16.1 Rotation of a rigid body

Preparatory Problems

16.1.1 Give the definition of each term below in words and, if possible, with an equation.

a) rotation angle

b) (x', y') *

c) (x, y) *

d) rotation matrix *

16.1.2 Assume that in the reference configuration, when $\theta = 0$, that the $x'y'$ axes attached to an object are aligned with the xy axes. Consider a point P attached to the moving frame (object) that has coordinates (x', y'). Find each of the quantities below in terms of some or all of $x', y', \hat{\imath}, \hat{\jmath}, \theta, \dot{\theta}$ and $\ddot{\theta}$.

a) $\vec{r}_P$ *

b) The rotation matrix $[R]$. *

16.1.3 Assume that in the reference configuration, when $\theta = 0$, that the $x'y'$ axes are aligned with the xy axes. Consider two points P_1 and P_2 attached to the moving frame. P_1 and P_2 have coordinates (x'_1, y'_1) (x'_2, y'_2). Find each of the quantities below in terms of some or all of $x'_1, y'_1, x'_2, y'_2, \hat{\imath}, \hat{\jmath}, \theta, \dot{\theta}$ and $\ddot{\theta}$.

a) $\vec{r}_{P_1/P_2}$ *

b) The rotation matrix $[R]$. *

16.1.4 The square plate OABC shown in the figure measures $\ell = 1$ m on a side. The coordinate axes $x'y'$ are fixed to the plate and the axes xy are fixed in space. The plate is rotated by an angle $\theta = 60°$ as shown.

a) Find the coordinates x' and y' of the corner point B in the rotated frame.

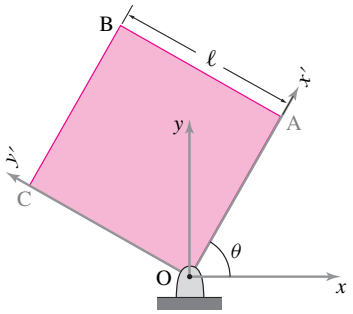
b) Find the coordinates x and y of the same point in the fixed frame without using the rotation matrix.

c) Write the rotation matrix R for the given rotation.

d) Find the coordinates of point B in the fixed frame using its (x', y') coordinates and the rotation matrix R.

e) Find the coordinates of point B in the rotated frame using its (x, y) coordinates and the rotation matrix R in its appropriate form.

*



Problem 16.1.4

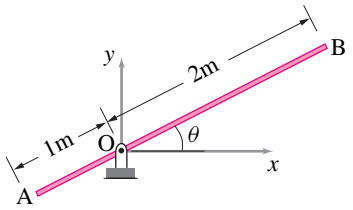
16.1.5 The rod shown in the figure rotates counterclockwise, starting from $\theta = 0$. Find the following quantities.

a) Write the position vector of points A and B as a list of numbers in an array, *e.g.*, $[\vec{r}_A]_{xy} = \begin{bmatrix} x_A \\ y_A \end{bmatrix}$, at $\theta = 0$. *

b) Write the rotation matrix R for $\theta = 45°$. *

c) Find the coordinates of points A and B using the rotation matrix R for $\theta = 45°$. *

d) If the rod rotates at a constant angular speed $\omega = \pi/3$ rad/s, find the coordinates of points A and B after 2 seconds, assuming the rod is at $\theta = 0$ when $t = 0$.



Problem 16.1.5

16.1.6 Find the angle of rotation corresponding to the following rotation matrices:

a) $\begin{bmatrix} 0.7071 & -0.7071 \\ 0.7071 & 0.7071 \end{bmatrix}$. *

b) $\begin{bmatrix} -0.7071 & -0.7071 \\ 0.7071 & -0.7071 \end{bmatrix}$. *

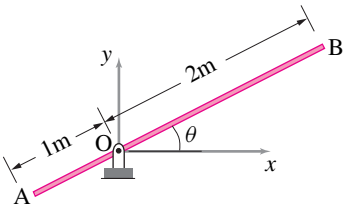
c) $\begin{bmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{bmatrix}$. *

More-Involved Problems

16.1.7 The rod shown in the figure rotates with constant angular speed $\omega = 10$ rad/s. $\theta = 0$ at $t = 0$ s.

a) Find the position vector $\vec{r}_B|_{t_1}$ and the velocity $\vec{v}_B|_{t_1}$ of point B at $t_1 = 1$ s. *

b) Find the position vector $\vec{r}_B|_{t_2}$ of point B at $t_2 = 1.2$ s. What is the net displacement of point B during the time interval $\Delta t = t_2 - t1$? Is this net displacement equal to $\vec{v}_B|_{t_1} \Delta t$? Why not? *



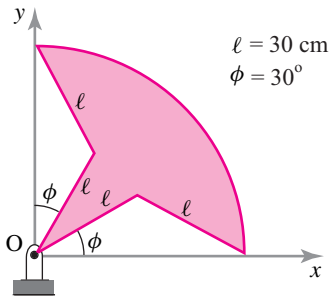
Problem 16.1.7

16.1.8 Write a computer program to animate the rotation of an object. Your input should be a set of x and y coordinates defining the object (such that `plot y vs x` draws the object on the screen) and the rotation angle θ. The output should be the rotated coordinates of the object.

a) From the geometric information given in the figure, generate coordinates of enough points to define the given object. *

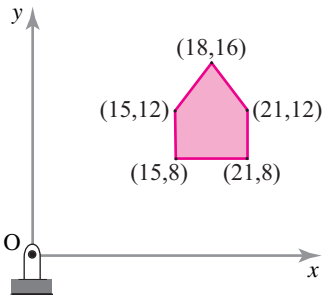
b) Using your program, plot the object at $\theta = 20°, 60°, 100°, 160°$, and $270°$.

c) Assume that the object rotates with constant angular speed $\omega = 2$ rad/s. Find and plot the position of the object at $t = 1$ s, 2 s, and 3 s.



Problem 16.1.8

16.1.9 The arrow shaped object shown in the figure is defined by the five points whose coordinates are given in some normalized units. The position shown is at $t = 0$. The time dependent angular position of the object is given by $\theta(t) = C_1 t^2 + C_2 t$ where $C_1 = 0.25\,\text{rad/s}^2$ and $C_2 = 0.1\,\text{rad/s}$. Animate the motion of the object and show its position from $t = 0$ to $t = 5\,\text{s}$ at every second.



Problem 16.1.9