SAMPLE 16.1 **Computing rotated position of an object:** A rigid object AOB is pinned at point O and is free to rotate about this point. Using the rotation matrix, find the coordinates of points A and B when $\theta = 30°$ and $110°$ respectively.

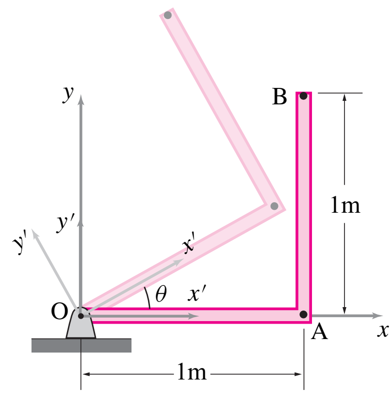

Figure 16.9

Solution Let $x'y'$ be a set of axes glued to the object with origin at O which is also the origin of the space-fixed coordinate axes xy. We first write the coordinates of points A and B in the body-fixed axes.

$$\vec{r}_{\mathrm{A}}|_{x'y'} = \begin{bmatrix} 1\,\mathrm{m} \\ 0 \end{bmatrix}, \qquad \vec{r}_{\mathrm{B}}|_{x'y'} = \begin{bmatrix} 1\,\mathrm{m} \\ 1\,\mathrm{m} \end{bmatrix}.$$

The rotation matrix for the $x'y'$ axes rotating counterclockwise along with the object is

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

We can find the coordinates of A and B in the rotated position by multiplying their $x'y'$ coordinates with the rotation matrix. We first write the coordinates of both points A and B in a single matrix, one column for each, and multiply with R to get the new coordinates:

$$[\ \vec{r}_{\mathrm{A}} \quad \vec{r}_{\mathrm{B}}\ ]_{xy} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x'_A & x'_B \\ y'_A & y'_B \end{bmatrix}.$$

Now, we calculate the new coordinates for the given values of θ.

- For $\theta = 30°$, we have,

$$\begin{aligned} [\ \vec{r}_{\mathrm{A}} \quad \vec{r}_{\mathrm{B}}\ ]_{xy} &= \begin{bmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{bmatrix} \begin{bmatrix} 1\,\mathrm{m} & 1\,\mathrm{m} \\ 0 & 1\,\mathrm{m} \end{bmatrix} \\ &= \begin{bmatrix} 0.866\,\mathrm{m} & 0.366\,\mathrm{m} \\ 0.5\,\mathrm{m} & 1.366\,\mathrm{m} \end{bmatrix}. \end{aligned}$$

  Thus,

$$[\vec{r}_{\mathrm{A}}]_{xy} = \begin{bmatrix} 0.866\,\mathrm{m} \\ 0.5\,\mathrm{m} \end{bmatrix}, \qquad [\vec{r}_{\mathrm{B}}]_{xy} = \begin{bmatrix} 0.366\,\mathrm{m} \\ 1.366\,\mathrm{m} \end{bmatrix}.$$

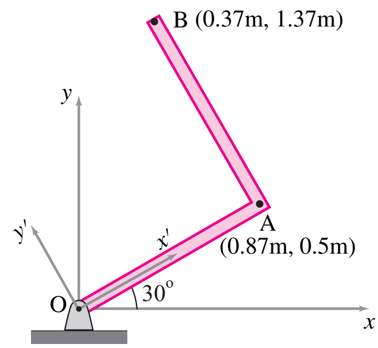

Figure 16.10: Coordinates of points A and B after a rotation of 30°.

- Similarly, for $\theta = 110°$, we get,

$$\begin{aligned} [\ \vec{r}_{\mathrm{A}} \quad \vec{r}_{\mathrm{B}}\ ]_{xy} &= \begin{bmatrix} -0.342 & -0.94 \\ 0.94 & -0.342 \end{bmatrix} \begin{bmatrix} 1\,\mathrm{m} & 1\,\mathrm{m} \\ 0 & 1\,\mathrm{m} \end{bmatrix} \\ &= \begin{bmatrix} -0.342\,\mathrm{m} & -1.282\,\mathrm{m} \\ 0.94\,\mathrm{m} & 0.598\,\mathrm{m} \end{bmatrix}. \end{aligned}$$

$$[\vec{r}_{\mathrm{A}}]_{xy} = \begin{bmatrix} -0.342\,\mathrm{m} \\ 0.94\,\mathrm{m} \end{bmatrix}, \qquad [\vec{r}_{\mathrm{B}}]_{xy} = \begin{bmatrix} -1.282\,\mathrm{m} \\ 0.598\,\mathrm{m} \end{bmatrix}.$$

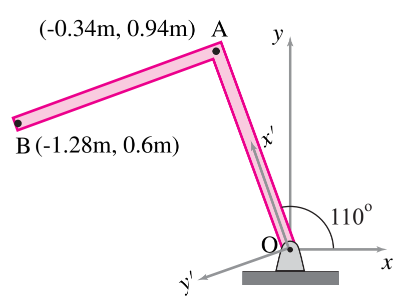

Figure 16.11: Coordinates of points A and B after a rotation of 110°.

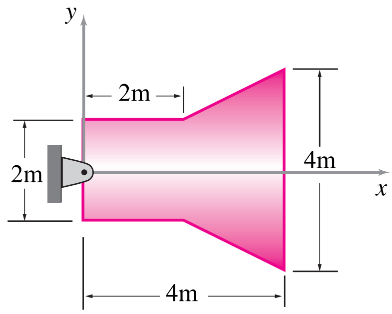

Figure 16.12

SAMPLE 16.2 Computer program for object rotation: The object shown in the figure rotates counterclockwise with its angular position θ given by $\theta(t) = \dot{\theta}_0 t$ where $\dot{\theta}_0 = 10^\circ/s$. Write a computer program to animate the motion of the object by plotting its position at specified time instants. Use your program to plot the position of the object every two seconds for a total of 18 seconds.

Solution In order to draw the object on the computer screen, we need to define it by taking a sufficient number of points on its boundary so that plotting all those points with connected line segments represents the object as closely as possible. In this case, we can select all the corner points on its boundary and define the object with the coordinates of these points. With the given geometry, it is fairly easy to find these coordinates. Denoting the coordinates of the kth point by (x_k, y_k), we can define this object by the following set of coordinates:

$$\text{object} = \begin{bmatrix} x_1 & x_2 & \dots & x_6 & x_1 \\ y_1 & y_2 & \dots & y_6 & y_1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 4 & 4 & 2 & 0 & 0 \\ -1 & -1 & -2 & 2 & 1 & 1 & -1 \end{bmatrix}.$$

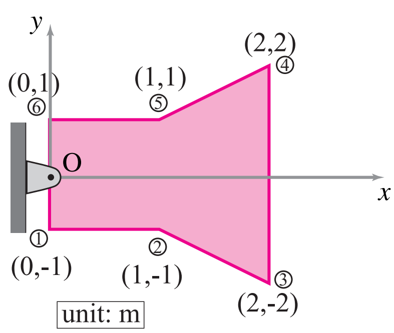

Figure 16.13: Coordinates of the six corner points are sufficient to define this object.

Note that the last column is a repeat of the first column. This is essential so that when we plot the object using the x and y coordinates listed here, we get a closed boundary.

Animation of the angular motion of this object basically requires determining the rotated position of the object and plotting it at sufficiently small time intervals. To do this, we need to define the initial orientation (position), define the time increment, find the angle of rotation θ corresponding to the new time, compute the corresponding rotation matrix, multiply the object coordinates with the rotation matrix, separate out the x and y coordinates, and plot x vs y. We then repeat the whole sequence until we reach the final time.

The following pseudocode can be easily adapted to a computer program to do the required animation.

```
object = [ 0  2  4  4  2  0  0
          -1 -1 -2  2  1  1 -1]     % coordinates of points
t = 0                               % specify initial time
t_final = 36                        % specify final time
delta_t = 2                         % specify time increment
thetadot0 = 10*pi/180               % specify thetadot0 in rad/s

while t < t_final
      t = t + delta_t               % increment the time
      theta = thetadot0*t           % compute current theta
      R = [ cos(theta) -sin(theta)  % find the rotation matrix
            sin(theta)  cos(theta)]
      new_position = R*object       % find new coordinates
      x = new_position(1st row)     % extract all x-coordinates
      y = new_position(2nd row)     % extract all y-coordinates
      plot y vs x                   % plot the new position
end
```

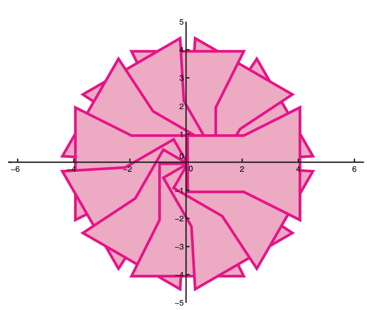

Figure 16.14: Rotated position of the object plotted at 2 second intervals.

The plot obtained by implementing this code is shown in fig. 16.14.