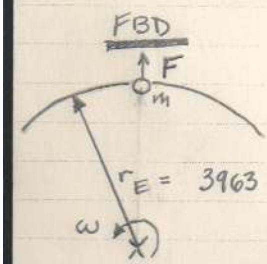
**15.2.1 Force on a person standing on the equator.** Find the magnitude of the total force acting on a 150 lbm person standing on the equator. The total force is the gravity force plus the force of the ground on the person (note that these two do not exactly cancel). Neglect the motion of the earth around the sun and of the sun around the solar system, etc. The radius of the earth is 3963 mi. Give your solution in both pounds ( lbf) and Newtons ( N).

**Answer:**
$F = 0.52\,\text{lbf} = 2.3\,\text{N}$

Page 2

**1.1 Force on a person standing on the equator.** The total force acting on an object of mass $m$, moving with a constant angular speed $\omega$ on a circular path with radius $r$, is given by $F = m\omega^2 r$. A person standing still on the earth is held in a circular path about the earth's axis by the sum of the reaction force of the ground and the force of earth's gravity. Find the magnitude of the total force acting on a 150 lbm person standing on the equator. Neglect the motion of the earth around the sun and of the sun around the solar system, etc. The radius of the earth is 3963 miles. Give your solution in both pounds ( lbf) and Newtons ( N).




we calculate force due to rotation of a person standing at the equator

$$F = m\omega^2 r$$

$$F = m\omega^2 r$$

$$= (150\,\text{lb}_m)\left[\frac{1\,\text{rev}}{\text{day}}\ \frac{2\pi\,\text{rad}}{\text{rev}}\ \frac{1\,\text{day}}{86400\,\text{sec}}\right]^2 (3963\,\text{miles})\left(\frac{5280\,\text{ft}}{\text{mile}}\right)$$

$$= 16.598984\ \frac{\text{lbm ft}}{\text{sec}^2}\left(\frac{1\,\text{lbf}}{32.2\,\frac{\text{lbm ft}}{\text{sec}^2}}\right) = 0.5154964\ \text{lbf}$$

$$= \left(16.598984\ \frac{\text{lbm ft}}{\text{sec}^2}\right)\left(\frac{1\,\text{lbf}}{32.2\,\frac{\text{lbm ft}}{\text{sec}^2}}\right)\left(\frac{4.448\,\text{N}}{\text{lbf}}\right) = 2.292928\ \text{N}$$

$$\boxed{\begin{aligned}F &= 0.52\ \text{lbf}\\ &\phantom{=}\ 2.3\ \text{N}\end{aligned}}$$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**15.2.2** Consider a mass $m$ in circular motion. Let $\sum \vec{F} = \sum F_r \hat{\boldsymbol{e}}_r + \sum F_\theta \hat{\boldsymbol{e}}_\theta$. Using $\sum \vec{F} = m\vec{a}$, express $\sum F_r$ and $\sum F_\theta$ in terms of some or all of $\theta, \dot{\theta}, \ddot{\theta},, r$, and $m$.

$$\Sigma \underset{\sim}{F} = \Sigma F_r \underset{\sim}{\hat{e}_r} + \Sigma F_\theta \underset{\sim}{\hat{e}_\theta} \qquad \text{Use } \Sigma \underset{\sim}{F} = m\underset{\sim}{a}$$

$$\Rightarrow \Sigma F_r \underset{\sim}{\hat{e}_r} + \Sigma F_\theta \underset{\sim}{\hat{e}_\theta} = m\left[-R\dot{\theta}^2 \underset{\sim}{\hat{e}_r} + R\ddot{\theta}\, \underset{\sim}{\hat{e}_\theta}\right]$$

$$\Rightarrow \Sigma F_r = -mR\dot{\theta}^2 \quad \text{and} \quad \Sigma F_\theta = mR\ddot{\theta}$$

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**15.2.3** Using $\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$, find the expressions for $\sum F_x$ and $\sum F_y$ in terms of $\ddot{\theta}$, $\dot{\theta}$, $r$, and $\theta$. [Hint $\hat{\boldsymbol{e}}_r = \cos\theta\hat{\boldsymbol{\imath}} + \sin\theta\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{e}}_\theta = -\sin\theta\hat{\boldsymbol{\imath}} + \cos\theta\hat{\boldsymbol{\jmath}}$].

14.2.3

$$\Sigma \underset{\sim}{F} = m\underset{\sim}{a}$$

$$\Sigma F_x \underset{\sim}{\hat{i}} + \Sigma F_y \underset{\sim}{\hat{j}} = m\left[-R\dot{\theta}^2 \left[\cos\theta\, \underset{\sim}{i} + \sin\theta\, \underset{\sim}{j}\right] + mR\ddot{\theta}\left[-\sin\theta\, \underset{\sim}{i} + \cos\theta\, \underset{\sim}{j}\right]\right.$$

$$\Rightarrow \quad \Sigma F_x = -mR\dot{\theta}^2\cos\theta - mR\ddot{\theta}\sin\theta$$

$$\Sigma F_y = -mR\dot{\theta}^2\sin\theta + mR\ddot{\theta}\cos\theta$$

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**15.2.4** A bead of mass $m$ goes around a circular path of radius $R$ in the $xy$-plane with angular acceleration $\ddot{\theta} = ct^3$. The bead starts from rest at $\theta = 0$.

a) What is the angular momentum of the bead about the origin at $t = t_1$?

b) What is the rate of change of angular momentum about the origin at $t = t_1$?

c) What is the kinetic energy of the bead at $t = t_1$?

d) Does the kinetic energy increase, decrease, or remain constant with time? Why?

14.2.4

$\ddot{\theta} = ct^3$ ~~at~~ $\theta = 0$ at $t = 0$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$\underset{\sim}{H}_{/O} = \underset{\sim}{r}_{/O} \times m \underset{\sim}{v}$$
$$= r\hat{\underset{\sim}{e}}_r \times m[r\dot{\theta}\,\hat{\underset{\sim}{e}}_\theta]$$
$$= mr^2\dot{\theta}\,\hat{\underset{\sim}{k}}$$

$$\ddot{\theta} = ct^3$$
$$\frac{d\dot{\theta}}{dt} = ct^3 \Rightarrow (\dot{\theta} - \dot{\theta}_0) = c\frac{t^4}{4}\Big|_{t_0}^{t}$$
$$\Rightarrow \quad \dot{\theta} = \frac{ct^4}{4}$$

$$\Rightarrow \underset{\sim}{H}_{/O} = \frac{mr^2ct^4}{4}\hat{\underset{\sim}{k}}$$

b)
$$\dot{\underset{\sim}{H}}_{/O} = \underset{\sim}{r}_{/O} \times m\underset{\sim}{a}$$
$$= r\hat{\underset{\sim}{e}}_r \times m[-R\dot{\theta}^2\hat{\underset{\sim}{e}}_r + R\ddot{\theta}\,\hat{\underset{\sim}{e}}_\theta]$$
$$= mR^2\ddot{\theta}\,\hat{\underset{\sim}{k}} \qquad (R \equiv r)$$
$$\dot{\underset{\sim}{H}}_{/O} = mr^2ct_1^3\,\hat{\underset{\sim}{k}} \text{ Nm}$$

c) Kinetic energy, $E_K = \frac{1}{2}mv^2 = \frac{1}{2}m[R\dot{\theta}]^2 \quad \dot{\theta} = \frac{ct_1^4}{4}$

$$\Rightarrow E_K = \frac{mr^2}{2}\left[\frac{c^2t_1^8}{16}\right]$$

d) Kinetic energy increases with time.

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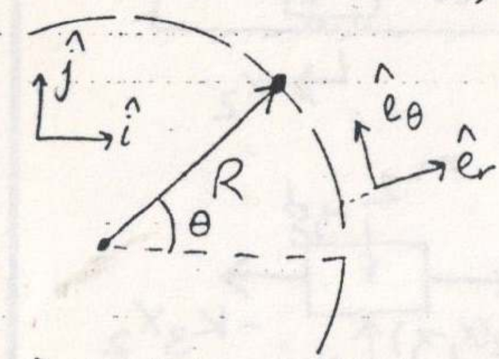
**15.2.5** A 200 gm particle goes in circles about a fixed center at a constant speed $v = 1.5\,\text{m/s}$. It takes 7.5 s to go around the circle once.

a) Find the angular speed of the particle.

b) Find the magnitude of acceleration of the particle.

c) Take center of the circle to be the origin of a $xy$-coordinate system. Find the net force on the particle when it is at $\theta = 30^\circ$ from the $x$-axis.

4.3 0.2 kg particle going in circles at a constant rate; $v = 1.5\,\text{m/s}$, $7.5\,\text{sec/rev}$.




What is the radius of the circle?

$$1.5\,\text{m/s} = v = \frac{\pi d\ \text{m}}{7.5\,\text{sec}}$$

$$\Rightarrow d = 3.58\,\text{m};\ R = 1.79\,\text{m}$$

a) Find the angular speed $\omega, \dot{\theta}$

$$\text{frequency} = \frac{1\,\text{rev}}{7.5\,\text{sec}} = .133\,\text{rev/sec};\ \therefore\ \omega = 2\pi f = \boxed{0.838\,\text{rad/s} = \omega}$$

b) find $|\underset{\sim}{a}|$: $|\underset{\sim}{a}| = |-\dot{\theta}^2 R \hat{e}_r| = (0.838/\text{s})^2(1.79\,\text{m})$

$$\therefore\ \boxed{a = 1.25\,\text{m/s}^2}\ ;\ \underset{\sim}{a} = -1.25\,\text{m/s}^2\,\hat{e}_r$$

c) Find $\Sigma \underset{\sim}{F}$ when $\theta = 30^\circ$:

$$\Sigma \underset{\sim}{F} = \dot{\underset{\sim}{L}} = m\underset{\sim}{a} = m(-1.25\,\text{m/s}^2\,\hat{e}_r)$$

where $\hat{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$$\Rightarrow \Sigma \underset{\sim}{F} = (0.2\,\text{kg})(-1.25\,\text{m/s}^2)(\cos 30^\circ\,\hat{i} + \sin 30^\circ\,\hat{j})$$

$$\Rightarrow \boxed{\Sigma \underset{\sim}{F} = (-0.216\,\hat{i} - 0.125\,\hat{j})\ \text{N}}$$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**15.2.6** A race car cruises on a circular track at a constant speed of 120 mph. It goes around the track once in three minutes. Find the magnitude of the centripetal force on the car. What applies this force on the car? Does the driver have any control over this force?

$V = 120$ mph $\qquad 2\pi\,(\text{rad}) \equiv 1\,\text{rev}$ in $3 \times 60\,\text{s} \Rightarrow \dot{\theta} = 0.0349\,\frac{\text{rad}}{\text{s}}$

$R = \dfrac{v}{\dot{\theta}} = 1.537\,\text{Km}.$

$\Sigma \underset{\sim}{F} = \underset{\sim}{\dot{L}} = m\underset{\sim}{a} = m(-R\dot{\theta}^2 \hat{\underset{\sim}{e}}_r) = -m(1537)(0.0349)^2\,(\text{N})$

$\Rightarrow |F| = (1.872\,m)\,\text{N}$

Force on car is due to change in direction of Velocity.

Less speed lowers the centripetal force.

14.2.7

**15.2.7** A particle moves on a counter-clockwise, origin-centered circular path in the $xy$-plane at a constant rate. The radius of the circle is $r$, the mass of the particle is $m$, and the particle completes one revolution in time $\tau$.

a) Neatly draw the following things:
   1. The path of the particle.
   2. A dot on the path when the particle is at $\theta = 0°$, $90°$, and $210°$, where $\theta$ is measured from the $x$-axis (positive counter-clockwise).
   3. Arrows representing $\hat{\boldsymbol{e}}_R$, $\hat{\boldsymbol{e}}_\theta$, $\vec{\boldsymbol{v}}$, and $\vec{\boldsymbol{a}}$ at each of these points.

b) Calculate all of the quantities in part (3) above at the points defined in part (2), (represent vector quantities in terms of the cartesian base vectors $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$).

c) If this motion was imposed by the tension in a string, what would that tension be?

d) Is radial tension enough to maintain this motion or is another force needed to keep the motion going (assuming no friction)?

e) Again, if this motion was imposed by the tension in a string, what is $F_x$, the $x$ component of the force in the string, when $\theta = 210°$? Ignore gravity.

**Answer:**
(b) For $\theta = 0°$,

$$\begin{aligned} \hat{\boldsymbol{e}}_r &= \hat{\boldsymbol{\imath}} \\ \hat{\boldsymbol{e}}_t &= \hat{\boldsymbol{\jmath}} \\ \vec{\boldsymbol{v}} &= \frac{2\pi r}{\tau}\hat{\boldsymbol{\jmath}} \\ \vec{\boldsymbol{a}} &= -\frac{4\pi^2 r}{\tau^2}\hat{\boldsymbol{\imath}}, \end{aligned}$$

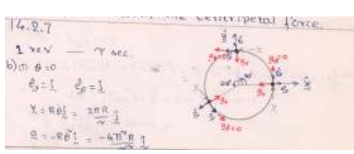
for $\theta = 90°$,

$$\begin{aligned} \hat{\boldsymbol{e}}_r &= \hat{\boldsymbol{\jmath}} \\ \hat{\boldsymbol{e}}_t &= -\hat{\boldsymbol{\imath}} \\ \vec{\boldsymbol{v}} &= -\frac{2\pi r}{\tau}\hat{\boldsymbol{\imath}} \\ \vec{\boldsymbol{a}} &= -\frac{4\pi^2 r}{\tau^2}\hat{\boldsymbol{\jmath}}, \end{aligned}$$

and for $\theta = 210°$,

$$\begin{aligned} \hat{\boldsymbol{e}}_r &= -\frac{\sqrt{3}}{2}\hat{\boldsymbol{\imath}} - \frac{1}{2}\hat{\boldsymbol{\jmath}} \\ \hat{\boldsymbol{e}}_t &= \frac{1}{2}\hat{\boldsymbol{\imath}} - \frac{\sqrt{3}}{2}\hat{\boldsymbol{\jmath}} \\ \vec{\boldsymbol{v}} &= -\frac{\sqrt{3}\pi r}{\tau}\hat{\boldsymbol{\jmath}} + \frac{\pi r}{\tau}\hat{\boldsymbol{\imath}} \\ \vec{\boldsymbol{a}} &= \frac{2\sqrt{3}\pi^2 r}{\tau^2}\hat{\boldsymbol{\imath}} + \frac{2\pi^2 r}{\tau^2}\hat{\boldsymbol{\jmath}}. \end{aligned}$$

(c) $T = \frac{4m\pi^2 r}{\tau^2}$.
(d) Tension is enough.




**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

(i) $\hat{e}_r = \hat{j}$ $\quad \hat{e}_\theta = -\hat{i}$ $\quad \underset{\sim}{a} = -\frac{R4\pi^2}{\tau^2}\hat{e}_r = -\frac{4\pi^2}{\tau^2}R\hat{j}$

$\underset{\sim}{v} = R\dot{\theta}\hat{e}_\theta = -\frac{R2\pi}{\tau}\hat{i}$

(ii) $\hat{e}_r = -\cos 30^\circ \hat{i} - \sin 30 \hat{j}$ and $\hat{e}_\theta = \sin 30 \hat{i} - \cos 30 \hat{j}$

$\hat{e}_r = -\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}$ and $\hat{e}_\theta = \frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}$

$\underset{\sim}{v} = R\dot{\theta}\hat{e}_\theta = \frac{2\pi R}{\tau}\left[\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right]$

$\underset{\sim}{a} = -\frac{4\pi^2 R}{\tau^2}\left[-\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right]$

c) $\underset{\sim}{F} = -mR\dot{\theta}^2\hat{e}_r$, Tension is $F = \frac{4\pi^2 mR}{\tau^2}$

d) Tension is enough.

e) $F_x\Big|_{\theta=210^\circ} = \frac{4\pi^2 mR}{\tau^2}\left(\frac{\sqrt{3}}{2}\right)$.

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**15.2.8** The velocity and acceleration of a 1 kg particle, undergoing constant rate circular motion, are known at some instant $t$:

$\vec{\boldsymbol{v}} = -10\,\mathrm{m/s}(\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}), \quad \vec{\boldsymbol{a}} = 2\,\mathrm{m/s^2}(\hat{\boldsymbol{\imath}} - \hat{\boldsymbol{\jmath}}).$

a) Write the position of the particle at time $t$ using $\hat{\boldsymbol{e}}_R$ and $\hat{\boldsymbol{e}}_\theta$ base vectors.

b) Find the net force on the particle at time $t$.

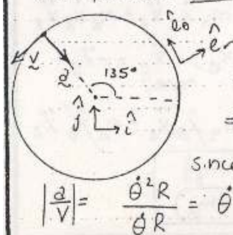
c) At some later time $t^*$, the net force on the particle is in the $-\hat{\boldsymbol{\jmath}}$ direction. Find the elapsed time $t - t^*$.

d) After how much time does the force on the particle reverse its direction.

4.9 1 kg particle; $\underset{\sim}{v} = -10\,\mathrm{m/s}\,(\hat{\imath}+\hat{\jmath})$

$\underset{\sim}{a} = 2\,\mathrm{m/s^2}(\hat{\imath}-\hat{\jmath})$

Since $\underset{\sim}{v} = v\hat{e}_\theta$ & $\underset{\sim}{a} = -a\hat{e}_r$, i.e. $\underset{\sim}{v}$ is tangent to the circle and $\underset{\sim}{a}$ points in toward the center of the circle the particle must be located at $\theta = 135°$:




need radius of circle.

$\Rightarrow |\underset{\sim}{v}| = \sqrt{10^2+10^2} = 10\sqrt{2}\,\mathrm{m/s}.$

$\Rightarrow |\underset{\sim}{a}| = \sqrt{2^2+2^2} = 2\sqrt{2}\,\mathrm{m/s^2}$

Since $a = -\dot\theta^2 R\hat{e}_r$ & $v = \dot\theta R\hat{e}_\theta$,

$\left|\frac{a}{v}\right| = \frac{\dot\theta^2 R}{\dot\theta R} = \dot\theta \Rightarrow \frac{2\sqrt{2}\,\mathrm{m/s^2}}{10\sqrt{2}\,\mathrm{m/s}} = 1/5\,\mathrm{rad/s} = \dot\theta$

then, $R = v/\dot\theta = (10\sqrt{2})5 = 70.7\,\mathrm{m}$

a) Thus, the position of the particle is

$R\underbrace{(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})}_{\hat{e}_r} = R\hat{e}_r = \boxed{70.7\,\mathrm{m}\;\hat{e}_r}$

b) Net force on particle at this time:

$\Sigma \underset{\sim}{F} = m\underset{\sim}{a} = m(-2\sqrt{2}\,\mathrm{m/s^2})\,\hat{e}_r = \boxed{-2\sqrt{2}\,\mathrm{N}\;\hat{e}_r}$

-or-

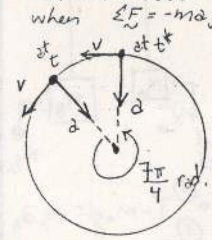
in cartesian coords,

$\Sigma \underset{\sim}{F} = m\underset{\sim}{a} = m(2\,\mathrm{m/s^2}(\hat{\imath}-\hat{\jmath})) = \boxed{2\,\mathrm{N}(\hat{\imath}-\hat{\jmath})}$

4.9 continued p.6

c) At some later time $t^*$, the net force on the particle is in the $-\hat{\jmath}$ direction. Find the elapsed time $t^*-t$.

Since $\Sigma \underset{\sim}{F} = m\underset{\sim}{a}$, the net force is always oriented in the direction of the acceleration vector for constant rate circular motion. Thus, when $\Sigma \underset{\sim}{F} = -ma\hat{\jmath}$, we must have $\theta = 90°$:

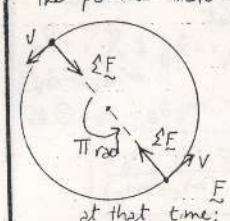



So since the particle started at $\theta = 135°$, it must travel thru $7\pi/4$ radians.

Since $\dot\theta = 1/5\,\mathrm{rad/sec}$, (found earlier),

$t^* - t = \frac{7\pi/4\,\mathrm{rad}}{1/5\,\mathrm{rad/sec}} = \boxed{27.5\,\mathrm{sec}}$

d) After how much time does the force on the particle reverse its direction?




By the same reasoning as in c) above:

$\underset{\sim}{a}$ will reverse direction when the particle is at the other edge of the circle. since $\Sigma \underset{\sim}{F} = m\underset{\sim}{a}$, $\underset{\sim}{F}$ will also reverse direction at that time:

$\Delta t = \frac{\pi\,\mathrm{rad}}{1/5\,\mathrm{rad/s}} = \boxed{15.7\,\mathrm{sec}}$

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**15.2.9** A particle of mass 3 kg moves in the $xy$-plane so that its position is given by

$$\vec{r}(t) = 4\,\mathrm{m}\left[\cos\left(\frac{2\pi t}{s}\right)\hat{\imath} + \sin\left(\frac{2\pi t}{s}\right)\hat{\jmath}\right]$$

with respect to point O, the origin of a fixed cartesian coordinate system.

a) What is the path of the particle? Show how you know what the path is.

b) What is the angular velocity of the particle? Is it constant? Show how you know if it is constant or not.

c) What is the velocity of the particle in polar coordinates?

d) What is the speed of the particle at $t = 3\,\mathrm{s}$?

e) What net force does it exert on its surroundings at $t = 0\,\mathrm{s}$? Assume the $x$ and $y$ axes are fixed.

f) What is the angular momentum of the particle at $t = 3\,\mathrm{s}$ about point $O$?

14.2.9

a) $\underset{\sim}{r}(t) = 4m\left[\cos\left(\frac{2\pi t}{s}\right)\hat{\underset{\sim}{i}} + \sin\left(\frac{2\pi t}{s}\right)\underset{\sim}{j}\right]$

$|\underset{\sim}{r}| = 4m\sqrt{\cos^2(\lambda t) + \sin^2(\lambda t)} = \underline{4m}$ $\therefore$ path is a circle.

b) $\theta = \lambda t = \dot{\theta} t \Rightarrow \dot{\theta} = 2\pi\,\mathrm{rad/s}$

c) $\underset{\sim}{v} = 4\lambda(-\sin\lambda t\,\underset{\sim}{i} + \cos\lambda t) = 8\pi\,\hat{\underset{\sim}{e}}_\theta\ \mathrm{m/s}$

d) $|\underset{\sim}{v}|_t = 8\pi\ \mathrm{m/s}$

e) $\Sigma\underset{\sim}{F} = m\underset{\sim}{a} = m\left[-R\dot{\theta}^2\hat{\underset{\sim}{e}}_r + R\cancel{\ddot{\theta}}\hat{\underset{\sim}{e}}_\theta\right] = -mR\dot{\theta}^2\hat{\underset{\sim}{e}}_r$

$F = -48\pi^2\hat{\underset{\sim}{e}}_r\,(N)$ at $t_0 = 0s$ $\hat{\underset{\sim}{e}}_r = \hat{\underset{\sim}{i}} \Rightarrow \underset{\sim}{F} = -48\pi^2\hat{\underset{\sim}{i}}\,(N)$

f) $\underset{\sim}{H}_{/0} = r_{/0} \times m\underset{\sim}{v}$

$= 4[\cos\lambda t\,\underset{\sim}{i} + \sin\lambda t\,\hat{\underset{\sim}{j}}] \times 3\cdot 4\lambda[-\sin\lambda t\,\hat{\underset{\sim}{i}} + \cos\lambda t\,\underset{\sim}{j}]$ Nms

$= 48\lambda[\cos^2\lambda t\,\hat{\underset{\sim}{k}} + \sin^2\lambda t\,\hat{\underset{\sim}{k}}]$ Nms

$\underset{\sim}{H}_{/0} = 96\pi\,\hat{\underset{\sim}{k}}$ Nms //

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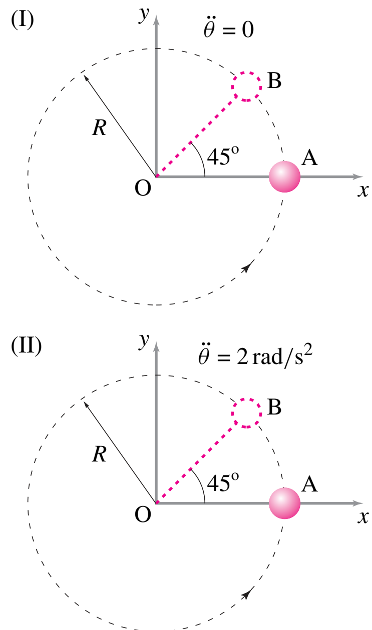
**15.2.10 A comparison of constant and nonconstant rate circular motion.** A 100 gm mass is going in circles of radius $R = 20\,\text{cm}$ at a constant rate $\dot{\theta} = 3\,\text{rad/s}$. Another identical mass is going in circles of the same radius but at a non-constant rate. The second mass is accelerating at $\ddot{\theta} = 2\,\text{rad/s}^2$ and at position A, it happens to have the same angular speed as the first mass.

a) Find and draw the accelerations of the two masses (call them I and II) at position A.

b) Find $\dot{\vec{\boldsymbol{H}}}_{/\text{O}}$ for both masses at position A.

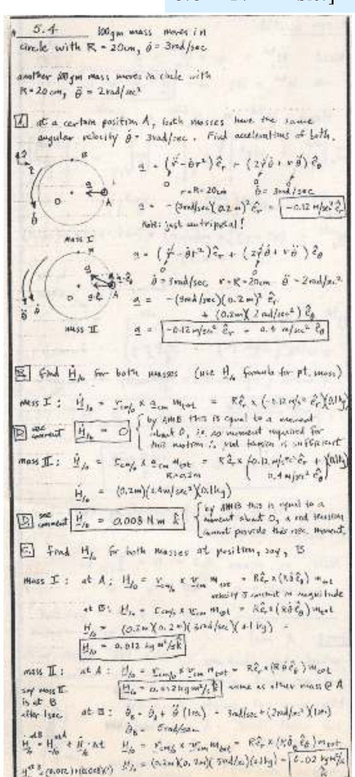
c) Find $\vec{\boldsymbol{H}}_{/\text{O}}$ for both masses at positions A and B. Do the changes in $\vec{\boldsymbol{H}}_{/\text{O}}$ between the two positions reflect (qualitatively) the results obtained in (b)?

d) If the masses are pinned to the center O by massless rigid rods, is tension in the rods enough to keep the two motions going? Explain.



Problem 15.10

**Answer:**
(b) $(\dot{\vec{\boldsymbol{H}}}_{/\text{O}})_I = \vec{\mathbf{0}}$, $(\dot{\vec{\boldsymbol{H}}}_{/\text{O}})_{II} = 0.0080\,\text{N·m}\hat{\boldsymbol{k}}$.
(c) Position-A: $(\vec{\boldsymbol{H}}_{/\text{O}})_I = 0.012\,\text{N·m}\cdot\text{s}\hat{\boldsymbol{k}}$,$(\vec{\boldsymbol{H}}_{/\text{O}})_{II} = 0.012\,\text{N·m}\cdot\text{s}\hat{\boldsymbol{k}}$, Position-B: $(\vec{\boldsymbol{H}}_{/\text{O}})_I 0.012\,\text{N·m}\cdot\text{s}\hat{\boldsymbol{k}}$, $(\vec{\boldsymbol{H}}_{/\text{O}})_{II} = 0.014\,\text{N·m}\cdot\text{s}\hat{\boldsymbol{k}}$.]


5.4 100gm mass moves in circle with $R = 20\text{cm}$, $\dot\theta = 3\text{rad/sec}$

another 100gm mass moves in circle with $R = 20\text{ cm}$, $\ddot\theta = 2\text{rad/sec}^2$

[A] at a certain position A, both masses have the same angular velocity $\dot\theta = 3\text{rad/sec}$. Find accelerations of both.

$\underline{a} = (\ddot r - \dot\theta r^2)\hat e_r + (2\dot r\dot\theta + r\ddot\theta)\hat e_\theta$

$r = R = 20\text{cm}$ $\dot\theta = 3\text{rad/sec}$

$\underline{a} = -(3\text{rad/sec})(0.2\text{ m})^2\hat e_r = \boxed{-0.12\text{ m/sec}^2\,\hat e_r}$

Note: just centripetal!

Mass I

$\underline{a} = (\ddot r - \dot\theta r^2)\hat e_r + (2\dot r\dot\theta + r\ddot\theta)\hat e_\theta$

$\dot\theta = 3\text{rad/sec}$ $r = R = 20\text{cm}$ $\ddot\theta = 2\text{rad/sec}^2$

$\underline{a} = -(3\text{rad/sec})(0.2\text{ m})^2\hat e_r + (0.2\text{m})(2\text{ rad/sec}^2)\hat e_\theta$

Mass II $\underline{a} = \boxed{-0.12\text{ m/sec}^2\,\hat e_r + 0.4\text{ m/sec}^2\,\hat e_\theta}$

[B] Find $\dot{\underline{H}}_{/o}$ for both masses (use $\underline{H}_{/o}$ formula for pt. mass)

Mass I: $\dot{\underline{H}}_{/o} = \underline{r}_{cm/o}\times\underline{a}_{cm}m_{tot} = R\hat e_r\times(-0.12\text{ m/s}^2\,\hat e_r)(0.1\text{kg})$

[D] see comment $\boxed{\dot{\underline{H}}_{/o} = 0}$ { by AMB this is equal to a moment about O, i.e. no moment required for this motion ∴ rod tension is sufficient

Mass II: $\dot{\underline{H}}_{/o} = \underline{r}_{cm/o}\times\underline{a}_{cm}m_{tot} = R\hat e_r\times(-0.12\text{ m/s}^2\,\hat e_r + 0.4\text{ m/sec}^2\,\hat e_\theta)(0.1\text{kg})$ $R = 0.2\text{m}$

$\dot{\underline{H}}_{/o} = (0.2\text{m})(0.4\text{m/sec}^2)(0.1\text{kg})$

[D] see comment $\boxed{\dot{\underline{H}}_{/o} = 0.008\text{ N m }\hat k}$ { by AMB this is equal to a moment about O, a rod tension cannot provide this nec. moment.

[C] Find $\underline{H}_{/o}$ for both masses at position, say, B

Mass I: at A: $\underline{H}_{/o} = \underline{r}_{cm/o}\times\underline{v}_{cm}m_{tot} = R\hat e_r\times(R\dot\theta\hat e_\theta)m_{tot}$

velocity is constant in magnitude

at B: $\underline{H}_{/o} = \underline{r}_{cm/o}\times\underline{v}_{cm}m_{tot} = R\hat e_r\times(R\dot\theta\hat e_\theta)m_{tot}$

$\underline{H}_{/o} = (0.2\text{m})(0.2\text{m})(3\text{rad/sec})(0.1\text{kg})$

$\boxed{\underline{H}_{/o} = 0.012\text{ kg m}^2/\text{s}\,\hat k}$

Mass II: at A: $\underline{H}_{/o} = \underline{r}_{cm/o}\times\underline{v}_{cm}m_{tot} = R\hat e_r\times(R\dot\theta\hat e_\theta)m_{tot}$

$\boxed{\underline{H}_{/o} = 0.012\text{kg m}^2/\text{s}\,\hat k}$ same as other mass @ A

say mass II is at B after 1sec

at B: $\dot\theta_B = \dot\theta_A + \ddot\theta(1\text{s}) = 3\text{rad/sec} + (2\text{rad/sec}^2)(1\text{s})$

$\dot\theta_B = 5\text{rad/sec}$

$\underline{H}^{at\,B}_{/o} = \underline{H}^{at\,A}_{/o} + \dot{\underline{H}}_{/o}\cdot\Delta t$

$\underline{H}_{/o} = \underline{r}_{cm/o}\times\underline{v}_{cm}m_{tot} = R\hat e_r\times(R\dot\theta_B\hat e_\theta)m_{tot}$

$\underline{H}^{at\,B}_{/o} = (0.012) + (0.008)(1)$

$\underline{H}_{/o} = (0.2\text{m})(0.2\text{m})(5\text{rad/s})(0.1\text{kg}) = \boxed{0.02\text{ kg m}^2/\text{s}\,\hat k}$

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**15.2.11** A small mass $m$ is connected to one end of a spring. The other end of the spring is fixed to the center of a circular track. The radius of the track is $R$, the unstretched length of the spring is $\ell_0$ (with $\ell_0 < R$), and the spring constant is $k$.

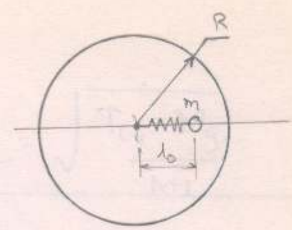
a) With what speed should the mass be launched in the track so that it keeps going at a constant speed?

b) If the spring is replaced by another spring of same relaxed length but twice the stiffness, what will be the new required launch speed of the particle?

14.2.11

$$\sum \underset{\sim}{F} = m\underset{\sim}{a}$$

$$= m(-R\dot{\theta}^2 \hat{\underset{\sim}{e}}_r)$$




$$\sum \underset{\sim}{F} = K(R-l_0)(-\hat{\imath}) = -mR\dot{\theta}^2 \hat{\underset{\sim}{e}}_r \,//\, \hat{\underset{\sim}{\imath}}$$

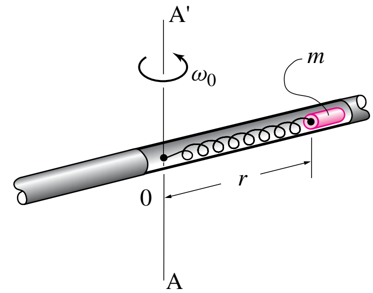
$$\Rightarrow K(R-l_0) = mR\dot{\theta}^2$$

$$\Rightarrow \dot{\theta} = \sqrt{\frac{K(R-l_0)}{mR}} \Rightarrow V = R\dot{\theta} \Rightarrow |V| = \sqrt{\frac{KR(R-l_0)}{m}}$$

b) $K_2 = 2K$

$\therefore |V_2| = \sqrt{2}\,|V|$

**15.2.12** A bead of mass $m$ is attached to a spring of stiffness $k$. The bead slides without friction in the tube shown. The tube is driven at a constant angular rate $\dot{\theta}_0$ about axis $AA'$ by a motor (not pictured). There is no gravity. The unstretched spring length is $r_0$. Find the radial position $r$ of the bead if it is stationary with respect to the rotating tube.




Problem 15.12

**Answer:**
$r = \frac{kr_o}{k-m\omega_o^2}$.

4.4 Bead in a constantly rotating tube attached to spring. No gravity. Bead at radial position, r

FBD $\hat{e}_\theta$ $\hat{e}_r$

$\{\sum \underline{F} = m\underline{a}\} \cdot \hat{e}_r$ $\underline{a} = -\omega_0^2 r\, \hat{e}_r$

$\underline{a}$ $\underline{T}$ $\underline{N}$

$\underline{T} = -k(r-r_0)\,\hat{e}_r$ Force in spring

$\underline{N} = N\,\hat{e}_\theta$

$-k(r-r_o) = -m\omega_0^2 r$

$$\boxed{r = \frac{kr_0}{k-m\omega_0^2}}$$

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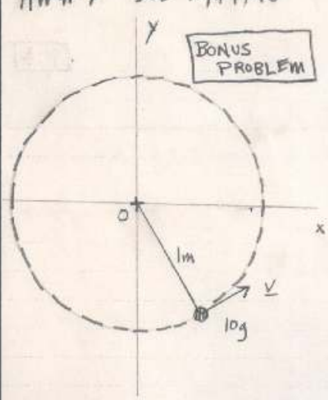
BONUS PROBLEM




A. Mass slides on frictionless surface in circle, as shown.

B. FBD $\{\sum \underline{F} = m\underline{a}\} \cdot \hat{e}_r$

$\underline{a}$ $\underline{T}$

$\omega = \frac{2\pi\,\text{rev}}{\text{sec}}\,\frac{2\pi\,\text{rad}}{1\,\text{rev}} = \frac{4\pi^2\,\text{rad}}{\text{sec}}$

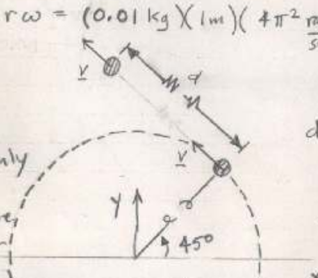
$a = \omega^2 r = \frac{16\pi^4\,\text{m}}{\text{sec}^2}$

$T = m\omega^2 r = \boxed{0.16\,\pi^4\ \text{N}}$

C. $\underline{H} = \underline{r} \times m\underline{V}$ $v = \omega r$

$H = mr\omega = (0.01\ \text{kg})(1\,\text{m})(4\pi^2\,\frac{\text{rad}}{\text{sec}}) = \boxed{0.04\pi^2\ \frac{\text{kg}\,\text{m}}{\text{sec}}}$

D.



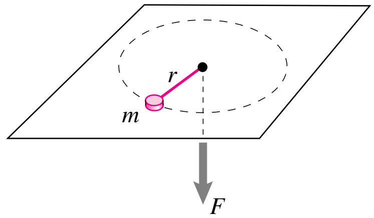

String suddenly breaks at $\theta = 45°$ where is mass after 1 sec?

$d = vt$
$= \left(4\pi^2\,\frac{\text{m}}{\text{sec}}\right)(1\,\text{sec})$
$= \boxed{4\pi^2\ \text{m}}$

In coordinates $\underline{d} = \left(\frac{\sqrt{2}}{2} - v\cos 45°\, t\right)\hat{\imath} + \left(\frac{\sqrt{2}}{2} + v\sin 45°\, t\right)\hat{\jmath}$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**15.2.13** A particle of mass $m$ is restrained by a string to move with a constant angular speed $\omega$ around a circle of radius $R$ on a horizontal frictionless table. If the radius of the circle is reduced slowly to $r$, by pulling the string with a slowly varying force $F$ through a hole in the table, what will the particle's angular velocity be in the final circular motion? Is kinetic energy changed in moving from circular motion at $R$ to circular motion at $r$? Why or why not?



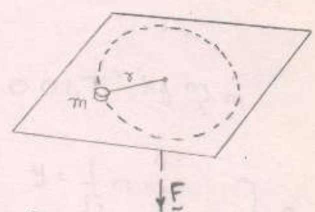

Problem 15.13

14.2.13

$\omega$ = Constant.

System is conserved.

$E_1 = E_2$




$$\frac{1}{2} m (R\omega)^2 = \frac{1}{2} F(R-r) + \frac{1}{2} m (r\dot{\theta})^2$$

$$R^2\omega^2 = \frac{F(R-r)}{m} + r^2\dot{\theta}^2$$

$$\dot{\theta}^2 = \frac{R^2}{r^2}\omega^2 - \frac{F}{m}\frac{(R-r)}{r^2} \Rightarrow \dot{\theta} = \sqrt{\left(\frac{R\omega}{r}\right)^2 - \frac{F(R-r)}{mr^2}}$$

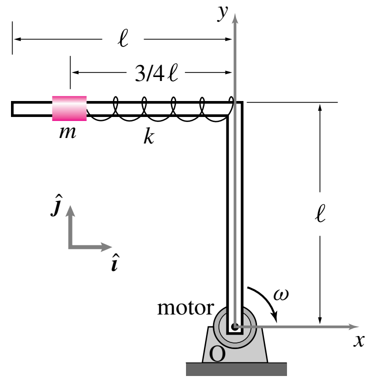
$$E_1 = \frac{1}{2} m R^2 \omega^2$$

$$E_2 = \frac{1}{2} m r^2 \dot{\theta}^2 = \frac{1}{2} m r^2 \left[\frac{R^2\omega^2}{r^2} - \frac{F(R-r)}{mr^2}\right] = \frac{1}{2} m R^2 \omega^2 - \frac{1}{2} F(R-r)$$

$\Rightarrow E_2 < E_1$

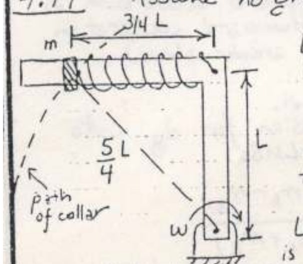
**15.2.14** An 'L' shaped rigid, massless, and frictionless bar is made up of two uniform segments of length $\ell = 0.4\,\mathrm{m}$ each. A collar of mass $m = 0.5\,\mathrm{kg}$, attached to a spring at one end, slides frictionlessly on one of the arms of the 'L'. The spring is fixed to the elbow of the 'L' and has a spring constant $k = 6\,\mathrm{N/m}$. The structure rotates clockwise at a constant rate $\omega = 2\,\mathrm{rad/s}$. If the collar is steady at a distance $\frac{3}{4}\ell = 0.3\,\mathrm{m}$ away from the elbow of the 'L', find the relaxed length of the spring, $\ell_0$. Neglect gravity.
.




Problem 15.14: **Forces in constant rate circular motion.**

**Answer:**
$\ell_0 = 0.2\,\mathrm{m}$

4.17 Assume no gravity.

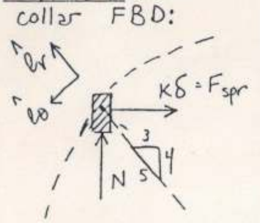



$L = 0.4\,\mathrm{m}$, $w = 2\ \mathrm{rad/s}$.
$K = 6\ \mathrm{N/m}$, $m = 0.5\ \mathrm{kg}$

Find the relaxed length of the spring given that the collar is steady at $3L/4$ from the elbow.
Let $\delta$ = amount spring is stretched. Then the relaxed length $L_o = L - \delta$.

· The acceleration of the collar is:
$\underset{\sim}{a} = -\dot{\theta}^2 R\,\hat{e}_r = -(4\ 1/s^2)(\frac{5}{4}L)\hat{e}_r = -5L\ \frac{1}{s^2}\,\hat{e}_r$

4.17 continued p. 7
collar FBD:




N = normal force exerted by rod on collar.

$\underset{\sim}{N} = (\frac{4}{5}\hat{e}_r - \frac{3}{5}\hat{e}_\theta)N$

$\underset{\sim}{F}_{spr} = (-\frac{3}{5}\hat{e}_r - \frac{4}{5}\hat{e}_\theta)K\delta$

Apply LMB:
$\sum \underset{\sim}{F} = \dot{\underset{\sim}{L}} \Rightarrow$

$N(\frac{4}{5}\hat{e}_r - \frac{3}{5}\hat{e}_\theta) + k\delta(-\frac{3}{5}\hat{e}_r - \frac{4}{5}\hat{e}_\theta) = m(-5L\frac{1}{s^2})\hat{e}_r$

$\hat{e}_r \cdot (\sum \underset{\sim}{F} = \dot{\underset{\sim}{L}}) \Rightarrow \frac{4}{5}N - \frac{3}{5}k\delta = -2.5L\ \frac{\mathrm{kg}}{s^2}$ (1)

$\hat{e}_\theta \cdot (\sum \underset{\sim}{F} = \dot{\underset{\sim}{L}}) \Rightarrow -\frac{3}{5}N - \frac{4}{5}k\delta = 0$ (2)

(2) gives $N = -\frac{4}{3}k\delta$. put this into (1):

$\frac{-16}{15}k\delta - \frac{9}{15}k\delta = -2.5L\ \frac{\mathrm{kg}}{s^2}$ recall $L = 0.4\,\mathrm{m}$, $K = 6\ \mathrm{N/m}$

$\therefore \frac{25}{15}(6\frac{N}{m})(\delta) = (-2.5)(0.4)\,[N]$

$\therefore \delta = 0.1\,\mathrm{m}$ and $L_o = L - \delta =$
$0.3\mathrm{m} - 0.1\mathrm{m} = \boxed{0.2\,\mathrm{m} = L_o}$

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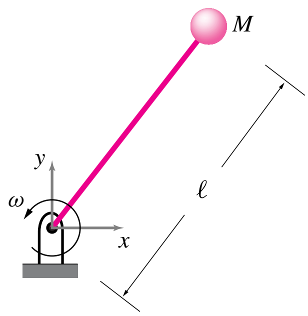
**15.2.15** A massless rigid rod with length $\ell$ attached to a ball of mass $M$ spins at a constant angular rate $\omega$ which is maintained by a motor (not shown) at the hinge point. The rod can only withstand a tension of $T_{cr}$ before breaking. Find the maximum angular speed of the ball so that the rod does not break assuming

a) there is no gravity, and

b) there is gravity (neglect bending stresses).




Problem 15.15

14.2.15 (a) No gravity

$\sum \underset{\sim}{F} = \dot{\underset{\sim}{L}} = m\underset{\sim}{a}$

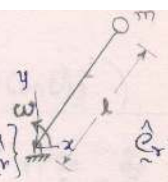
$\sum \underset{\sim}{F} = M(-\ell\dot\theta^2 \hat{\underset{\sim}{e}}_r)$

$T_{cr}(-\hat{\underset{\sim}{e}}_r) = -M\ell\dot\theta^2 \hat{\underset{\sim}{e}}_r$

$\Rightarrow \dot\theta = \sqrt{\frac{T_{cr}}{M\ell}}$

(b) With gravity

$\sum \underset{\sim}{F} = M\underset{\sim}{a}$




$\left\{-T\hat{\underset{\sim}{e}}_r - mg\underset{\sim}{j} = -M\ell\dot\theta^2\hat{\underset{\sim}{e}}_r\right\}$

$\hat{\underset{\sim}{e}}_r = \cos\theta\,\underset{\sim}{i} + \sin\theta\,\underset{\sim}{j}$

$\{\ \}\cdot\hat{\underset{\sim}{e}}_r$

$T = M\ell\dot\theta^2 - mg\sin\theta$

$T_{max} = M\ell\dot\theta^2 + Mg$

$\Rightarrow T_{cr} = Ml\omega^2 + Mg$

$\Rightarrow \omega_{max}^2 = \dfrac{T_{cr} - Mg}{Ml}$ $\quad \therefore \omega_{max} = \sqrt{\dfrac{T_{cr} - Mg}{Ml}}$

14.2.16

**15.2.16** A 1 m long massless string has a particle of 10 grams mass at one end and is tied to a stationary point $O$ at the other end. The particle rotates counter-clockwise in circles on a frictionless horizontal plane. The rotation rate is $2\pi$ rev/sec. Assume an $xy$-coordinate system in the plane with its origin at $O$.

a) Make a clear sketch of the system.

b) What is the tension in the string (in Newtons)?

c) What is the angular momentum of the mass about $O$?

d) When the string makes a 45° angle with the positive $x$ and $y$ axis on the plane, the string is quickly and cleanly cut. What is the position of the mass 1 sec later? Make a sketch of the particle's trajectory.

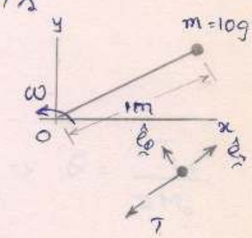
**Answer:**
(b) $T = 0.16\pi^4 N$.
(c) $\vec{\boldsymbol{H}}_{/O} = 0.04\pi^2\,\mathrm{kg{\cdot}m/s}\hat{\boldsymbol{k}}$
(d) $\vec{\boldsymbol{r}} = [\frac{\sqrt{2}}{2} - v\cos(\frac{\pi t}{4})]\hat{\boldsymbol{\imath}} + [\frac{\sqrt{2}}{2} + v\sin(\frac{\pi t}{4})]\hat{\boldsymbol{\jmath}}$.

14.2.16

$m = 0.01\,\mathrm{kg}$ $\quad \dot\theta = 2\pi\ \mathrm{rev/s} = 4\pi^2\ \mathrm{rad/s}$

$\sum \underset{\sim}{F} = \underset{\sim}{\dot L} = m\underset{\sim}{a}$

b)




$T(-\hat{e}_r) = m[-r\dot\theta^2 \hat{e}_r]$

$T = m r\dot\theta^2 \Rightarrow T = 0.16\pi^4\,\mathrm{N}$

c) $\underset{\sim}{H}_{/o} = \underset{\sim}{r}_{/o} \times m\underset{\sim}{v}$ $\qquad \underset{\sim}{v} = R\dot\theta\hat{e}_\theta$

$= \hat{e}_r \times mR\dot\theta\hat{e}_\theta$

$= 0.01 \times 4\pi^2 (\hat{k})(\mathrm{Nms}) \Rightarrow \underset{\sim}{H}_{/o} = 0.04\pi^2\ \mathrm{kg}\,\frac{\mathrm{m}}{\mathrm{s}}\cdot\mathrm{m}.$

d) $\theta = 45°$ at $t = 0$, $x = \frac{1}{\sqrt2}\,\mathrm{m}$; $y = \frac{1}{\sqrt2}\,\mathrm{m}$

$\sum \underset{\sim}{F} = m\underset{\sim}{a}$

$0 = m[\ddot x\,\hat\imath + \ddot y\,\hat\jmath] \Rightarrow \dot x = C_1 \Rightarrow x = C_1 t + C_2$

$\dot y = C_3 \Rightarrow y = C_3 t + C_4$

$C_2 = \frac{1}{\sqrt2}\,(\mathrm{m})$

$C_4 = \frac{1}{\sqrt2}\,(\mathrm{m})$

$e_\theta = -\sin\theta\,\hat\imath + \cos\theta\,\hat\jmath$

$\underset{\sim}{v}\Big|_{\theta=45°} = v\hat{e}_\theta = (-v\sin\theta)\,\hat\imath + (v\cos\theta)\,\hat\jmath \Rightarrow C_1 = -v\sin\theta$, $\quad C_2 = v\cos\theta$

$\Rightarrow x = \frac{1}{\sqrt2}(\mathrm{m}) - v\sin\theta\, t$ and $y = \frac{1}{\sqrt2}(\mathrm{m}) + v\cos\theta\, t$

at $t = 1\,\mathrm{s}$ $\quad \underset{\sim}{r} = \left(\left(\frac{1(\mathrm{m})}{\sqrt2} - v\sin\theta\right)\hat\imath + \left(\frac{1(\mathrm{m})}{\sqrt2} + v\cos\theta\right)\hat\jmath\right)$

14.2.17

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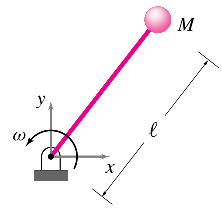
**15.2.17** A ball of mass $M$ fixed to an inextensible rod of length $\ell$ and negligible mass rotates about a frictionless hinge as shown in the figure. A motor (not shown) at the hinge point accelerates the mass-rod system from rest by applying a constant torque $M_O$. The rod is initially lined up with the positive $x$-axis. The rod can only withstand a tension of $T_{cr}$ before breaking. At what time will the rod break and after how many revolutions? Neglect bending stresses.

a) Neglect gravity.

b) Include gravity.

$\gamma$




Problem 15.17

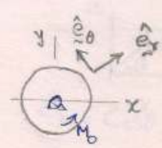
$\sqrt{2}$ ... $+\left(\frac{T(m)}{\sqrt{2}} + V\cos\theta\right)\hat{j}$ )

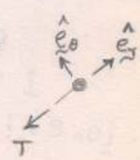
14.2.17

a) Neglect gravity

$\sum \underline{F} = \dot{\underline{L}} = m\underline{a}$

$-T\hat{e}_r = m[-r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}\hat{e}_\theta] \Rightarrow \dot{\theta} = \sqrt{\frac{T_{cr}}{mr}}$



$\sum M_{/O} = \dot{H}_{/O}$

$\dot{H}_{/O} = r_{/O} \times ma = mr^2 \ddot{\theta} \hat{k}$

$M_o = mr^2\ddot{\theta} \Rightarrow \ddot{\theta} = \frac{M_o}{mr^2}$

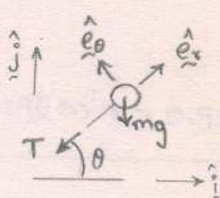
$\frac{d\dot{\theta}}{dt} = \frac{M_o}{mr^2} \Rightarrow \int d\dot{\theta} = \int \frac{M_o}{mr^2} dt$

$\dot{\theta} = \frac{M_o(t)}{mr^2}$ and $\theta = \frac{M_o t^2}{2mr^2}$

$\Rightarrow \sqrt{\frac{T_{cr}}{mr}} = \frac{M_o t}{mr^2}$ $\qquad \theta = \frac{M_o r^2 mrT_{cr}}{2mr^2 M_o} \Rightarrow \theta = \frac{rT_{cr}}{2M_o}$

$\Rightarrow t = \frac{r\sqrt{mrT_{cr}}}{M_o}$ $\qquad \theta_{rev} = \frac{rT_{cr}}{4\pi M_o}$

b) Consider Gravity




$\sum F = \dot{L} = ma$

$-T\hat{e}_r - mg\hat{j} = m[-r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}\hat{e}_\theta]$ $\qquad e_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\{\}\cdot\hat{e}_r \Rightarrow T = mr\dot{\theta}^2 - mg\sin\theta$ $\qquad e_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

$\sum M_{/O} = \dot{H}_{/O}$

$\sum M_{/O} = M_o + r \times mg(-\hat{j}) = M_o - mgr\cos\theta\,\hat{k}$

$\dot{H}_{/O} = mr^2\ddot{\theta}\hat{k} \Rightarrow M_o - mgr\cos\theta\Big|_{\theta=-90^\circ} = mr^2\ddot{\theta}$

We have $T_{max} = mr\dot{\theta}^2 + mg\Big|_{\theta=-90^\circ}$ $\qquad mr^2\ddot{\theta} = M_o$

$\dot{\theta} = \sqrt{\frac{T_{cr}-mg}{mr}} = \frac{M_o t}{mr^2}$ $\qquad \Rightarrow \dot{\theta} = \frac{M_o t}{mr^2} \Rightarrow \theta = \frac{M_o t^2}{2mr^2}$

$\Rightarrow t = \frac{r\sqrt{mr(T_{cr}-mg)}}{M_o}$ $\qquad \Rightarrow \theta = \frac{r(T_{cr}-mg)}{2M_o} \Rightarrow \theta_{rev} = \frac{r(T_{cr}-mg)}{4\pi M_o}$

14.2.18

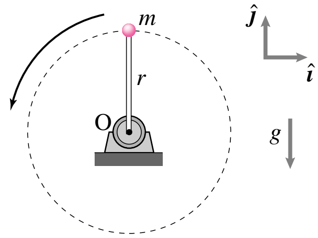
**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**15.2.18** A particle of mass $m$, tied to one end of a rod whose other end is fixed at point $O$ to a motor, moves in a circular path in the vertical plane at a constant rate. Gravity acts in the $-\hat{\jmath}$ direction.

a) Find the difference between the maximum and minimum tension in the rod.

b) Find the ratio $\frac{\Delta T}{T_{max}}$ where $\Delta T = T_{max} - T_{min}$. A criterion for ignoring gravity might be if the variation in tension is less than 2% of the maximum tension; i.e., when $\frac{\Delta T}{T_{max}} < 0.02$. For a given length $r$ of the rod, find the rotation rate $\omega$ for which this condition is met. .

c) For $\omega = 300\,\text{rpm}$, what would be the length of the rod for the condition in part (b) to be satisfied? .



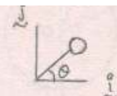

Problem 15.18

**Answer:**
(a) $2mg$.
(b) $\omega = \sqrt{99g/r}$
(c) $r \approx 1\,\text{m}$ ($r > 0.98\,\text{m}$)

$\Sigma \underset{\sim}{F} = \dot{\underset{\sim}{L}} = m\underset{\sim}{a}$ $\dot{\theta} = \text{Constant}$

$\{T(-\hat{e}_r) + mg(-\hat{\jmath}) = m(-r\dot{\theta}^2\hat{e}_r)\} \cdot \hat{e}_r$

$T = mr\dot{\theta}^2 - mg\sin\theta$




$\hat{\underset{\sim}{e}}_r = \cos\theta\, \underset{\sim}{i} + \sin\theta\, \underset{\sim}{j}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$M_A = -l\,\underset{\sim}{\hat{e}_\theta} \times m(\underset{\sim}{a})$$

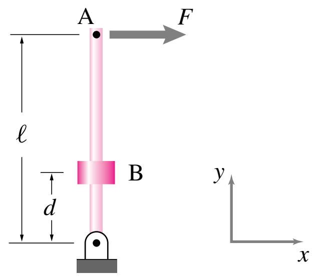
$$= ml\,(l\dot{\omega} - R\omega^2)$$

a) If $\dot{\theta} = 0$ & $\ddot{\theta} > 0$ => $M_A$ is +ve

b) If $\ddot{\theta} = 0$ & $\dot{\theta} > 0$ => $M_A$ is −ve.

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**15.2.19** A massless rigid bar of length $L$ is hinged at the bottom. A force $F$ is applied at point A at the end of the bar. A mass $m$ is glued to the bar at point B, a distance $d$ from the hinge. There is no gravity. What is the acceleration of point A at the instant shown? Assume the angular velocity is initially zero.




Problem 15.19

.2.19

$$\underset{\sim}{a}_A = -L\dot{\theta}^2 \hat{\underset{\sim}{e}}_r + L\frac{FL}{md^2}\hat{\underset{\sim}{e}}_\theta$$

$$\Sigma \underset{\sim}{M}_{/O} = \dot{\underset{\sim}{H}}_{/O}$$

$$\underset{\sim}{r} \times \underset{\sim}{F} = \underset{\sim}{r}_{/O} \times m\underset{\sim}{a}$$

$$FL\hat{\underset{\sim}{k}} = d\,\underset{\sim}{\hat{\imath}} \times m\left[-d\dot{\theta}^2 \hat{\underset{\sim}{e}}_r + d\ddot{\theta}\hat{\underset{\sim}{e}}_\theta\right]$$

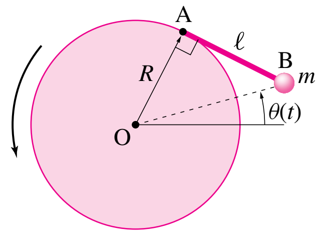
$$FL\hat{\underset{\sim}{k}} = -d^2 m\ddot{\theta}\hat{\underset{\sim}{k}}$$

$$\ddot{\theta} = \frac{FL}{md^2} \Rightarrow \dot{\theta}\Big|_{\dot{\theta}_0}^{\dot{\theta}} = \frac{FL}{md^2}t\Big|_{t_0}^{t} \Rightarrow \dot{\theta} = \frac{FLt}{md^2}$$

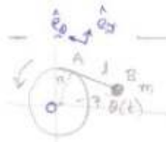
$$\underset{\sim}{a}_A = -L\left[\frac{FLt}{md^2}\right]^2 \hat{\underset{\sim}{e}}_r + L\frac{FL}{md^2}\underset{\sim}{\hat{\imath}} \quad //$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**15.2.20** The mass $m$ is attached rigidly to the rotating disk by the light rod AB of length $\ell$. Neglect gravity. Find $M_A$ (the moment on the rod $AB$ from its support point at $A$) in terms of $\dot{\theta}$ and $\ddot{\theta}$. What is the sign of $M_A$ if $\dot{\theta} = 0$ and $\ddot{\theta} > 0$? What is the sign if $\ddot{\theta} = 0$ and $\dot{\theta} > 0$?




Problem 15.20




$\underset{\sim}{r} = R\hat{\underset{\sim}{e}}_r - l\hat{\underset{\sim}{e}}_\theta$

$R\omega\hat{\underset{\sim}{e}}_\theta + l\omega\hat{\underset{\sim}{e}}_r$

$= \dot{\underset{\sim}{H}}_{/o}$ $\quad \underset{\sim}{a} = R\dot{\omega}\hat{e}_\theta - R\omega^2\hat{e}_r + l\dot{\omega}\hat{e}_r + l\omega^2\hat{\underset{\sim}{e}}_\theta$

$(l\dot{\omega} - R\omega^2)\hat{\underset{\sim}{e}}_r + (l\omega^2 + R\dot{\omega})\hat{\underset{\sim}{e}}_\theta$

$-l\hat{\underset{\sim}{e}}_\theta \times m(\underset{\sim}{a})$

$= ml(l\dot{\omega} - R\omega^2)$

$\dot{\theta} = 0$ & $\ddot{\theta} > 0 \Rightarrow M_A$ is +ve

$\ddot{\theta} = 0$ & $\dot{\theta} > 0 \Rightarrow M_A$ is −ve.

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**15.2.21 Simple pendulum, comprehensive version.** This problem covers many aspects of a simple pendulum. A point mass $M$ hangs on a massless string or rod of length $L$. The gravitational force is $Mg$. The pendulum is in a vertical plane. At any time $t$, the angle between the straight down line and the pendulum, measured counter clockwise, is $\theta(t)$. Neglect air friction. When numbers are called for use $M = 1\,\text{kg}$, $L = 1\,\text{m}$ and $g = 10\,\text{m/s}^2$.

a) **Find the equations of motion.** That is, assume that you know both $\theta$ and $\dot{\theta}$, find $\ddot{\theta}$. There are several ways to do this problem. Find the equations using

   1. Linear momentum balance
   2. Angular momentum balance
   3. Conservation of energy

b) **Tension.** Assuming that you know $\theta$ and $\dot{\theta}$, find the tension $T$ in the string.

c) **Reaction components.** Assuming you know $\theta$ and $\dot{\theta}$, find the $x$ and $y$ components of the force that the hinge support causes on the pendulum. Clearly define the directions of positive $x$ and $y$ with a sketch.

d) **Reduction to first order equations.** The equation that you found in (a) is a nonlinear second order ordinary differential equation. It can be changed to a pair of first order equations by defining a new variable $\omega \equiv \dot{\theta}$. Write the equation from (a) as a pair of first order equations.

e) **Numerical solution.** Given the initial conditions $\theta(t = 0) = \pi/2$ and $\omega(t = 0) = \dot{\theta}(t = 0) = 0$. Using numerical integration, find: $\theta(t)$, $\dot{\theta}(t)$ & $T(t)$. Make a single plot, or three vertically aligned plots, of these variables for one full oscillation of the pendulum.

f) **Maximum tension.** Using your numerical solutions, find the maximum value of the tension in the rod as the mass swings.

g) Plot the $x$ and $y$ reaction components as a function of time.

h) **Period of oscillation.** How long does it take to make one oscillation?

i) **Other observations.** Some questions:

   1. Does the solution to (f) depend on the length of the string?
   2. Is the solution to (f) exactly 30 or just a number near 30? If it is exact can you find the result analytically?
   3. Is the period found in (h) longer or shorter than the period found by solving the linear equation $\ddot{\theta} + (g/l)\theta = 0$, based on the (inappropriate-to-use in this case) small angle approximation $\sin\theta = \theta$? Explain intuitively why you expect the period to be longer or shorter?

**Answer:**
(a) $\ddot{\theta} = -(g/L)\sin\theta$
(d) $\dot{\omega} = -(g/L)\sin\theta, \quad \dot{\theta} = \omega$
(f) $T_{max} = 30N$


Pendulum Problems
14.2.21
a) 1) Linear momentum Balance
2) Angular Momentum Balance
3) Conservation of energy
b) $T = mr\dot{\theta}^2 + mg\cos\theta$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

d) $\omega = \dot{\theta}$

$\dot{\omega} = \frac{-g}{l} \sin\theta$

e) $\theta(t=0) = \pi/2$ ; $\omega(t=0) = \dot{\theta}(t=0) = 0 \frac{rad}{s}$

h) $T = 2.36\,s$

$x_1 = \theta \quad \dot{x}_1 = \dot{\theta} = x_2$

$x_2 = \dot{\theta} = \dot{x}_1 \quad \dot{x}_2 = -\frac{g}{l} \sin x_1$

i) 1) No

2) $E_1 = E_2$

$$mgl - mgl\cos\theta_{max} = \frac{1}{2} m (l\dot{\theta})^2 + mgl - mgl\cos\theta$$

$$2gl\,[\cos\theta - \cos\theta_{max}] = l^2\dot{\theta}^2$$

$$\Rightarrow \dot{\theta} = \sqrt{\frac{2g}{l}(\cos\theta - \cos\theta_{max})}$$

$\theta = 0, \quad \theta_{max} = \pi/2 \text{ (rad)} \Rightarrow \dot{\theta} = \sqrt{\frac{2g}{l}}$

f) $T = ml\dot{\theta}^2 + mg\cos\theta$

$T = ml\left(\sqrt{\frac{2g}{l}}\right)^2 + mg(1) \Rightarrow T = 3mg$

3) $t = 2\pi\sqrt{\frac{l}{g}} = 1.98\,s$. is shorter than actual $t = 2.36\,s$

Accelerations in the system with $\theta|_{t=0} = \pi/2$ are larger than in system with small $\theta_0$ values.

14. 2.21

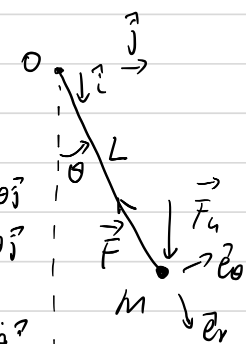
Simple pendulum dynamics

A point mass M hangs on a massless string or rod of length L. The pendulum is verticle under gravitational force Mg. neglect air friction

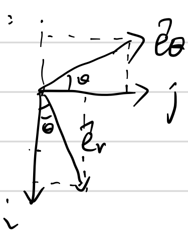
Given: $M = 1\,kg$

$L = 1\,m$

$g = 10\,m/s^2$




Coordinate system:




$\vec{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\vec{e}_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

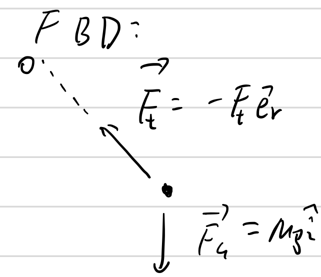
$\dot{\vec{e}}_r = -\sin\theta\dot{\theta}\,\hat{i} + \cos\theta\dot{\theta}\,\hat{j}$

$= \dot{\theta}\,\vec{e}_\theta$

$\dot{\vec{e}}_\theta = -\cos\theta\dot{\theta}\,\hat{i} - \sin\theta\dot{\theta}\,\hat{j}$

$= -\dot{\theta}\,\vec{e}_r$

FBD:




a. Find EoM

For circular motion:

$\vec{v} = \omega L\,\vec{e}_\theta = \dot{\theta} L\,\vec{e}_\theta$

$\vec{a} = \dot{\vec{v}} = (\ddot{\theta} L)\,\vec{e}_\theta + (\dot{\theta} L)\,\dot{\vec{e}}_\theta$

$= \ddot{\theta} L\,\vec{e}_\theta - \dot{\theta}^2 L\,\vec{e}_r$

① Using Linear Momentum Balance:

$\sum \vec{F} = m\vec{a}$

$-F_t\vec{e}_r + Mg\,\hat{i} = M(\ddot{\theta} L\vec{e}_\theta - \dot{\theta}^2 L\vec{e}_r) = \{LMB\}$

$\vec{e}_\theta \cdot \{LMB\}$: ( Dot $\vec{e}_\theta$ with LMB to eliminate unknown F
Same as dot $\vec{v}$ with LMB )

$\vec{e}_\theta \cdot (-F_t\vec{e}_r + Mg\,\hat{i}) = \vec{e}_\theta \cdot (M\ddot{\theta} L\vec{e}_\theta - M\dot{\theta}^2 L\vec{e}_r)$

$$-F_t\underbrace{(\vec{e}_\theta \cdot \vec{e}_r)}_{0} + Mg(\vec{e}_\theta \cdot \hat{i}) = M\ddot{\theta} L\underbrace{(\vec{e}_\theta \cdot \vec{e}_\theta)}_{1} - M\dot{\theta}^2 L\underbrace{(\vec{e}_\theta \cdot \vec{e}_r)}_{0}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$Mg\,\vec{e}_\theta \cdot \hat{i} = m\dot{v}$$

$$Mg\,(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) \cdot \hat{i} = m\dot{v}$$

$$-Mg\sin\theta = M\dot{v}$$

$$\dot{v} = -g\sin\theta$$

$$\dot{v} = \frac{d}{dt}(\dot{\theta}L) = \ddot{\theta}L \Rightarrow \ddot{\theta}L = -g\sin\theta \Rightarrow \boxed{\ddot{\theta} = -\frac{g}{L}\sin\theta}$$

② Using Angular Momentum Balance

$$\sum \vec{M}_{/0} = \dot{\vec{H}}_{/0}$$

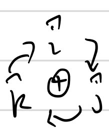
$$\vec{r}_{/0} \times (\vec{F}_t + \vec{G}) = M\,\vec{r}_{/0} \times \vec{a} + \underbrace{I^{G}\dot{\omega}\hat{k}}_{0 \text{ for point mass}}$$

$$L\vec{e}_r \times (-F_t\vec{e}_r + Mg\hat{i}) = mL(\vec{e}_r \times \vec{a})$$

$$MgL(\vec{e}_r \times \hat{i}) = ML(\vec{e}_r \times (\dot{v}\vec{e}_\theta + v\dot{\theta}\vec{e}_r))$$

$$MgL(\vec{e}_r \times \hat{i}) = ML\dot{v}(\vec{e}_r \times \vec{e}_\theta) + MLv\dot{\theta}(\vec{e}_r \times \vec{e}_r)$$

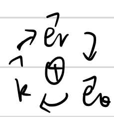
$$\vec{e}_r \times \hat{i} = (\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) \times \hat{i} = -\sin\theta\,\hat{k}$$



$$-MgL\sin\theta\,\hat{k} = ML\dot{v}\,\hat{k}$$

$$-MgL\sin\theta = ML\ddot{\theta}L \quad (\text{See part 1 for } \dot{v} = \ddot{\theta}L)$$

$$-g\sin\theta = \ddot{\theta}L$$



$$\boxed{\ddot{\theta} = -\frac{g}{L}\sin\theta}$$

③ Conservation of Energy

Select zero potential energy to be the same hight as point 0.

$$E_p = -mgL\cos\theta$$

$$E_k = \frac{1}{2}mv^2$$

$$E_{tot} = E_k + E_p = -mgL\cos\theta + \frac{1}{2}mv^2$$

Since the work done by gravitational force has been considered and tangential pulling force of the rod does zero work:

$$\Delta E_{tot} = 0$$

$$\frac{d}{dt}(E_{tot}) = 0$$

$$\frac{d}{dt}(E_{tot}) = MgL\sin\theta\dot{\theta} + \frac{1}{2}\cdot 2MV\dot{V}$$

$$= MgL\sin\theta\dot{\theta} + M\dot{\theta}L\,\ddot{\theta}L = 0$$

$$g\sin\theta\dot{\theta} = -\dot{\theta}\ddot{\theta}L \qquad \text{assume } \dot{\theta} \neq 0$$

$$g\sin\theta = -\ddot{\theta}L$$

$$\boxed{\ddot{\theta} = -\frac{g}{L}\sin\theta}$$

b. find tension force given $\theta$ & $\dot{\theta}$

$\{LMB\}: -F_t\vec{e}_r + Mg\hat{\imath} = M(\dot{V}\vec{e}_\theta - V\dot{\theta}\vec{e}_r)$

$\vec{e}_r\cdot\{LMB\}: \quad -F_t + Mg(\vec{e}_r\cdot\hat{\imath}) = -MV\dot{\theta}$

$$-F_t + Mg\cos\theta = -\dot{\theta}^2LM$$

$$\boxed{F_t = Mg\cos\theta + \dot{\theta}^2LM}$$

c. Find the x and y components of force that the hinger causes on pendulum.

The force caused by the hinger on pendulum is $\vec{F}_t$.

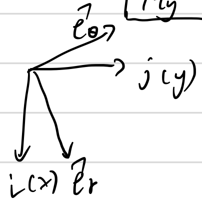
So $F_{tx} = \vec{F}_t\cdot\hat{\imath} = -F_t\vec{e}_r\cdot\hat{\imath}$

$$\boxed{F_{tx} = -(Mg\cos\theta + \dot{\theta}^2LM)\cos\theta}$$

$F_{ty} = \vec{F}_t\cdot\hat{\jmath} = -F_t\vec{e}_r\cdot\hat{\jmath}$

$$\boxed{F_{ty} = -(Mg\cos\theta + \dot{\theta}^2LM)\sin\theta}$$

Coordinate System:



d. Reduce the second order eqn. $\ddot{\theta} = -\frac{g}{L}\sin\theta$ to first order equations:

$$\omega = \dot{\theta} \qquad \ddot{\theta} = \frac{d}{dt}\dot{\theta} = \dot{\omega}$$

$$\boxed{\begin{cases} \omega = \dot{\theta} \\ \dot{\omega} = -\frac{g}{L}\sin\theta \end{cases}}$$

e,h. See code

i. 1. We have EoM:

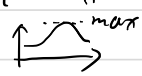
$$\ddot{\theta} = -\frac{g}{L}\sin\theta$$

when $F_t = F_{t\,max}$

$g\cos\theta$ and $\dot{\theta}^2 L$ are all at max.

$g\cos\theta$ is at max when $\theta = 0$

$\dot{\theta}^2 L$ is at max when $\dot{\theta}$ is at max

for $\dot{\theta}$ to be at max, $\ddot{\theta} = 0$. ( 

)

$$-\frac{g}{L}\sin\theta = 0 \Rightarrow \theta = 0$$

At $\theta = 0$:

$$E_{tot,\theta=0} = E_{tot,0} = 0$$

$$-mgl\cos(0) + \frac{1}{2}mV_{\theta=0}^2 = 0$$

$$V_{\theta=0} = \sqrt{2gL}$$

$$\dot{\theta}_{\theta=0} = \frac{V_{\theta=0}}{L} = \frac{\sqrt{2gL}}{L}$$

$$F_{t\,max} = Mg\cos(0) + \left(\frac{\sqrt{2gL}}{L}\right)^2 LM$$

$$= Mg + \frac{2gL}{L^2} LM$$

$$= Mg + 2gM \Rightarrow \text{Independent from } L.$$

2. When $F_t = F_{tmax}$, $\theta = 0$

Energy balance:

$E_{tot} = -mgL\cos\theta + \frac{1}{2}mv^2$

starting from $\theta_0 = \pi/2$, $\dot{\theta}_0 = 0$:

$E_{tot,0} = 0$

At $\theta = 0$:

$E_{tot,\theta=0} = E_{tot,0} = 0$

$-mgL\cos(0) + \frac{1}{2}mV_{\theta=0}^2 = 0$

$V_{\theta=0} = \sqrt{2gL}$

$\dot{\theta}_{\theta=0} = \frac{V_{\theta=0}}{L} = \frac{\sqrt{2gL}}{L}$

$F_{tmax} = Mg\cos(0) + \left(\frac{\sqrt{2gL}}{L}\right)^2 LM$

$= Mg + \frac{2gL}{L^2}LM$

$= Mg + 2gM = 3gM$

$M = 1\,kg$ $g = 10\,m/s^2$ : $F_{tmax} = 30\,N$

3. The solution for $\ddot{\theta} + (g/L)\theta = 0$

is:

$\theta = C_1\sin\left(\sqrt{\frac{g}{L}}t\right) + C_2\cos\left(\sqrt{\frac{g}{L}}t\right)$

$\omega$

$T = 2\pi/\omega = 2\pi\sqrt{\frac{L}{g}} = 1.99\,s < 2.3445\,s$ of real $T$ found by numerical solution

At large angle from $-\pi/2$ to $\pi/2$,

$|\sin(\theta)| < |\theta|$ Assuming $\frac{\sin\theta}{\theta} = c$, $0<c<1$

$\ddot{\theta} + c(g/L)\theta = 0$ $\theta_{real} = C_1\sin\left(\sqrt{c\frac{g}{L}}t\right) + C_2\cos\left(\sqrt{c\frac{g}{L}}t\right)$

$T_{real} = 2\pi\sqrt{\frac{L}{cg}} > 2\pi\sqrt{\frac{L}{g}} = T \Rightarrow T_{real} > T$

# HW8-P43-15.2.21

Tuesday, March 30, 2021 6:10 PM

Solution by Michael Zakoworotny

**15.2.21 Simple pendulum, comprehensive version.** This problem covers many aspects of a simple pendulum. A point mass $M$ hangs on a massless string or rod of length $L$. The gravitational force is $Mg$. The pendulum is in a vertical plane. At any time $t$, the angle between the straight down line and the pendulum, measured counter clockwise, is $\theta(t)$. Neglect air friction. When numbers are called for use $M = 1\,\text{kg}$, $L = 1\,\text{m}$ and $g = 10\,\text{m/s}^2$.

a) **Find the equations of motion.** That is, assume that you know both $\theta$ and $\dot{\theta}$, find $\ddot{\theta}$. There are several ways to do this problem.* Find the equations using
   1. Linear momentum balance
   2. Angular momentum balance
   3. Conservation of energy

b) **Tension.** Assuming that you know $\theta$ and $\dot{\theta}$, find the tension $T$ in the string.

c) **Reaction components.** Assuming you know $\theta$ and $\dot{\theta}$, find the $x$ and $y$ components of the force that the hinge support causes on the pendulum. Clearly define the directions of positive $x$ and $y$ with a sketch.

d) **Reduction to first order equations.** The equation that you found in (a) is a nonlinear second order ordinary differential equation. It can be changed to a pair of first order equations by defining a new variable $\omega \equiv \dot{\theta}$. Write the equation from (a) as a pair of first order equations. *

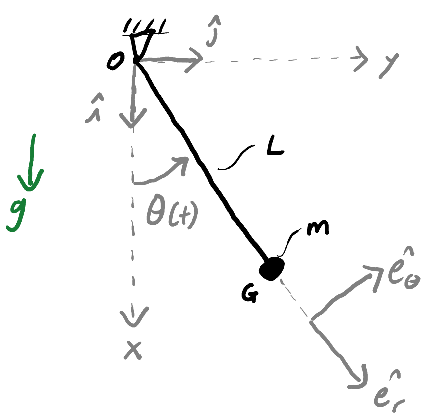
e) **Numerical solution.** Given the initial conditions $\theta(t=0) = \pi/2$ and $\omega(t=0) = \dot{\theta}(t=0) = 0$. Using numerical integration, find: $\theta(t)$, $\dot{\theta}(t)$ & $T(t)$. Make a single plot, or three vertically aligned plots, of these variables for one full oscillation of the pendulum.

f) **Maximum tension.** Using your numerical solutions, find the maximum value of the tension in the rod as the mass swings. *

g) Plot the $x$ and $y$ reaction components as a function of time.

h) **Period of oscillation.** How long does it take to make one oscillation?

i) **Other observations.** Some questions:
   1. Does the solution to (f) depend on the length of the string?
   2. Is the solution to (f) exactly 30 or just a number near 30? If it is exact can you find the result analytically?
   3. Is the period found in (h) longer or shorter than the period found by solving the linear equation $\ddot{\theta} + (g/l)\theta = 0$, based on the (inappropriate-to-use in this case) small angle approximation $\sin\theta = \theta$? Explain intuitively why you expect the period to be longer or shorter?




Given: $M = 1\,kg$
$L = 1\,m$
$g = 10\,m/s^2$

Assume:
- No friction
- Pendulum in vertical plane

FBD:


a) Find equation of motion, given $\theta, \dot{\theta}, \ddot{\theta}$

1) Use linear momentum balance

$$\sum \vec{F} = m\vec{a}$$

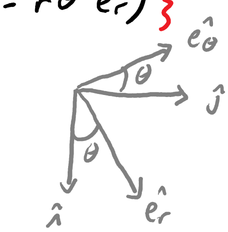
$$\{ -T\hat{e}_r + Mg\hat{\imath} = M(r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r) \}$$

$$\{\}\cdot\hat{e}_\theta : \cancel{M}g\,\hat{\imath}\cdot\hat{e}_\theta = \cancel{M}r\ddot{\theta}$$

$r = L$

$$-g\sin\theta = L\ddot{\theta}$$

$$\boxed{\ddot{\theta} = -\frac{g}{L}\sin\theta}$$




$\hat{e}_\theta \cdot \hat{\imath} = -\sin\theta$

2) Use angular momentum balance

$$\sum M_{/O} = \dot{\vec{H}}_{/O} = \dot{\vec{H}}_{G/O} + \dot{\vec{H}}_{/G}$$

$\sum M_{/O} = \vec{r}_{G/O} \times Mg\hat{\imath}$, $\vec{r}_{G/O} = L\hat{e}_r$

$\dot{\vec{H}}_{G/O} = \vec{r}_{G/O} \times m\vec{a}_{G/O}$, $\vec{r}_{G/O} = L\hat{e}_r$, $\vec{a}_{G/O} = L\ddot{\theta}\hat{e}_\theta - L\dot{\theta}^2\hat{e}_r$

$\dot{\vec{H}}_{/G} = \cancel{I}^G\ddot{\theta}\hat{k}$ O, point mass

$$L\hat{e}_r \times Mg\hat{\imath} = L\hat{e}_r \times M(L\ddot{\theta}\hat{e}_\theta - L\dot{\theta}^2\hat{e}_r)$$

$\hat{e}_r \times \hat{e}_\theta = \hat{k}$

$\hat{e}_r \times \hat{e}_r = 0$

$$\{ -LMg\sin\theta\,\hat{k} = L^2M\ddot{\theta}\,\hat{k} \}$$

$$\{\}\cdot\hat{k} : -\cancel{L}\cancel{M}g\sin\theta = L^{\cancel{2}}\cancel{M}\ddot{\theta}$$

$$-g\sin\theta = L\ddot{\theta}$$

$$\boxed{\ddot{\theta} = \frac{-g}{L}\sin\theta}$$

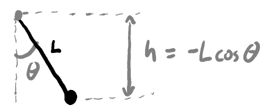
3) Use conservation of energy

$$E_{TOT} = E_K + E_P = const$$

$$\frac{1}{2} M |\vec{v}|^2 + Mgh = const$$

$$\frac{1}{2} M (L\dot{\theta})^2 - MgL\cos\theta = const$$

$$\left\{ \frac{1}{2} M L^2 \dot{\theta}^2 - MgL\cos\theta = const \right\}$$




$\vec{v} = L\dot{\theta}\,\hat{e}_\theta$

$$\frac{d}{dt}\{\}: \cancel{M}L^{\cancel{2}}\cancel{\dot{\theta}}\ddot{\theta} + \cancel{M}g\cancel{L}\cancel{\dot{\theta}}\sin\theta = 0$$

$$\boxed{\ddot{\theta} = \frac{-g}{L}\sin\theta}$$

b) Find tension in string, $T$, given $\theta$, $\dot{\theta}$

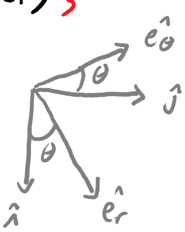
Consider LMB equation

$$\left\{ -T\hat{e}_r + Mg\hat{n} = M(r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r) \right\}$$

$$\{\}\cdot\hat{e}_r: \quad -T + Mg\,\hat{n}\cdot\hat{e}_r = -M\underbrace{r}_{L}\dot{\theta}^2$$

$$-T + Mg\cos\theta = -ML\dot{\theta}^2$$

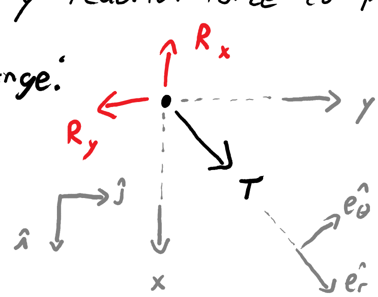
$$\boxed{T = M(g\cos\theta + L\dot{\theta}^2)}$$




$\hat{n}\cdot\hat{e}_r = \cos\theta$

c) Find x and y reaction force components at hinge

FBD of hinge:



$\Sigma F_{hinge} = 0$

$\{ T\hat{e}_r - R_x \hat{\imath} - R_y \hat{\jmath} = 0 \}$ $\hat{e}_r \cdot \hat{\imath} = \cos\theta$

$\quad = M(g\cos\theta + L\dot{\theta}^2)$ $\hat{e}_r \cdot \hat{\jmath} = \sin\theta$

$\{\} \cdot \hat{\imath}:\ M(g\cos\theta + L\dot{\theta}^2)\cos\theta - R_x = 0$

$$\boxed{R_x = M(g\cos\theta + L\dot{\theta}^2)\cos\theta}$$

$\{\} \cdot \hat{\jmath}:\ M(g\cos\theta + L\dot{\theta}^2)\sin\theta - R_y = 0$

$$\boxed{R_y = M(g\cos\theta + L\dot{\theta}^2)\sin\theta}$$

d) Write $2^{nd}$ order ODE as a pair of $1^{st}$ order ODE's

$\ddot{\theta} = -\frac{g}{L}\sin\theta$ Let $w \equiv \dot{\theta}$

$$\boxed{\begin{aligned} \dot{\theta} &= w \\ \dot{w} &= -\frac{g}{L}\sin\theta \end{aligned}}$$

$1^{st}$ order form

e) Find $\theta(t)$, $\dot{\theta}(t)$ and $T(t)$ using numerical integration

IC's: $\theta(0) = \pi/2$

$w(0) = \dot{\theta}(0) = 0$ pg. 6

Implement the $1^{st}$ order form from d) in MATLAB using ode45, and the tension formula from b)

f) Find max tension from numerical solution:

The maximum tension can be obtaned as the maximum of the tension array over one period

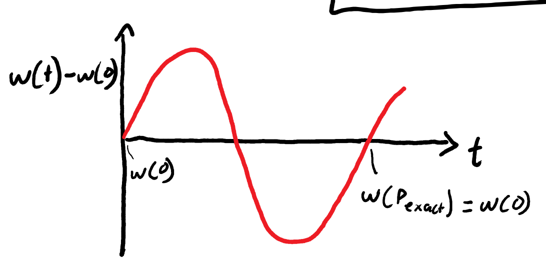
$$\boxed{T_{max} = 30N}$$

g) The reaction components, $R_x$ and $R_y$, from c), are plotted in MATLAB.

h) Find the period of one oscillation:

The period is found numerically in MATLAB as the time for $\theta$, or $\omega$, to complete one full cycle (ie. return to initial value). To get an exact value, the zero of the function $\omega(t) - \omega(0)$ closest to $P_{approx} = 2\pi\sqrt{L/g}$, the small angle period, is found using fzero.

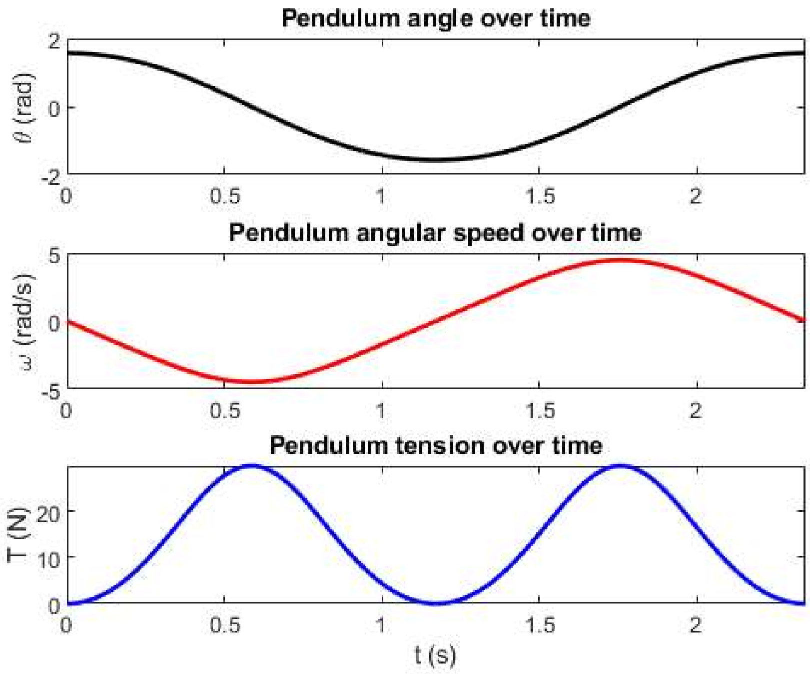
$$\boxed{P_{exact} = 2.345\ s}$$

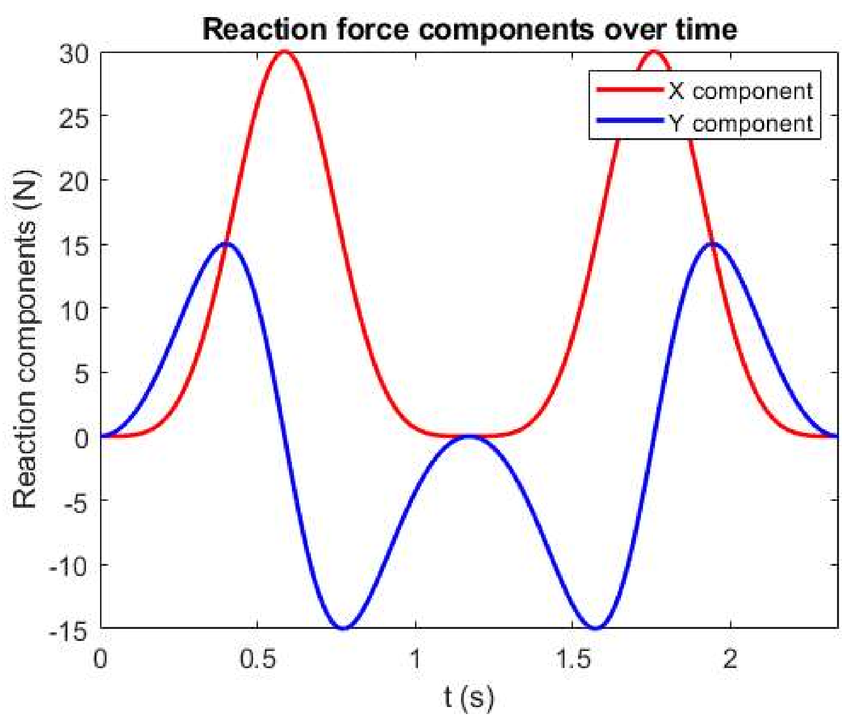


```
% % MAE 2030
% HW 8, Q43 part d Solution
% Solution by Michael Zakoworotny

clear all; clc; close all;
set(0,'DefaultAxesFontSize',12); % set plot font size

% Define pendulum parameters and gravity
M = 1; L = 1; g = 10;
p.M = M; p.L = L; p.g = g;

% Set initial conditions
th0 = pi/2; w0 = 0;
z0 = [th0 w0]';

% Set run time
t0 = 0; tend = 10;
tspan = [t0 tend];

% Integate over time
fun = @(t,z) myrhs(t,z,p); % function defining the ode
tol = 1e-6;
opts = odeset('RelTol',tol,'AbsTol',tol);
soln = ode45(fun, tspan, z0, opts); % get a solution object for accurate
                                    % interpolation

% Cut off the plot after one oscillation - find the exact time for one
% oscillation. Use small angle approximation of period for guess. Find the
% time around T_approx for which the velocity returns to its initial value.
% (Use the velocity because there are no sign changes when finding the
% zeros associated with theta)
T_approx = 2*pi*sqrt(L/g);
T_exact = fzero(@(t) deval(soln,t,2) - w0, T_approx);

% Extract th and w, over one period.
% deval evaluates the soln structure at all points in t_array, where 1 is
% the index for th, and 2 is the index for w
t_array = linspace(t0, T_exact, 500); % 500 time points from 0 to 1 period
th = deval(soln, t_array, 1);
w  = deval(soln, t_array, 2);
Tension = M*(g*cos(th) + L*w.^2); % tension formula from b)

figure;
% Plot theta vs time
subplot(3,1,1); % create 3 vertically stacked subplots, plot in uppermost
plot(t_array, th,'-k','LineWidth',2);
xlim([0,T_exact]); % set x limits to 0 - T_exact
ylabel('\theta (rad)','interpreter','tex');
title('Pendulum angle over time');

% Plot omega vs time
subplot(3,1,2); % plot in middle subplot
plot(t_array, w,'-r','LineWidth',2);
xlim([0,T_exact]); % set x limits to 0 - T_exact
ylabel('\omega (rad/s)','interpreter','tex');
```

```
title('Pendulum angular speed over time');

% Plot tension vs time
subplot(3,1,3); % plot in lower subplot
plot(t_array, Tension,'-b','LineWidth',2);
xlim([0,T_exact]); % set x limits to 0 - T_exact
ylabel('T (N)');
xlabel('t (s)');
title('Pendulum tension over time');

% f - Maximum tension
max_Tension = max(Tension);
fprintf('The maximum tension is %.3f N\n',max_Tension);

% g - x and y reaction components
R_x = Tension.*cos(th);
R_y = Tension.*sin(th);
% Plot reaction components
figure; box on; hold on;
plot(t_array, R_x, '-r', 'LineWidth', 2);
plot(t_array, R_y, '-b', 'LineWidth', 2);
xlim([0,T_exact]); % set x limits to 0 - T_exact
xlabel('t (s)');
ylabel('Reaction components (N)');
title('Reaction force components over time');
legend({'X component','Y component'});

% h - Print the exact period, found in line 32
fprintf('The period is %.4f s\n', T_exact);

function zdot = myrhs(t,z,p)
    M = p.M; g = p.g; L = p.L;
    th = z(1); w = z(2);

    thdot = w;
    wdot  = -g/L*sin(th);
    zdot = [thdot wdot]';
end
```

```
The maximum tension is 30.000 N
The period is 2.3452 s
```


Pendulum angle over time
θ (rad)
Pendulum angular speed over time
ω (rad/s)
Pendulum tension over time
T (N)
t (s)


Reaction force components over time
X component
Y component
Reaction components (N)
t (s)

- We see that the pendulum angle varies periodically, though since this oscillates at large angles, the motion is not precisely a sinusoid. The angular velocity oscillates similarly with a phase shift, as expected.
- The tension oscillates between 0 and the maximum value of 30, with twice the frequency of the angle θ. This is representative of the fact that tension in a string is strictly positive, and reaches its maximum value whenever θ = 0° (the lowest point in arc).
- The x reaction component (vertical) follows a similar trend as the tension, though it is slightly distorted when θ is at its maximum and minimum values from the introduction of the additional cos function.
- The y reaction component (horizontal) oscillates between positive and negative values, consistent with whether θ is positive or negative. However, it exhibits two intermediate peaks, which corresponding to angles between 0 and 90∞ This reflects the cosθ*sinθ term in the equation for $R_y$, which is maximized at θ=45°. A more intuitive way to understand this is to recall that the horizontal component of tension is 0 when θ = 0° (string is vertical), and when θ = +/-90∞for these initial conditions ($\dot{\theta}$=0 momentarily at highest point in arc); the tension must therefore be maximized between these angles.

i) 1) Does maximum tension depend on $L$?

By varying $L$ in MATLAB code, we see that the maximum tension does not change. This can be shown analytically in part 2)

2) Is max tension exactly 30?

We know maximum tension occurs at $\theta = 0$ (bottom of arc), since rope must support full weight of mass and greater centripetal acceleration

$$T_{max} = M(g\cos\theta + L\dot{\theta}^2)$$

Solve for $\dot{\theta}$ using energy: evaluate at $\theta = 0$

$$E(\theta = 0) = E(\theta = \pi/2)$$

at rest at $\theta = 0$

$$\cancel{M} g(-L) + \tfrac{1}{2}\cancel{M}(L\dot{\theta})^2 = 0 + 0$$

$$g\cancel{L} = \tfrac{1}{2} L^{\cancel{2}} \dot{\theta}^2$$

$$\dot{\theta} = \sqrt{2g/L}$$

$$T_{max} = M\left(g\cos 0 + L\cdot\left(\sqrt{\tfrac{2g}{L}}\right)^2\right)$$

$$T_{max} = M(g + 2g)$$

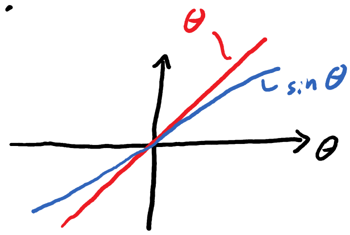
$$* \quad \boxed{T_{max} = 3Mg} \quad *$$

We see that $T_{max} = 30$ for our given $M$ and $g$, and is independent of $L$, supporting the result from 1)

3) How does period from small angle approximation compare to this exact period?:

Exact: $\ddot{\theta} = -\frac{g}{L} \sin\theta$

Approx: $\ddot{\theta} = -\frac{g}{L} \theta$




For a given value of $\theta$, the approximation overpredicts the magnitude of $\ddot{\theta}$, since $|\theta| > |\sin\theta|$. The pendulum governed by the approximate model will therefore have a

$\boxed{\text{shorter period.}}$

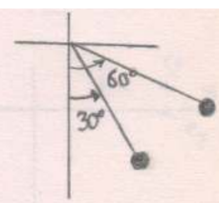
**15.2.22 Tension in a simple pendulum string.** A simple pendulum of length 2 m with mass 3 kg is released from rest at an initial angle of 60° from the vertically down position.

a) What is the tension in the string just after the pendulum is released?

b) What is the tension in the string when the pendulum has reached 30° from the vertical?

$l = 2m$ $m = 3Kg$ $\theta(t=0) = 60^\circ$

$\theta(t) = 30^\circ$




$$\dot\theta = \sqrt{\frac{2g}{l}(\cos\theta - \cos\theta_{max})}$$

a) Just released, $\dot\theta = 0$ $T = ml\dot\theta^2 + mg\cos\theta$

$\therefore T = mg\cos\theta = mg\cos(\pi/3) = 15\,N.$

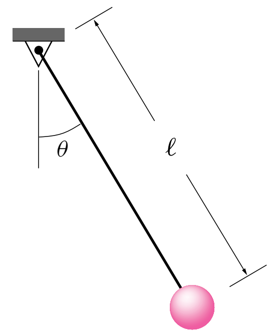
b) $\dot\theta = \sqrt{\frac{2g}{l}\left[\cos\frac{\pi}{6} - \cos\frac{\pi}{3}\right]} \Rightarrow \dot\theta^2 = \frac{20}{2}\left\{\frac{\sqrt3}{2} - \frac12\right\}\frac{rad}{s^2} = 5(\sqrt3 - 1)\frac{rad}{s^2}$

$T = \left\{3 \cdot 2 \cdot 5(\sqrt3 - 1) + 3 \times 10 \cdot \frac{\sqrt3}{2}\right\} N = 47.94\,N$

**15.2.23 Cartesian coordinates** Find the nonlinear governing differential equation for a simple pendulum

$$\ddot{\theta} = -\frac{g}{\ell}\sin\theta$$

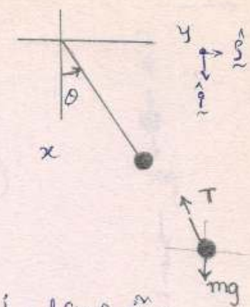
using linear momentum in Cartesian coordinates and without using the polar coordinate formulas for velocity and acceleration. Of course you can use that $x = \ell\cos\theta$ and $y = \ell\sin\theta$ for $x$ pointing down.



Problem 15.23

14.2.23

$x = l\cos\theta \qquad y = l\sin\theta$




$\underset{\sim}{r} = l\cos\theta\,\underset{\sim}{i} + l\sin\theta\,\underset{\sim}{j}$

$\dot{\underset{\sim}{r}} = -l\sin\theta\,\dot{\theta}\,\underset{\sim}{i} + l\cos\theta\,\dot{\theta}\,\underset{\sim}{j}$

$\ddot{\underset{\sim}{r}} = -l\sin\theta\,\ddot{\theta}\,\underset{\sim}{i} - l\cos\theta\,\dot{\theta}^2\,\underset{\sim}{i} + l\cos\theta\,\ddot{\theta}\,\underset{\sim}{j} - l\sin\theta\,\dot{\theta}^2\,\underset{\sim}{j}$

$\Sigma\underset{\sim}{F} = m\underset{\sim}{a}$

$(-T\cos\theta\,\underset{\sim}{i} - T\sin\theta\,\underset{\sim}{j} + mg\,\underset{\sim}{i}) = m(-l\sin\theta\,\ddot{\theta} - l\cos\theta\,\dot{\theta}^2)\underset{\sim}{i} + m(l\cos\theta\,\ddot{\theta} - l\sin\theta\,\dot{\theta}^2)\underset{\sim}{j}$

$\{-T\cos\theta + mg = -ml(\sin\theta\,\ddot{\theta} + \cos\theta\,\dot{\theta}^2)\}\,\sin\theta$

$\{-T\sin\theta = ml(\cos\theta\,\ddot{\theta} - \sin\theta\,\dot{\theta}^2)\}\,\cos\theta$

$mg\sin\theta = -ml\ddot{\theta}(\sin^2\theta + \cos^2\theta)$

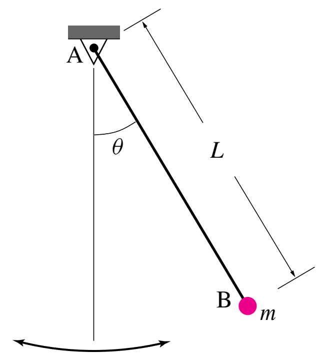
$\Rightarrow \quad \ddot{\theta} + \frac{g}{l}\sin\theta = 0$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**15.2.24 Tension in a rope-swing rope.** Model a swinging person as a point mass. The swing starts from rest at an angle $\theta = 90°$. When the rope passes through vertical the tension in the rope is higher (it is hard to hang on). A person wants to know ahead of time if she is strong enough to hold on. How hard does she have to hang on compared, say, to her own weight? You are to find the solution two ways. Use the same m, g, and L for both solutions.

a) Find $\ddot{\theta}$ as a function of $g$, $L$, $\theta$, and $m$. This equation is the governing differential equation. Write it as a system of first order equations. Solve them numerically. Once you know $\dot{\theta}$ at the time the rope is vertical you can use other mechanics relations to find the tension. If you like, you can plot the tension as a function of time as the mass falls.

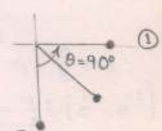
b) Use conservation of energy to find $\dot{\theta}$ at $\theta = 0$. Then use other mechanics relations to find the tension.




Problem 15.24

**Answer:**
(b) The maximum tension is 3 times the person's weight.

14.2.24




$E_1 = mgl \qquad E_2 = \frac{1}{2}mv^2 + mg(l - l\cos\theta)$

$mgl = \frac{1}{2}m(l\dot{\theta})^2 \Rightarrow \dot{\theta}^2 = \frac{2gl}{l^2} \Rightarrow \dot{\theta} = \sqrt{\frac{2g}{l}}$

$T = ml\dot{\theta}^2 + mg \Rightarrow T = ml\frac{2g}{l} + mg \Rightarrow T = 3mg$

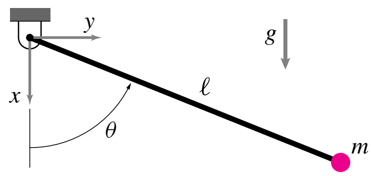
14.2.25

**15.2.25 Pendulum.** A pendulum with a negligible-mass rod and point mass $m$ is released from rest at the horizontal position $\theta = \pi/2$.

a) Find the acceleration (a vector) of the mass just after it is released at $\theta = \pi/2$ in terms of $\ell, m, g$ and any base vectors you define clearly.

b) Find the acceleration (a vector) of the mass when the pendulum passes through the vertical at $\theta = 0$ in terms of $\ell, m, g$ and any base vectors you define clearly

c) Find the string tension when the pendulum passes through the vertical at $\theta = 0$ (in terms of $\ell, m$ and $g$).

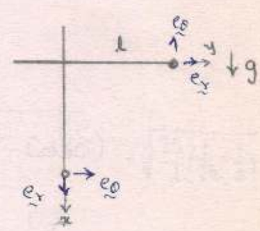



Problem 15.25

$\underset{\sim}{r} = l\hat{\underset{\sim}{e}}_r$

$\dot{\underset{\sim}{r}} = l\,[\underset{\sim}{\omega} \times \hat{\underset{\sim}{e}}_r]$

$\ddot{\underset{\sim}{r}} = l\,(\dot{\underset{\sim}{\omega}} \times \hat{\underset{\sim}{e}}_r) + l\underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \hat{\underset{\sim}{e}}_r)$




$\ddot{\underset{\sim}{r}} = -l\dot{\theta}^2\hat{\underset{\sim}{e}}_r + l\ddot{\theta}\hat{\underset{\sim}{e}}_\theta$

$\Sigma\underset{\sim}{F} = m\underset{\sim}{a}$

$\{-mg\hat{\underset{\sim}{e}}_\theta = m(-l\dot{\theta}^2\hat{\underset{\sim}{e}}_r + l\ddot{\theta}\hat{\underset{\sim}{e}}_\theta)\} \Rightarrow \{\,\}\cdot\hat{\underset{\sim}{e}}_\theta \Rightarrow -mg = ml\ddot{\theta}$

$\Rightarrow \ddot{\theta} = \dfrac{-g}{l}$

$\underset{\sim}{a} = (-l\dot{\theta}^2 \underset{\sim}{\hat{e}_r} + l\ddot{\theta}\hat{e}_\theta)$

$\sum \underset{\sim}{F} = m\underset{\sim}{a}$

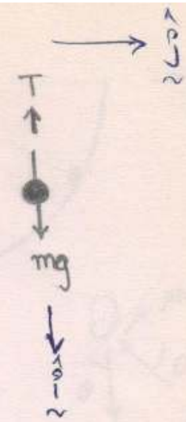
$T(-\underset{\sim}{\hat{i}}) + mg\,\underset{\sim}{\hat{i}} = m(-l\dot{\theta}^2\underset{\sim}{\hat{i}} + l\ddot{\theta}(-\underset{\sim}{\hat{j}}))$

$mg - T = -ml\dot{\theta}^2 \Rightarrow \dot{\theta}^2 = \left(\frac{T/m - g}{l}\right)$

$\underset{\sim}{a} = -l\dot{\theta}^2\underset{\sim}{\hat{e}_r} = -\left(\frac{T}{m} - g\right)\underset{\sim}{\hat{e}_r}$

c)

$\left.T = ml\dot{\theta}^2 + mg \quad \left(\dot{\theta} = \sqrt{\frac{2g}{l}}\right) \Rightarrow T = 3mg\right\} \quad \underset{\sim}{a} = -2g\,\underset{\sim}{\hat{i}}$




**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**15.2.26** Write a computer program to solve the nonlinear pendulum equation, $\ddot{\theta} = -\frac{g}{\ell}\sin\theta$, over a given time interval $(0, t)$, and initial conditions $\theta(0)$, and $\dot{\theta}(0)$. The output should be a vector of time instants, $t_i$, in the given time interval and the corresponding $\theta_i$ and $\dot{\theta}_i$.

Now use your computer program to find the solution of

$$\ddot{\theta} = -\sin\theta, \qquad \theta(0) = \pi/4,\ \dot{\theta}(0) = 0.$$

Compare the solution obtained with the analytical solution of the corresponding simple pendulum equation, $\ddot{\theta} = -\theta$ with the same initial conditions. In particular,

a) Find the difference in the time period of oscillations of the two systems.

b) Plot $\dot{\theta}$ obtained from the two solutions against time and comment on the differences.

c) Plot $\dot{\theta}$ against $\theta$ from the two solutions on the same plot and compare the two phase portraits. Comment on the differences.

$\ddot{\theta} = -\theta$

$\theta = C_1 e^{it} + C_2 e^{-it}$ & $\dot{\theta} = C_1 i e^{it} + C_2(-i)e^{-it}$

$\frac{\pi}{4} = C_1 + C_2$ $\qquad 0 = iC_1 - iC_2 \Rightarrow C_1 = C_2$

$\Rightarrow \cancel{\frac{\pi}{4} = iC_1 - iC_2}$ & $\cancel{0 = C_1 + C_2} \Rightarrow \cancel{C_1 = -C_2}$

$2iC_1 = \frac{\pi}{4} \Rightarrow C_1 = \frac{\pi}{8i}$ & $C_2 = \frac{-\pi}{8i}$ $\qquad \Rightarrow C_1 = \frac{\pi}{8} = C_2$

$\dot{\theta} = \frac{\pi}{8} i \left[e^{it} - e^{-it}\right]$

$\Rightarrow \theta = \frac{\pi}{8\cancel{i}} e^{it} + \frac{\pi}{8\cancel{i}} e^{-it} = \frac{\pi}{8} 2\cos t = \frac{\pi}{4}\cos t.$ $\qquad = -\frac{\pi}{4}\sin t$

**15.2.28** A pendulum of mass $m$ and length $\ell$ is released from rest at $\theta(0) = 60°$. It executes oscillatory motion. If the pendulum were to be released from two different positions, $\theta(0) = 45°$ and $\theta(0) = 0°$, with some corresponding initial angular speed such that the ensuing motion were exactly the same as that with $\theta(0) = 60°$ and $\dot{\theta}(0) = 0$, find the required initial angular speeds.

a) First, find the corresponding $\dot{\theta}(0)$ without any computer simulation.

b) Verify your answer by plotting computer generated solutions for the three different initial conditions.

c) What is the general relationship between $\theta(0)$ and $\dot{\theta}(0)$ that produces a predefined motion generated by, say, a given set of $\theta(0) = \theta_0$ and $\dot{\theta}(0) = \dot{\theta}_0$.

we have $\dot{\theta}^2 = \frac{2g}{\ell}(\cos\theta - \cos\theta_{max})$

① $\theta(0) = 60°$, $\dot{\theta}(0) = 0$

② $\theta(0) = 45°$, $\dot{\theta}(0) = \dot{\theta}_1$ $\quad \dot{\theta}_1 = \sqrt{\frac{2g}{\ell}(\cos 45° - \cos 60°)} = \sqrt{\frac{2g}{\ell}\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)}$

③ $\theta(0) = 0$, $\dot{\theta}(0) = \dot{\theta}_2$ $\quad \dot{\theta}_2 = \sqrt{\frac{2g}{\ell}(\cos 0 - \cos 60)} = \sqrt{\frac{g}{\ell}}$

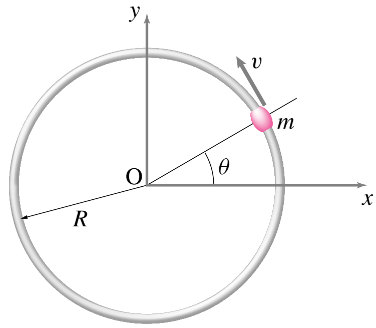
c) $\dot{\theta}^2 = \frac{2g}{\ell}\left[\cos\theta - \cos\theta_{max}\right]$.

$\dot{\theta}_0^2 = \frac{2g}{\ell}\left[\cos\theta_0 - \cos\theta_{max}\right]$.

**15.2.30 Bead on a hoop with friction.** A bead slides on a rigid, stationary, circular wire. The coefficient of friction between the bead and the wire is $\mu$. The bead is loose on the wire (not a tight fit but not so loose that you have to worry about rattling). Assume gravity is negligible.

a) Given $v$, $m$, $R$, & $\mu$; what is $\dot{v}$?

b) If $v(\theta = 0) = v_0$, how does $v$ depend on $\theta$, $\mu$, $v_0$ and $m$?

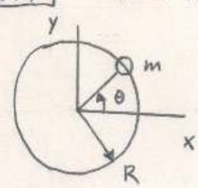



Problem 15.30

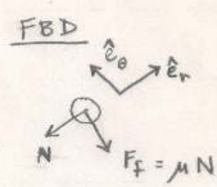
**Answer:**
(a) $\dot{v} = -\mu \frac{v^2}{R}$.
(b) $v = v_0 e^{-\mu\theta}$.

TAM 203 HW 7 DUE 3/14 WEISENFELD Page 1

$\boxed{5.7}$ bead on a hoop with friction, velocity v




FBD




N is just centripetal force of hoop on bead $N = mR\omega^2$

so $F_f = \mu m R\omega^2$

A. LINEAR MOMENTUM BALANCE

$\{\Sigma \underline{F} = m\underline{a}\} \cdot \hat{e}_\theta$ $\quad a = \dot{v}$

$-F_f = m\dot{v}$

$\dot{v} = -\mu R\omega^2 \quad \omega = \frac{V}{R}$

$$\boxed{\dot{V} = -\mu \frac{V^2}{R}}$$

B. $\frac{dV}{dt} = -\mu R\omega \frac{V}{R}$ $\quad$ but $\omega = \frac{d\theta}{dt}$

$\frac{dV}{d\theta} = -\mu V$ $\quad \frac{dv}{v} = -\mu d\theta$

$$\boxed{V = V_0 e^{-\mu\theta}}$$

$\ln V = -\mu\theta + C_1$

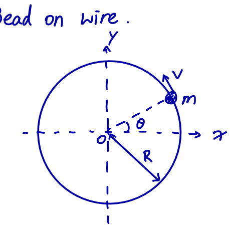
$V = e^{C_1} e^{-\mu\theta}$ at $\theta = 0$ $V = V_0$ ✓

$V = V_0 e^{-\mu\theta}$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

HW #8 P44 15.2.30 Teaya Yang

Bead on wire.

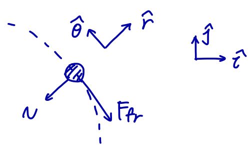



$\mu$ between bead and wire.
negligible gravity.

a) Given $v, m, R, \mu$. find $\dot{v}$.

* The $\hat{r}$ and $\hat{\theta}$ used below are unit vectors for polar coordinates. Same as the $\hat{e}_r$ and $\hat{e}_\theta$ used in lectures.

FBD of bead:




LMB: $\Sigma \vec{F} = m\vec{a}$

$-N\hat{r} - F_{fr}\,\hat{\theta} = m\vec{a}$

$-N\hat{r} - \mu N\hat{\theta} = m\vec{a}$ └ what's $\vec{a}$? See kinematics section.

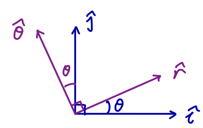
For polar coords:

$\hat{r} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$

$\hat{\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$

$\dot{\hat{r}} = \dot{\theta}\,\hat{\theta}$

$\dot{\hat{\theta}} = -\dot{\theta}\,\hat{r}$




Kinematics:

$\vec{a} = \frac{d^2}{dt^2}\vec{r}$

$\vec{v} = \frac{d}{dt}(r\hat{r})$

$= \cancel{\dot{r}}^{\,0}\hat{r} + r\dot{\hat{r}}$ (since $r = R$)

$= \underbrace{r\dot{\theta}}_{\equiv v \text{ from problem statement}}\hat{\theta}$

then $\dot{v} = r\ddot{\theta} = R\ddot{\theta}$

$\vec{a} = \frac{d}{dt}(v\hat{\theta})$

$= \dot{v}\hat{\theta} + v\dot{\hat{\theta}}$

$= \dot{v}\hat{\theta} - v\dot{\theta}\hat{r}$

$= \dot{v}\hat{\theta} - \frac{v^2}{R}\hat{r}$

Plug $\vec{a}$ back into LMB:

$\{-N\hat{r} - \mu N\hat{\theta} = m(\dot{v}\hat{\theta} - \frac{v^2}{R}\hat{r})\}$

$\{\ \}\cdot\hat{r}$: $-N = -m\frac{v^2}{R}$

$N = m\frac{v^2}{R}$

$\{\ \}\cdot\hat{\theta} = -\mu N = m\dot{v}$

$\dot{v} = -\frac{\mu N}{m}$

$= -\frac{\mu}{m}(m\frac{v^2}{R})$

$\boxed{\dot{v} = -\frac{\mu v^2}{R}}$

b) $v(\theta=0) = v_0$, how does $v$ depend on $\theta, \mu, v_0$, and $m$?

We have ODE $\frac{dv}{dt} = -\frac{\mu}{R}v^2$, but we are trying to solve for $v(\theta)$.

Recall that $v = R\dot{\theta}$:

$\frac{dv}{dt} = -\frac{\mu}{\cancel{R}}v(\cancel{R}\dot{\theta})$

$\frac{dv}{\cancel{dt}} = -\mu v\frac{d\theta}{\cancel{dt}}$

$\frac{dv}{d\theta} = -\mu v$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Now solve ODE :

$$\frac{1}{v}\,dv = -\mu\,d\theta$$

$$\ln|v| = -\mu\theta + C_0$$

$$v(\theta) = C_1 e^{-\mu\theta}$$

Applying I.C. : $v(\theta=0) = C_1 e^0 = v_0$

$$C_1 = v_0$$

$$\Rightarrow \boxed{v(\theta) = v_0 e^{-\mu\theta}}$$

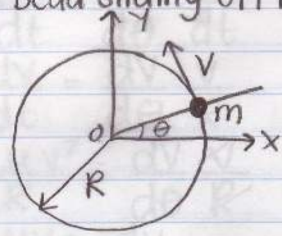
$v$ is not dependent on mass $m$.

* I worked with Catherine Eiben & went to Rong Long's office hours

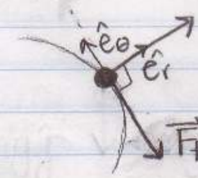
Kim Dunphy
HW#1-3 due 10/2/08
TAM 2030
Section 3 at 1:25
TA: Rong Long

SOLUTION

1) 13.45 - Bead sliding on rigid, stationary, circular wire; assume gravity is negligible




FBD:




a. Given v, m, R, & u, find $\dot{v}$

$\Sigma\vec{F} = m\vec{a}$ $\quad\quad \vec{a} = R\ddot{\theta}\,\hat{e}_\theta - R\dot{\theta}^2\hat{e}_r$

$\{N\hat{e}_r - \mu N\hat{e}_\theta = mR\ddot{\theta}\hat{e}_\theta - mR\dot{\theta}^2\hat{e}_r\}$

$\{\}\cdot\hat{e}_r = N = -mR\dot{\theta}^2 \quad\quad \dot{\theta} = \frac{v}{R}$

$N = -\frac{m|\vec{v}|^2}{R} = \frac{-mv^2}{R}$

$\Sigma\vec{M}_o = \Sigma\dot{\vec{H}}_{/o}$

$(-\mu N\hat{e}_\theta \times R\hat{e}_r) + \cancel{(N\hat{e}_r \times R\hat{e}_r)}^{0} = (R\hat{e}_r \times m\dot{v})$

$\{\mu NR\hat{k} = Rm\dot{v}\,\hat{k}\}$

$\{\}\cdot\hat{k} \Rightarrow \mu NR = Rm\dot{v}$

$\dot{v} = \frac{\mu N}{m}$

$\dot{v} = \frac{\mu}{m}\left(\frac{-mv^2}{R}\right)$

$\boxed{\dot{v} = -\frac{\mu v^2}{R}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

b. $v(\theta=0)=v_0$, how does $v$ depend on $\theta, \mu, v_0, \& m$?

$$\frac{dv}{dt} = -\frac{\mu}{R} v^2$$

$v(\theta) = v(\theta(t)) \rightarrow$ chain rule

$$\frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt} \qquad \frac{d\theta}{dt} = \dot{\theta} = \frac{v}{R}$$

$$\frac{dv}{dt} = \frac{dv}{d\theta}\frac{v}{R}$$

$$-\frac{\mu}{\cancel{R}} v^{\cancel{2}} = \frac{dv}{d\theta}\frac{\cancel{v}}{\cancel{R}}$$

$$-\mu v = \frac{dv}{d\theta}$$

$$\int -\mu \, d\theta = \int \frac{dv}{v} \qquad v(0) = v_0$$

$$-\mu\theta = \ln v + c$$

$$v = Ce^{-\mu\theta}$$

$$v(0) = v_0 = C\cancel{e^{-\mu(0)}}^{1}$$

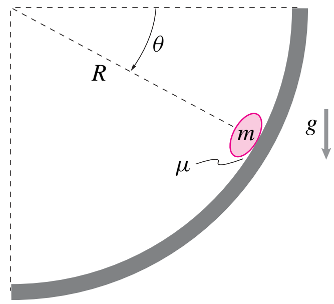
$$C = v_0$$

$$\boxed{v = v_0 e^{-\mu\theta}}$$

**15.2.31 Particle in a chute.** One of a million non-interacting rice grains is sliding in a circular chute with radius $R$. Its mass is $m$ and it slides with coefficient of friction $\mu$ (Actually it slides, rolls and tumbles — $\mu$ is just the effective coefficient of friction from all of these interactions.) Gravity $g$ acts downwards.

a) Find a differential equation that is satisfied by $\theta$ that governs the speed of the rice as it slides down the hoop. Parameters in this equation can be $m$, $g$, $R$ and $\mu$ [Hint: Draw FBD, write eqs of mechanics, express as ODE.]

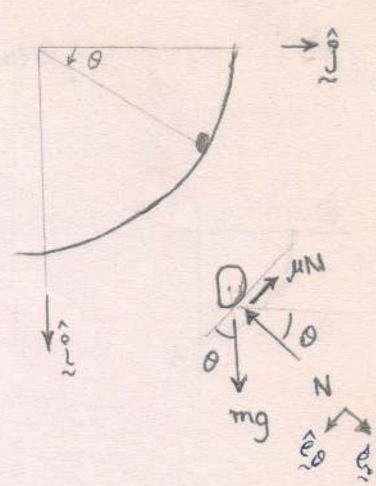
b) Find the particle speed at the bottom of the chute if $R = 0.5m$, $m = 0.1$ grams, $g = 10\,\mathrm{m/s^2}$, and $\mu = 0.2$ as well as the initial values of $\theta_0 = 0$ and its initial downward speed is $v_0 = 10\,\mathrm{m/s}$. [Hint: you are probably best off using a numerical solution.]




Problem 15.31

14.2.31

$$\sum \underset{\sim}{F} = m\underset{\sim}{a}$$




$$\left\{-N\hat{e}_r - \mu N\hat{e}_\theta + mg\hat{\imath} = m(-R\dot{\theta}^2\hat{e}_r + R\ddot{\theta}\hat{e}_\theta)\right\}$$

$$\{\}\cdot\hat{e}_r \Rightarrow -N + mg\sin\theta = -mR\dot{\theta}^2$$

$$\{\}\cdot\hat{e}_\theta \Rightarrow -\mu N + mg\cos\theta = mR\ddot{\theta}$$

$$N = mR\dot{\theta}^2 + mg\sin\theta$$

$$\Rightarrow -\mu(mR\dot{\theta}^2 + mg\sin\theta) + mg\cos\theta = mR\ddot{\theta}$$

$$\Rightarrow mR\ddot{\theta} + \mu mR\dot{\theta}^2 + \mu mg\sin\theta - mg\cos\theta = 0$$

b) Let $\theta = x_1$, $\dot{\theta} = x_2$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = (mg\cos x_1 - \mu mg\sin x_1 - \mu mR x_2^2)\frac{1}{mR}$$

$$\omega\big|_{\theta=90^\circ} = 19.98\,\mathrm{rad/s} \qquad V = 10\,\mathrm{m/s}.$$

$$\left[\because R = 0.5\,m,\ g = 10\,m/s^2,\ m = 0.1g,\ \mu = 0.2\right]$$

14.2.32

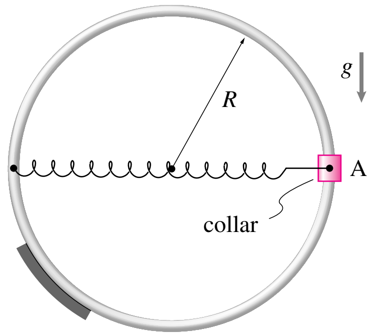
$\Sigma F = m$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**15.2.32** Due to a push which happened in the past, the collar with mass $m$ is sliding up at speed $v_0$ on the circular ring when it passes through the point $A$. The ring is frictionless. A spring of constant $k$ and unstretched length $R$ is also pulling on the collar.

a) What is the acceleration of the collar at $A$. Solve in terms of $R$, $v_0$, $m$, $k$, $g$ and any base vectors you define.

b) What is the force on the collar from the ring when it passes point A? Solve in terms of $R$, $v_0$, $m$, $k$, $g$ and any base vectors you define.



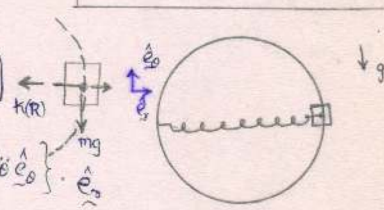

Problem 15.32

14.2.32

$m = 0.1g$ , $\mu = 0.2$

$\sum \underset{\sim}{F} = \dot{\underset{\sim}{L}} = m\underset{\sim}{a}$

$-KR\,\hat{\underset{\sim}{e}}_r + N\,\hat{\underset{\sim}{e}}_r - mg\,e_\theta = m\left[-R\dot{\theta}^2\hat{\underset{\sim}{e}}_r + R\ddot{\theta}\,\hat{\underset{\sim}{e}}_\theta\right]$




$\left\{-KR\hat{\underset{\sim}{e}}_r + N\hat{\underset{\sim}{e}}_r - mg\,e_\theta = -m\dfrac{v^2}{R}\hat{\underset{\sim}{e}}_r + mR\ddot{\theta}\,\hat{\underset{\sim}{e}}_\theta\right\}\cdot\hat{\underset{\sim}{e}}_r$

$-KR + N = -\dfrac{mv^2}{R}$

$\{\ \}\cdot\hat{\underset{\sim}{e}}_\theta \Rightarrow -mg = mR\ddot{\theta}$

$\underset{\sim}{a} = -R\dot{\theta}^2\hat{\underset{\sim}{e}}_r + R\ddot{\theta}\,\hat{\underset{\sim}{e}}_\theta$

$\Rightarrow \ddot{\theta} = \dfrac{-g}{R}$

$\underset{\sim}{a} = -\dfrac{v_0^2}{R}\hat{\underset{\sim}{e}}_r - g\,\hat{\underset{\sim}{e}}_\theta$ //

b) Force on the collar from the ring, $\underset{\sim}{N} = \left(KR - \dfrac{mv^2}{R}\right)\hat{\underset{\sim}{e}}_r$

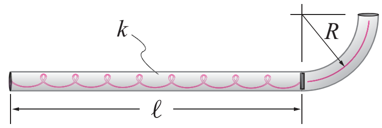
14.2.34

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**15.2.33** A toy used to shoot pellets is made out of a thin tube which has a spring of spring constant $k$ on one end. The spring is placed in a straight section of length $\ell$; it is unstretched when its length is $\ell$. The straight part is attached to a (quarter) circular tube of radius $R$, which points up in the air.

a) A pellet of mass $m$ is placed in the device and the spring is pulled to the left by an amount $\Delta\ell$. Ignoring friction along the travel path, what is the pellet's velocity $\vec{v}$ as it leaves the tube?

b) What force acts on the pellet just prior to its departure from the tube? What about just after?



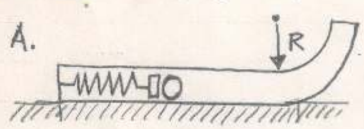

Problem 15.33

**Answer:**
(a) The velocity of departure is $\vec{v}_{dep} = \sqrt{\frac{k(\Delta\ell)^2}{m} - 2GR}\,\hat{\jmath}$, where $\hat{\jmath}$ is perpendicular to the curved end of the tube.
(b) Just before leaving the tube the net force on the pellet is due to the wall and gravity, $\vec{F}_{net} = -mg\hat{\jmath} - m\frac{|\vec{v}_{dep}|^2}{R}\hat{\imath}$; Just after leaving the tube, the net force on the pellet is only due to gravity, $\vec{F}_{net} = -mg\hat{\jmath}$.

Page 2

5.9 toy gun, assume gun is fixed to ground

A.




We'll do this using energy

Initial P.E. + K.E. = Final P.E. + K.E. when we ignore friction

$$\frac{1}{2}k(\Delta L)^2 + 0 = mgR + \frac{1}{2}mv^2$$

$$\boxed{\sqrt{\frac{k(\Delta L)^2}{m} - 2gR} = V}$$

B. Just before leaving the tube, particle is acted on by tube walls (centripetal force), and gravity.

Just after leaving the tube, particle is acted on only by gravity.

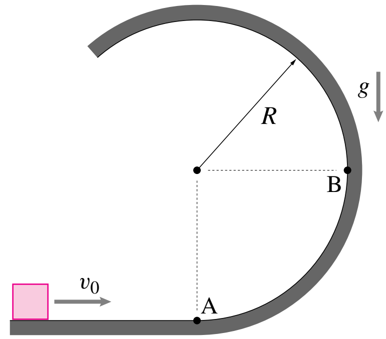
NOTE: Problem gets really interesting if you do experiment in the space shuttle in orbit and don't fix gun to anything (ie. weightlessness)

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

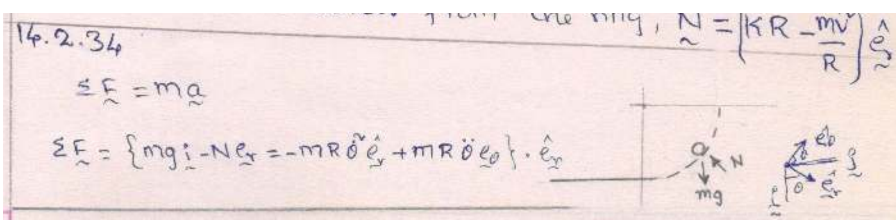
**15.2.34** A block with mass $m$ is moving to the right at speed $v_0$ when it reaches a circular frictionless portion of the ramp.

a) What is the speed of the block when it reaches point B? Solve in terms of $R$, $v_0$, $m$ and $g$.

b) What is the force on the block from the ramp just after it gets onto the ramp at point A? Solve in terms of $R$, $v_0$, $m$ and $g$. Remember, force is a vector.




Problem 15.34



$N = \left\{KR - \frac{mv}{R}\right\}\hat{e}$

14.2.34

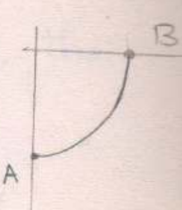
$\Sigma F = ma$

$\Sigma F = \{mg\,i - N e_r = -mR\dot{\theta}^2 \hat{e}_r + mR\ddot{\theta} e_\theta\} \cdot \hat{e}_r$

$\underset{\sim}{e}_r = \cos\theta\, \underset{\sim}{i} + \sin\theta\, \underset{\sim}{j}$ $\quad\quad \underset{\sim}{e}_\theta = -\sin\theta\, \underset{\sim}{i} + \cos\theta\, \underset{\sim}{j}$

$\{\ \}\cdot \hat{\underset{\sim}{e}}_r \Rightarrow mg\cos\theta - N = -mR\dot{\theta}^2$ — ①

$\{\ \}\cdot \hat{\underset{\sim}{e}}_\theta \Rightarrow -mg\sin\theta = mR\ddot{\theta}$ — ②




(a) Consider energy at A & B.

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgh$

$\frac{v^2}{2} = \frac{v_0^2}{2} - gh \Rightarrow v^2 = v_0^2 - 2gh \Rightarrow v = \sqrt{v_0^2 - 2gh}$

b) $N = mg\cos\theta + mR\dot{\theta}^2$

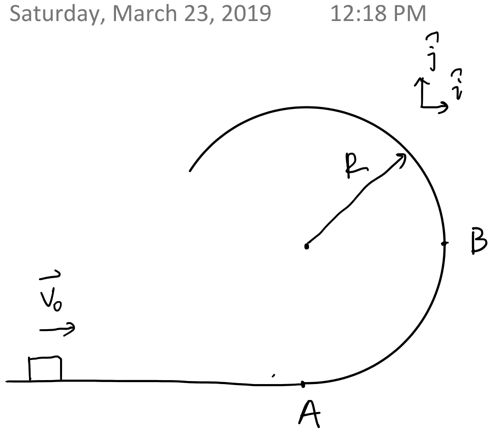
Force on block at 'A' $\quad$ at $\theta = 0$, $\dot{\theta} = \frac{v_0}{R}$

$N = mg + m\frac{v_0^2}{R} \Rightarrow \underset{\sim}{N} = -N\hat{\underset{\sim}{e}}_r \Rightarrow \underset{\sim}{N} = (-mg - \frac{mv_0^2}{R})\,\underset{\sim}{i}$

14.2.35

14.2.34

Saturday, March 23, 2019 12:18 PM




A block with mass $m$
moving to the right
initial velocity $v_0$
ramp is circular and frictionless

(a) Speed at point B $= V_B = ?$

The ramp is frictionless : $V_A = V_0$ ①

Conservation of energy : $\frac{1}{2} m V_A^2 - \frac{1}{2} m V_B^2 = mgR$ ②

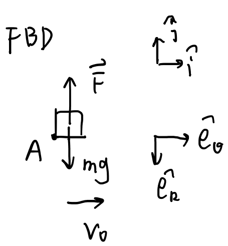
Solve for ①② $\Rightarrow V_B = \sqrt{V_0^2 - 2gR}$ , $V_0^2 \geq 2gR$

$\Rightarrow \vec{V_B} = \sqrt{V_0^2 - 2gR}\ \hat{j}$

If $v_0 < \sqrt{2gR}$, the block can never reach B.

(b) Force from the ramp when the block gets A.
(Just to the right of A)

FBD




The ramp is frictionless : $V_A = V_0$ ③

EOM : $\{\vec{F} - mg\hat{j} = m\vec{a}\}$

$\{\ \} \cdot \hat{e}_R \Rightarrow -F + mg = -m\frac{V_A^2}{R}$ ④

Solve for ③④ $\Rightarrow \vec{F} = mg\hat{j} - m\frac{V_0^2}{R}\hat{e}_R = (mg + m\frac{V_0^2}{R})\hat{j}$

### 15.2.35

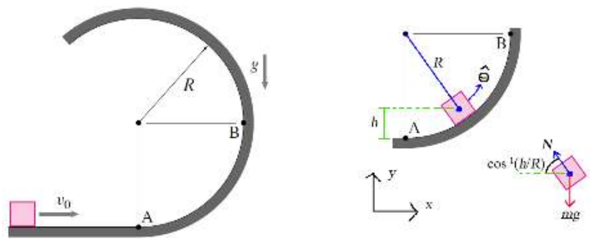



Figure 15.2.1: (a) Block with mass $m$ at speed $v_0$ approaching the circular frictionless ramp of radius $R$; gravity $g$ and points of interest A and B are shown. (b) Free body diagram of the block at point A.

A block with mass $m$ is moving to the right at speed $v_0$ when it reaches a circular frictionless portion of the ramp, as shown in shown in Fig. 15.2.1(a).

(a) What is the speed of the block when it reaches point B? Solve in terms of $R$, $v_0$, $m$, and $g$.

(b) What is the force on the block from the ramp just after it gets onto the ramp at point A? Solve in terms of $R$, $v_0$, $m$, and $g$. Remember, force is a vector.

---

**Part a**

Since the ramp is frictionless, only the gravitational force and the normal force from the ramp act on the block: see the free-body diagram in Fig. 15.2.1(b). The normal force from the ramp is a constraint force that does no work (always acts perpendicular to the block trajectory) and the gravitational force is conservative, therefore the total energy (kinetic plus potential) is constant,

$$E = \underbrace{E_P}_{mgh} + \underbrace{E_K}_{mv^2/2} = \text{constant}. \tag{15.2.1}$$

The potential energy is $mgh$ due to the gravitational force, where $h$ is the height of the block, and the kinetic energy is $mv^2/2$, where $v$ is the speed of the block. We do not need to consider the normal force provided by the ramp because it does no work (a constraint force; it acts orthogonal to the path of the block). Taking $h = 0$ at A, the initial energy of the block is $E_\text{A} = mv_0^2/2$. The height is then $h = R$ at B, where the total energy is

$$E_\text{B} = mgR + \frac{mv_\text{B}^2}{2}. \tag{15.2.2}$$

April 2, 2021, M. R. Buche

Since the total energy is conserved, we can take $E_\mathrm{A} = E_\mathrm{B}$,

$$\frac{mv_0^2}{2} = mgR + \frac{mv_\mathrm{B}^2}{2}, \tag{15.2.3}$$

and then solve for the speed at point B,

$$v_\mathrm{B} = \sqrt{v_0^2 - 2gR}. \tag{15.2.4}$$

Note that our result is independent of $m$, which is because all work-related forces (gravitational, here) are proportional to $m$. Additionally, note that this result holds for arbitrary curves shapes leading from point A to point B, which is because all all work-related forces are conservative. Finally, note that we obtain a complex (imaginary) result if the initial speed is not large enough (when $v_0 < \sqrt{2gR}$), which is because in that case the block would not reach point B (it would reach a maximum height of $h = v_0^2/2g < R$).

### Part b

The free body diagram of the block is shown in Fig. 15.2.1(b), showing the gravitational force $-mg\hat{\mathbf{j}}$ and the normal force from the ramp $\vec{\mathbf{N}}$. After the mass gets to point A, the normal force from the ramp $\vec{\mathbf{N}}$ always acts towards the center of the circular ramp, $\vec{\mathbf{N}} = -N\hat{\mathbf{e}}_r$. The linear momentum balance equation is then

$$-N\hat{\mathbf{e}}_r - mg\hat{\mathbf{j}} = m\mathbf{a}. \tag{15.2.5}$$

The acceleration of the block after it reaches A is given by

$$\vec{\mathbf{a}} = -R\dot{\theta}^2\,\hat{\mathbf{e}}_r + R\ddot{\theta}\,\hat{\mathbf{e}}_\theta, \tag{15.2.6}$$

so dotting Eq. (15.2.5) with $\hat{\mathbf{e}}_r$ (and canceling each negative signs) results in

$$N + mg\hat{\mathbf{j}}\cdot\hat{\mathbf{e}}_r = mR\dot{\theta}^2. \tag{15.2.7}$$

At point A, we have $\dot{\theta} = v_0/R$ and $\hat{\mathbf{j}} = -\hat{\mathbf{e}}_r$, which obtains (written as a vector)

$$\vec{\mathbf{N}} = m\left(g + \frac{v_0^2}{R}\right)\hat{\mathbf{j}} \quad \text{at A}. \tag{15.2.8}$$

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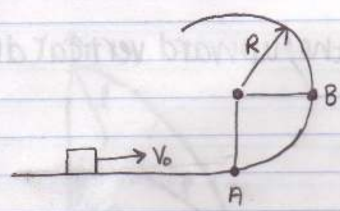
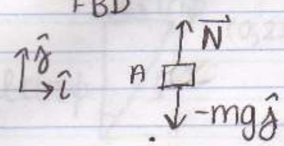
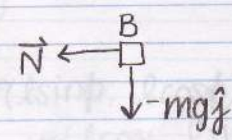
2) 13.49 → block with mass m, speed $V_0$, frictionless ramp




a. speed of block at B in terms of R, $V_0$, m, & g

FBD




Energy method: $E_{K_A} = \frac{1}{2}mV_0^2$ at A

$E_{K_B} = \frac{1}{2}mV_B^2$, $E_p = mgh$ at B

$= mgR$

$\vec{N} \Rightarrow \vec{N} \perp \vec{V} \Rightarrow \vec{N}\cdot\vec{V} = 0 \rightarrow \vec{N}$ does no work → no energy

$E_{K_A} + E_{K_B} + E_p =$ constant

$\frac{1}{2}\cancel{m}V_0^2 = \frac{1}{2}\cancel{m}V_B^2 + \cancel{m}gR$

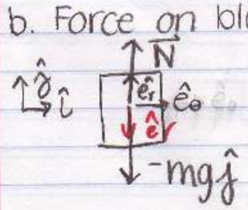
$\frac{1}{2}V_B^2 = \frac{1}{2}V_0^2 - gR$

$V_B^2 = V_0^2 - 2gR$

$\boxed{V_B = \sqrt{V_0^2 - 2gR}}$

✓

b. Force on block from ramp at A




$\vec{a} = R\dot{\theta}\hat{e}_\theta - R\dot{\theta}^2\hat{e}_r$ $\qquad \dot{\theta} = \frac{V_0}{R}$ at A

$\Sigma\vec{F} = m\vec{a}$

$\{N\hat{e}_r - mg\hat{e}_r = mR\dot{\theta}\hat{e}_\theta - mR\dot{\theta}^2\hat{e}_r\}$

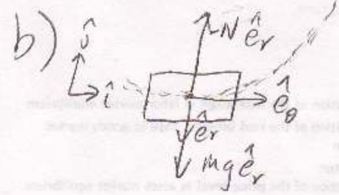
$\{\}\cdot\hat{e}_r \Rightarrow N - mg = -mR\dot{\theta}^2$

$= -m\cancel{R}\frac{V_0^2}{R^{\cancel{2}}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

13.49 continued

b)




$$\vec{a} = R\ddot{\theta}\hat{e}_\theta - R\dot{\theta}^2\hat{e}_r$$

$$\sum \vec{F} = m\vec{a}$$

$$-N\hat{e}_r + mg\,\hat{e}_r = m(R\ddot{\theta}\hat{e}_\theta - R\dot{\theta}^2\hat{e}_r)$$

$\{\ \}\cdot\hat{e}_r$ :

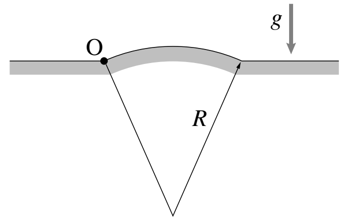
$$-N + mg = -mR\dot{\theta}^2$$

$$= -\frac{m v_0^2}{R}$$

$$N = m\left(\frac{v_0^2}{R} + g\right)$$

$$\boxed{\vec{N} = m\left(\frac{v_0^2}{R} + g\right)\hat{\jmath}}$$

**15.2.35** A car moves with speed $v$ along the surface of the hill shown which can be approximated as a circle of radius R. The car starts at a point on the hill at point $O$. Compute the magnitude of the speed $v$ such that the car just leaves the ground at the top of the hill.




Problem 15.35

14.2.35

$$\sum \underset{\sim}{F} = m\underset{\sim}{a}$$

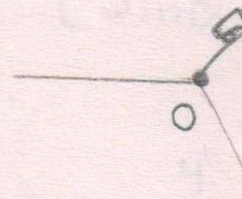
$$\left\{ -mg\,\underset{\sim}{\hat{\imath}} = m\left[-R\dot{\theta}^2\underset{\sim}{\hat{e}_r} + \cancelto{0}{R\ddot{\theta}\,\underset{\sim}{e}_\theta}\right] \right\}$$

$$\{\ \}\cdot\underset{\sim}{\hat{\imath}}$$

$$-mg = -mR\dot{\theta}^2$$

$$\Rightarrow \quad -g = -R\frac{v^2}{R^2}$$

$$\Rightarrow \quad v^2 = gR$$




Speed at which car just leaves the grou
top of the hill is $v = \sqrt{gR}$ //

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*