that define the object
% angle is the rotation angle
% Output p are coordinates that define the object after rotation

    % Rotation matrix
    R = [cos(angle) -sin(angle);
         sin(angle)  cos(angle)];
    p = R*p0;
end
```

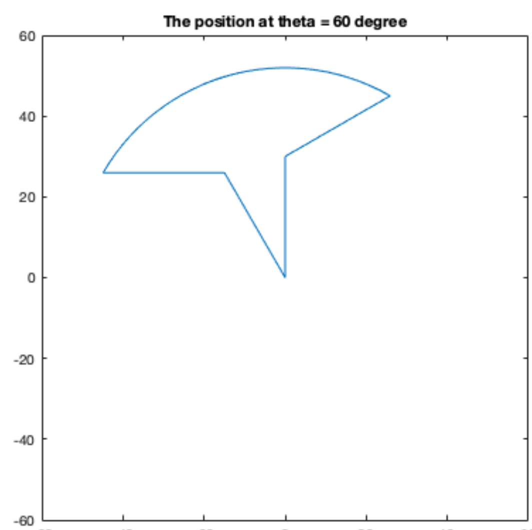
```
function Animate(p0,theta)
% This function animates the rotation of the given figure
% with rotation angle from 0 to theta
% theta is the final rotation angle through the whole process

    angle = linspace(0,theta,100);
    figure
    for k = 1:length(angle)
        % Calculate the x-y coordinates after rotation
        p = RotCoor(p0,angle(k));
        plot(p(1,:),p(2,:))
        axis equal
        axis([-60 60 -60 60])
        pause(0.01);
    end
end
```

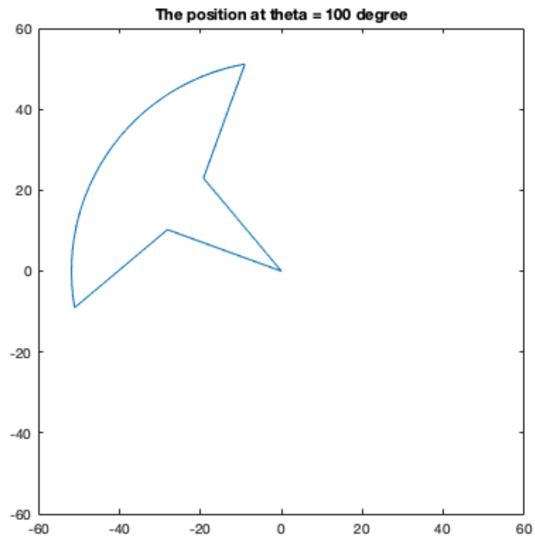
```
% Problem 15.1.8
% Animate the rotation of a given object
clc;
clear;
close all;

% Given length and angle as shown in the figure
l = 30;
phi = deg2rad(30);
% The radius of arc part
R = 2*l*cos(phi);
% Coordinates of points labeled on the plot
point1 = [0,0]';
point2 = l*[cos(phi)     , sin(phi)]';
point3 = l*[cos(pi/2-phi), sin(pi/2-phi)]';
% Calculate the x-y coordinates of arc part
arc_angle = linspace(0,pi/2,30);
point_arc = R*[cos(arc_angle);sin(arc_angle)];
% points that define the figure
p0 = [point1, point2, point_arc, point3, point1];
```

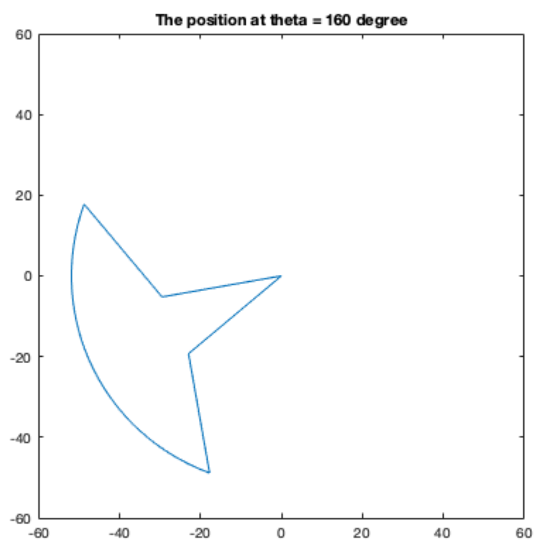
Part b plot the object at theta = 20 60 100 160 270 degrees

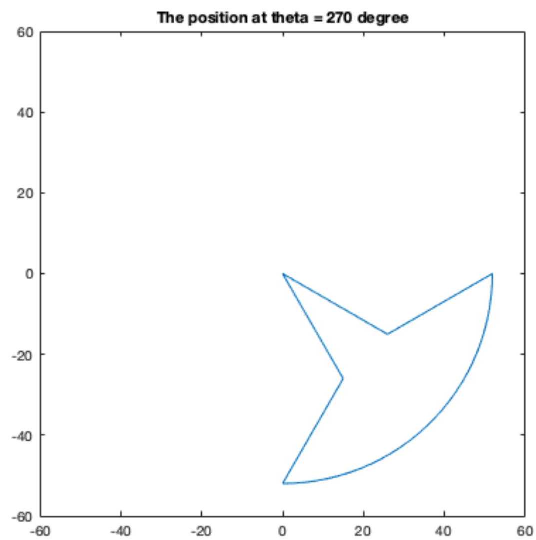
```
theta = [20 60 100 160 270];
for k = 1:length(theta)
    Animate(p0,deg2rad(theta(k)));
    msg = sprintf('The position at theta = %d degree',theta(k));
    title(msg)
end
```


The position at theta = 20 degree
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60


The position at theta = 60 degree
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60


The position at theta = 100 degree
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60

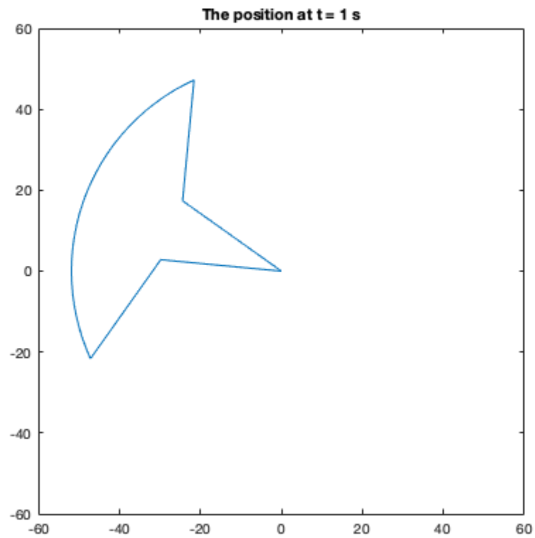

The position at theta = 160 degree
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60

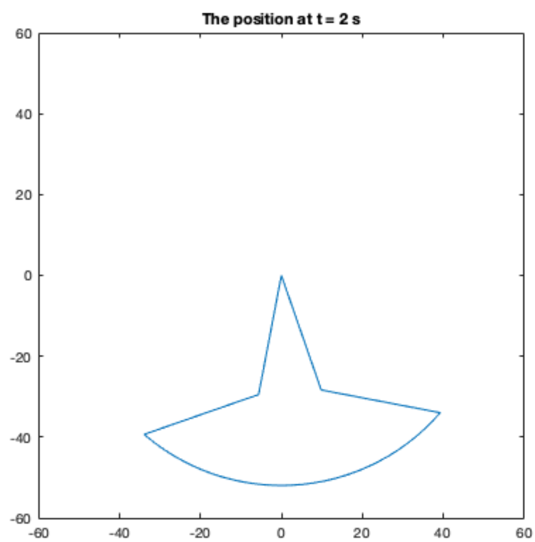


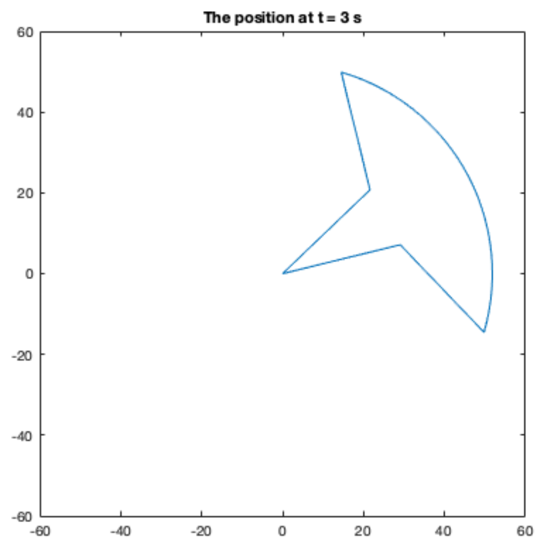
Part c plot the position of the object at t = 1,2 and 3s

The object rotates with constant angular speed omega = 2m/s

```
omega = 2;
for t = 1:3
        Animate(p0,omega*t);
        msg = sprintf('The position at t = %d s',t);
        title(msg)
end
```


The position at t = 1 s
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60


The position at t = 2 s
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60



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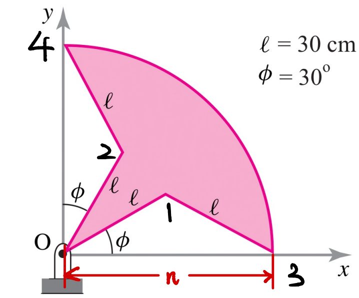
Katherine Cao

16.1.8 Write a computer program to animate the rotation of an object. Your input should be a set of x and y coordinates defining the object (such that `plot y vs x` draws the object on the screen) and the rotation angle θ. The output should be the rotated coordinates of the object.

a) From the geometric information given in the figure, generate coordinates of enough points to define the given object. *

b) Using your program, plot the object at $\theta = 20°, 60°, 100°, 160°$, and $270°$.

c) Assume that the object rotates with constant angular speed $\omega = 2\,\mathrm{rad/s}$. Find and plot the position of the object at $t = 1\,\mathrm{s}, 2\,\mathrm{s}$, and $3\,\mathrm{s}$.



Filename:pfigure13-3-objectrotation
Problem 16.1.8

Define θ as the counterclockwise angle rotated

2D rotation matrix [R] from P755 of textbook

$$\begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}_{[R]} \begin{bmatrix} x' \\ y' \end{bmatrix}, \quad \text{or}$$

$$[\vec{r}_P]_{xy} = [R]\,[\vec{r}_P]_{x'y'}, \quad \text{or} \qquad (16.8)$$

$$[\vec{r}_P]_{xy} = [R]\left[\vec{r}_P^{\,\mathrm{ref}}\right]_{xy}.$$

a) Generate coordinates of enough points to define the object

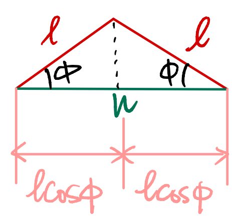
Point O : $(0, 0)$

Point 1 : $(\ell\cos\phi, \ell\sin\phi)$

Point 2 : $(\ell\sin\phi, \ell\cos\phi)$

Point 3 : $(n, 0) = (2\ell\cos\phi, 0)$

Point 4 : $(0, n) = (0, 2\ell\cos\phi)$



$\frac{\pi}{2}$ arc from 3 to 4

a group of points $(x_i, y_i) = (n\cos\alpha_i, n\sin\alpha_i)$

for $0 \le \alpha_i \le \frac{\pi}{2}$, in MATLAB, generate a group of α using linspace (0, pi/2, itotal)

In order to plot, in MATLAB, arrange points as:

Point O ⟶ Point 1 ⟶ arc 3→4 ⟶ Point 2 ⟶ Point O

b) Use Program to plot object at

$\theta = 20°, 60°, 100°, 160°$, and $270°$

With rotation matrix $[R] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

rotate all the points $\begin{bmatrix} x \\ y \end{bmatrix}$ used to define the object by θ:

$$\begin{bmatrix} x_{rotated} \\ y_{rotated} \end{bmatrix} = [R] \begin{bmatrix} x \\ y \end{bmatrix}$$

c) object rotates with constant angular velocity $\omega = 2\,rad/s$

Find & plot object at $t = 1s, 2s$, and $3s$

rotated angle at some time t,

since $\omega = 2\,rad/s$, constant

$$\theta = \omega t$$

about Animation:

dividing up total rotated angle for each "step",

plot for each step, pause

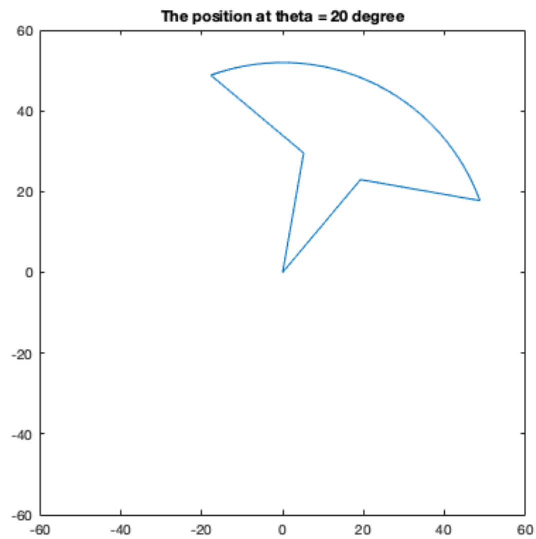
Table of Contents

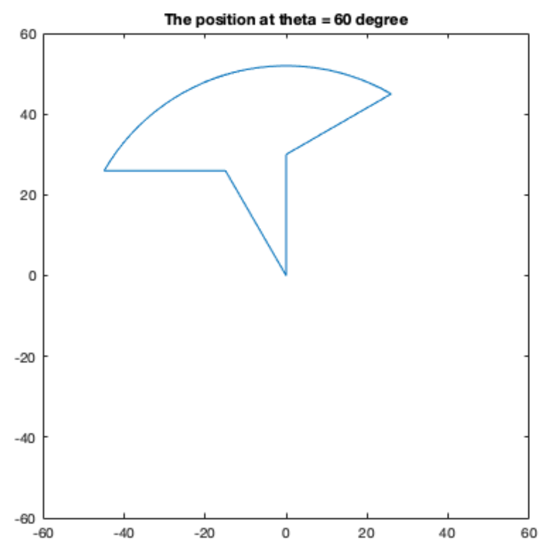

```
% MAE2030
% HW9_P46_16.1.8
% Animate the rotation of a given object
% Katherine Cao edited from Shuwen Lu 2019 version

clc;
clear;
close all;

% Given length and angle as shown in the figure
l = 30; %cm
phi = deg2rad(30); %rad

%The radius of arc part
n = 2*l*cos(phi);

% Coordinates of points labeled on the plot
point0 = [0;0];
point1 = [l*cos(phi);l*sin(phi)];
point2 = [l*sin(phi);l*cos(phi)];

% Generate the group of points
% To plot the pi/2 arc between point3 and point4
alpha = linspace(0,pi/2,30);
arc_xiyi = [n*cos(alpha);n*sin(alpha)];

% points that define the figure
% tracing between these points, order is deliberate
plot_points = [point0, point1, arc_xiyi, point2, point0];
```

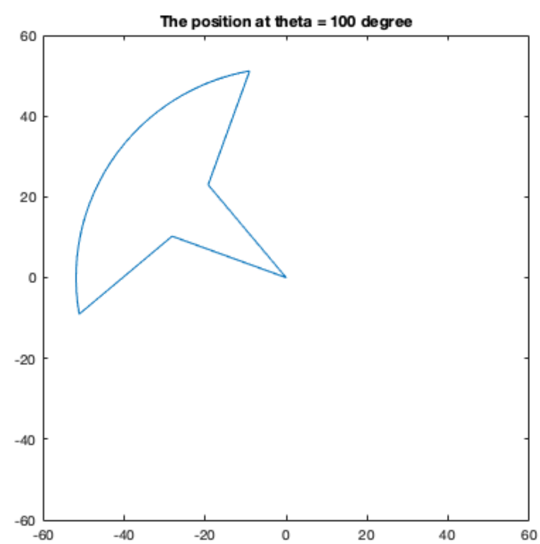
Part b plot the object at theta = 20 60 100 160 270 degrees

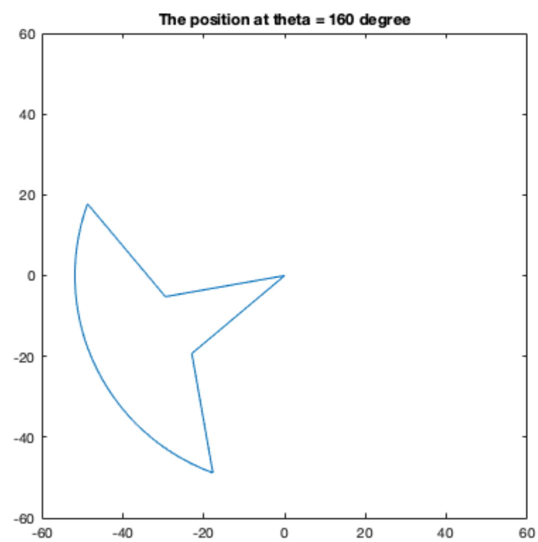
```
theta = [20 60 100 160 270];
for k = 1:length(theta)
    newpoints = rotate(plot_points,deg2rad(theta(k)));
    figure
    plot(newpoints(1,:),newpoints(2,:))
    axis equal
    axis([-60 60 -60 60])
    msg = sprintf('The position at theta = %d degree', theta(k));
```

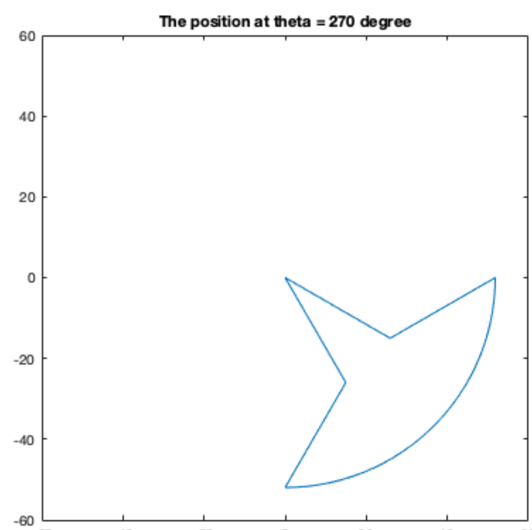
```
    title(msg)
end
```




The position at theta = 60 degree
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60


The position at theta = 100 degree
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60





Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Part c plot the position of the object at t=1,2, and 3s

The object rotates with constant angular speed omega = 2rad/s

```
omega = 2;

for t=1:3
    theta = omega*t; %constant angular velocity, rotated angle at t
    newpoints = rotate(plot_points,theta);
    figure
    plot(newpoints(1,:),newpoints(2,:))
    axis equal
    axis([-60 60 -60 60])
    msg = sprintf('The position at t = %d s',t);
    title(msg)
end
```

helper functions

```
function newpoints = rotate(points,angle)
% This function calculates the coordinates after
% 2D rotation of angle radians, angle is positive counterclockwise
% Input points are the group of x,y coordinates that define the object
% points(1,:) are all the x coordinates
% points(2,:) are all the y coordinates
% Output newpoints are coordinates that define the object after
 rotation

    % Rotation matrix
    R = [cos(angle) -sin(angle);
         sin(angle)  cos(angle)];
    newpoints = R*points;
end

function Animateb(points,theta,totalstep)
% This function animates the rotation of the given figure
% with rotation angle from 0 to theta
% theta is positive counterclockwise
% theta is the final rotation angle through the whole process
% totalstep defines how many points in between 0 and theta in
 animation

    angle = linspace(0,theta,totalstep);
    figure
    for k = 1:length(angle)
        % Calculate the x-y coordinates after rotation
        newpoints = rotate(points,angle(k));
        plot(newpoints(1,:),newpoints(2,:))
        axis equal
        axis([-60 60 -60 60])
```

```
        msg = sprintf('rotated %.2f deg',rad2deg(angle(k)));
        title(msg)
        pause(0.05);
    end
end

function Animatec(points,omega,t,totalstep)
% This function animates the rotation of the given figure
% with constant angular velocity omega rad/s from 0 to t seconds
% omega positive counterclockwise
% totalstep defines how many points in between 0 and theta in
 animation
    theta = omega*t;
    angle = linspace(0,theta,totalstep);
    timestep = linspace(0,t,totalstep);
    figure
    for k=1:length(angle)
        % Calculate the x-y coordinates after rotation
        newpoints = rotate(points,angle(k));
        plot(newpoints(1,:),newpoints(2,:))
        axis equal
        axis([-60 60 -60 60])
        msg = sprintf('rotated %.2f s %.2f rad',timestep(k),angle(k));
        title(msg)
        pause(0.05);
    end
end
```

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Animation for b

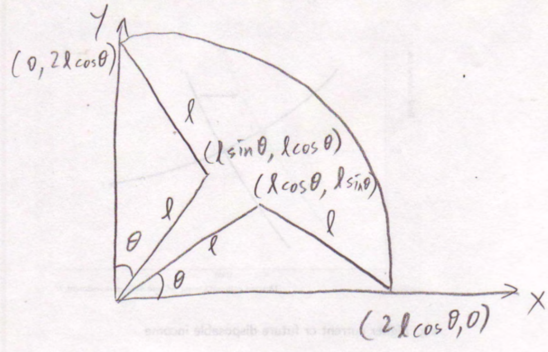
```
theta = [20 60 100 160 270];
totalstep = 40; % can be changed
%can swap in different theta
Animateb(plot_points,deg2rad(theta(1)),totalstep)
```

Animation for c

```
omega = 2; %rad/s
t = 3; %s, can swap in any time
totalstep = 40; %can be changed
Animatec(plot_points,omega,t,totalstep)
```

13.58

a)



$l = 30\,cm$

$\theta = 30^\circ$

b) see plots.

c) At $t = 1s$

object rotates $(2\,rad/s)\cdot(1s) = 2\,rad = 114.6^\circ$

At $t = 2s$

object rotates $4\,rad = 229.18^\circ$

At $t = 3s$

object rotates $6\,rad = 343.77^\circ$

```
% Johannes Feng
% Solution to 13.58b, due 10/21/08

function animation_b

% CLEAN UP
close all;
clc;

% USER INPUT (angle of rotation)
theta = input('Angle of rotation (in degrees):\n');

% CONSTANTS
l = 0.3;
phi = 30; % geometry of shape, in degrees

% DEFINE COORDINATES FOR NON-ROUND PART
x = [0 l*sind(phi) 0 l*cosd(phi) 2*l*cosd(phi)];
y = [2*l*cosd(phi) l*cosd(phi) 0 l*sind(phi) 0];
point = [x; y];

% DEFINE COORDINATES FOR QUARTER-CIRCLE
pts = linspace(0, pi/2, 1000);
xcirc = 2*l*cosd(phi)*cos(pts);
ycirc = 2*l*cosd(phi)*sin(pts);
circle = [xcirc; ycirc;];

% DEFINE ROTATION MATRIX
R = [cosd(theta) -sind(theta); sind(theta) cosd(theta)];

% DEFINE COORDINATES FOR ROTATED SHAPE
point = R*point;
xrot = point(1,:);
yrot = point(2,:);
circle = R*circle;
xcircrot = circle(1,:);
ycircrot = circle(2,:);
% DISPLAY THE RESULTS
figure;
title('Rotating object, by Johannes Feng');
hold on;
plot(x, y, ':b'); % original shape
plot(xcirc, ycirc, ':b');
plot(xrot, yrot, 'r'); % rotated shape
plot(xcircrot, ycircrot, 'r');
axis([-.8 .8 -.6 .6]);
grid on;
hold off;

end
```

```
% Johannes Feng
% Solution to 13.58c, due 10/21/08

function animation_c

% CLEAN UP
close all;
clc;

% USER INPUT (time of rotation, given angular speed)
t = input('Time of rotation (s):\n');

% CONSTANTS
l = 0.3;
phi = 30; % geometry of shape, in degrees
w = 2; % angular speed, in radians

% DEFINE COORDINATES FOR NON-ROUND PART
x = [0 l*sind(phi) 0 l*cosd(phi) 2*l*cosd(phi)];
y = [2*l*cosd(phi) l*cosd(phi) 0 l*sind(phi) 0];
point = [x; y];

% DEFINE COORDINATES FOR QUARTER-CIRCLE
pts = linspace(0, pi/2, 1000);
xcirc = 2*l*cosd(phi)*cos(pts);
ycirc = 2*l*cosd(phi)*sin(pts);
circle = [xcirc; ycirc;];

% DEFINE ROTATION MATRIX
R = [cos(t*w) -sin(t*w); sin(t*w) cos(t*w)];

% DEFINE COORDINATES FOR ROTATED SHAPE
point = R*point;
xrot = point(1,:);
yrot = point(2,:);
circle = R*circle;
xcircrot = circle(1,:);
ycircrot = circle(2,:);

% DISPLAY THE RESULTS
figure;
title('Rotating object, by Johannes Feng');
hold on;
plot(x, y, ':b'); % original shape
plot(xcirc, ycirc, ':b');
plot(xrot, yrot, 'r'); % rotated shape
plot(xcircrot, ycircrot, 'r');
axis([-.8 .8 -.6 .6]);
grid on;
hold off;
```

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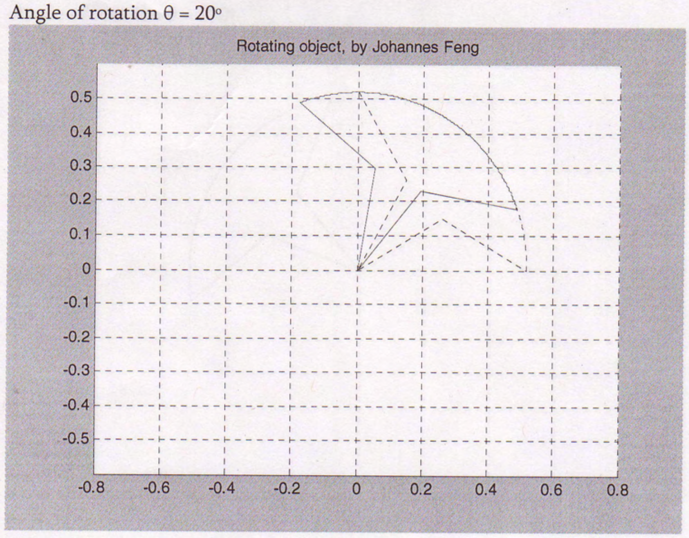
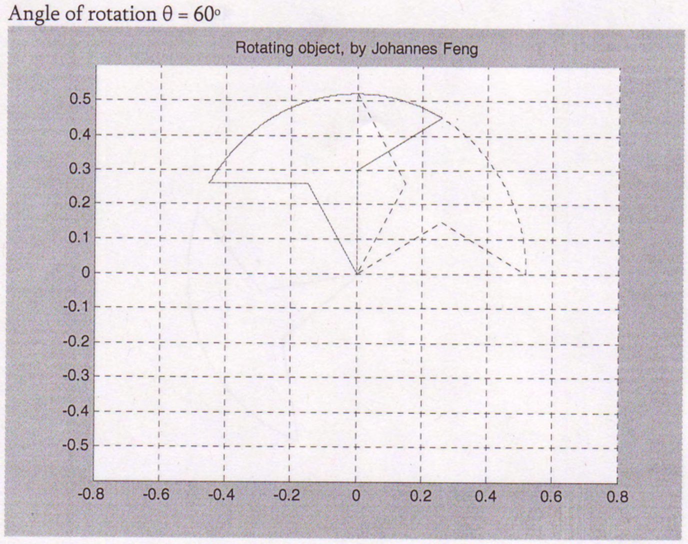
Angle of rotation θ = 20°



Angle of rotation θ = 60°



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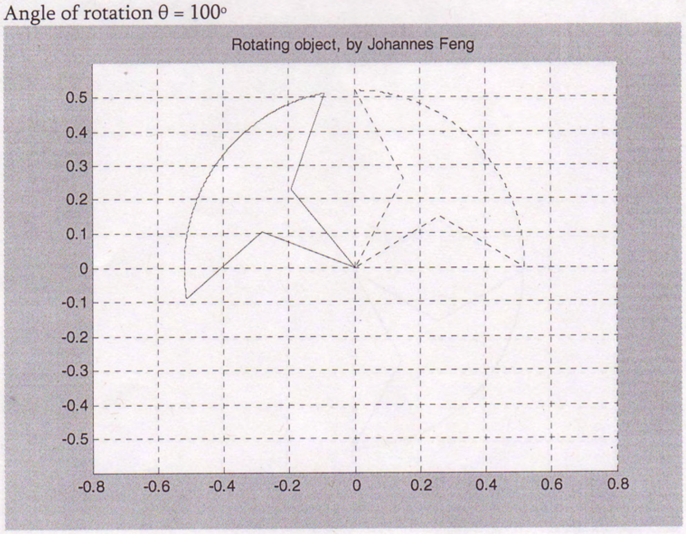
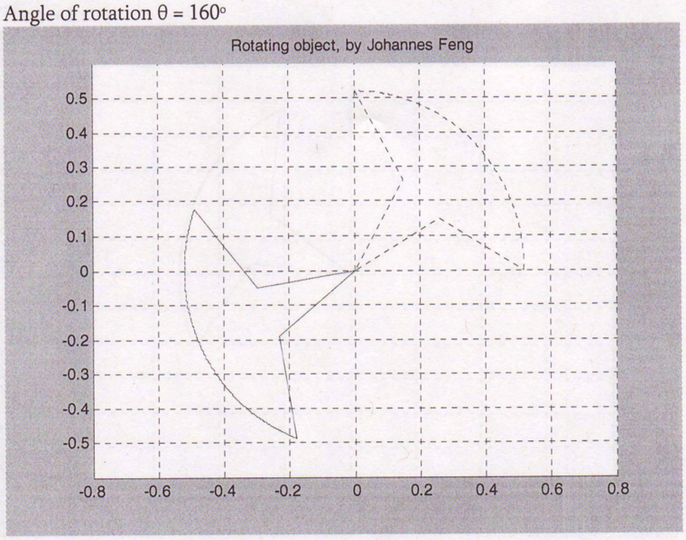
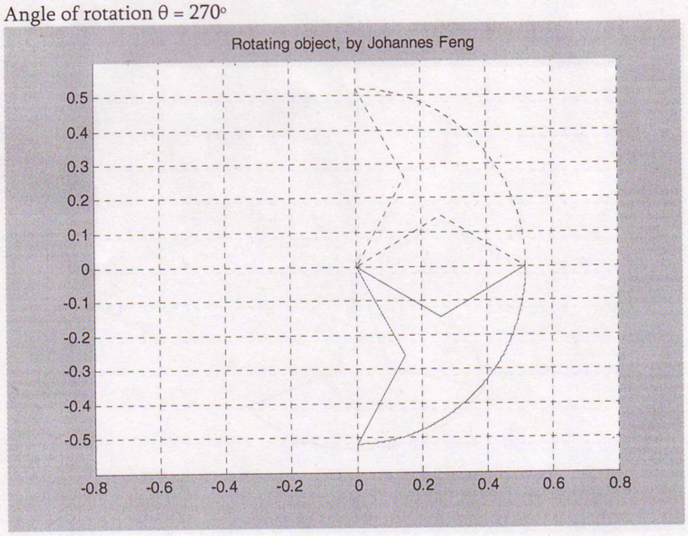
Angle of rotation θ = 100°



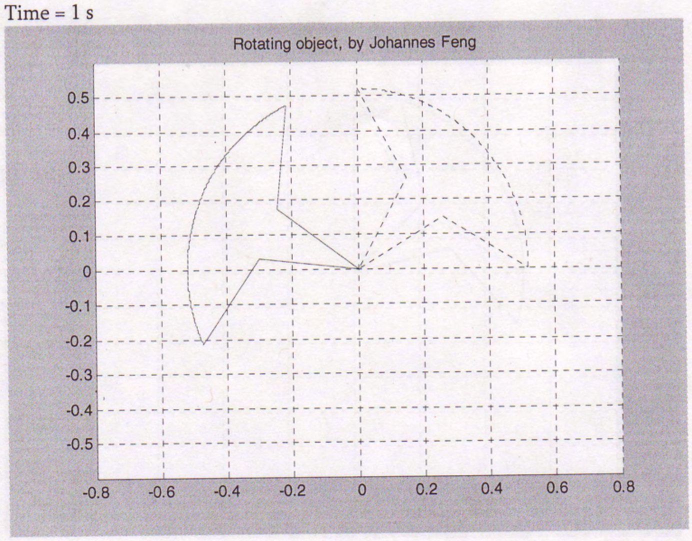
Angle of rotation θ = 160°



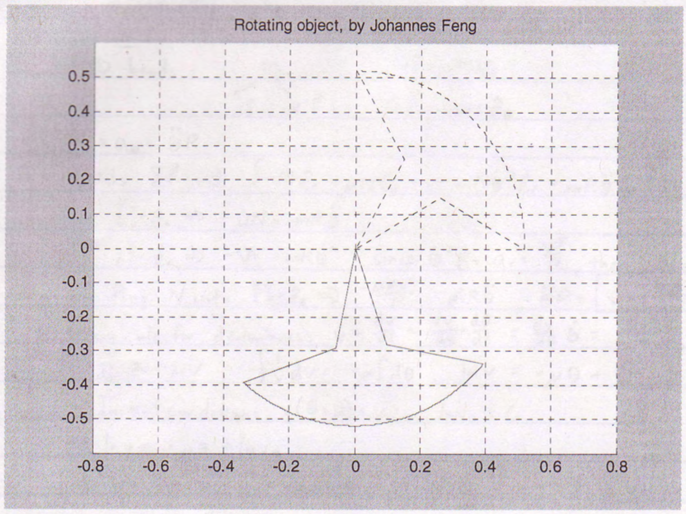
Angle of rotation $\theta = 270°$



Time = 1 s



Time = 2 s



Time = 3 s