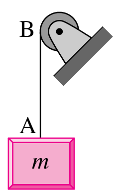
14.1 1D constrained motion and pulleys

For all problems, unless stated otherwise, treat all strings as inextensible, flexible and massless. Treat all pulleys and wheels as round, frictionless and massless. Assume all massive objects are prevented from rotating (e.g., wheels stay on the ground, *etc.*). When numbers are called for use $g = 10\,\mathrm{m/s^2}$ or $g = 32\,\mathrm{ft/s^2}$.

Preparatory Problems

14.1.1 A motor at B allows the block of mass $m = 3\,\mathrm{kg}$ shown in the figure to accelerate downwards at $2\,\mathrm{m/s^2}$. There is gravity. What is the tension in the string AB?

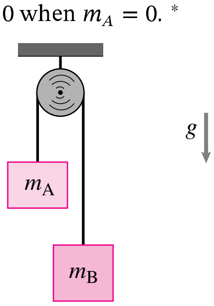


Problem 14.1.1

14.1.2 Two masses connected by an inextensible string hang from an ideal pulley.

a) Find the downward acceleration of mass B. Answer in terms of any or all of m_A, m_B, g, and the present velocities of the blocks. As a check, your answer should give $a_B = g$ when $m_A = 0$ and $a_B = 0$ when $m_A = m_B$. *.

b) Find the tension in the string. As a check, your answer should give $T = m_B g = m_A g$ when $m_A = m_B$ and $T = 0$ when $m_A = 0$. *

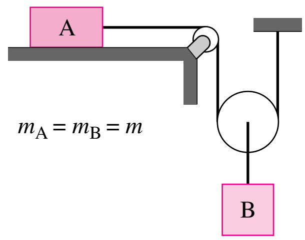


Problem 14.1.2

14.1.3 The blocks shown are released from rest.

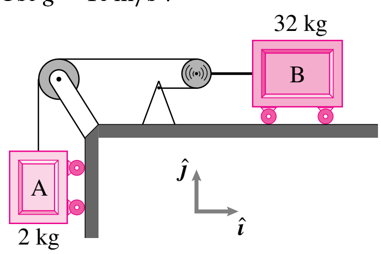
a) What is the acceleration of block A at $t = 0^+$ (just after release)?

b) What is the speed of block B after it has fallen 2 meters?



Problem 14.1.3

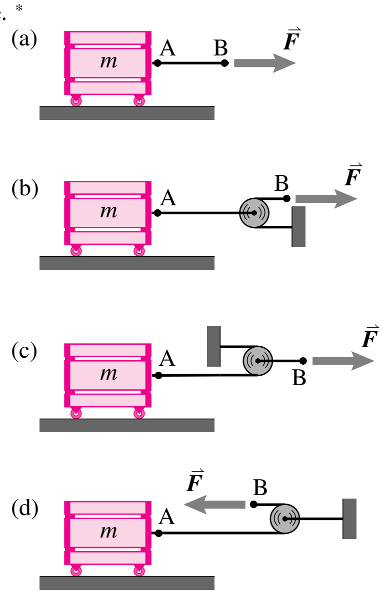
14.1.4 What is the acceleration of block A? Use $g = 10\,\mathrm{m/s^2}$. *



Problem 14.1.4

14.1.5 For the system shown in problem 14.1.2, find the acceleration of mass B using energy balance ($P = \dot{E}_K$).

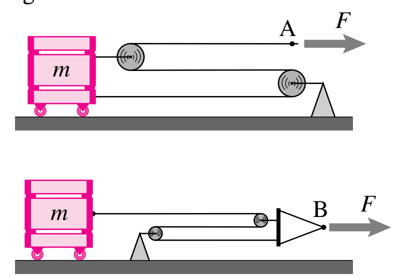
14.1.6 For the various situations pictured, find the acceleration of mass A and point B. Clearly define any variables, coordinates or sign conventions that you use. *



Problem 14.1.6: Four different ways to pull a mass.

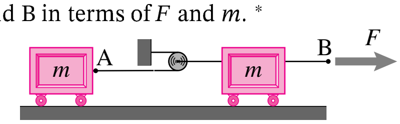
14.1.7 For each of the situations in problem 14.1.6 find the acceleration of the mass using energy balance ($P = \dot{E}_K$). Define any variables, coordinates, or sign conventions that you need to do your calculations and to define your solution.

14.1.8 What is the ratio of the acceleration of point A to that of point B in each configuration? $m = m$ and $F = F$. *



Problem 14.1.8

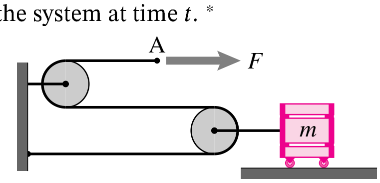
14.1.9 Find the acceleration of points A and B in terms of F and m. *



Problem 14.1.9

14.1.10 For the situation pictured in problem 14.1.9 find the accelerations of the two masses using energy balance ($P = \dot{E}_K$). Define any variables, coordinates, or sign conventions that you need to do your calculations and to define your solution.

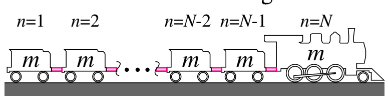
14.1.11 The point of application A of the force moves twice as fast as the mass. At some instant in time t, the speed of the mass is $\dot{x}$ to the left. Find the input power to the system at time t. *



Problem 14.1.11

More-Involved Problems

14.1.12 A train engine of mass m pulls and accelerates N cars, each of mass m. The power of the engine is P_t and its speed is v_t. Find the tension T_n between car n and car $n+1$. Assume there is no resistance and the ground is level. Assume the cars are connected with rigid links. *



Problem 14.1.12

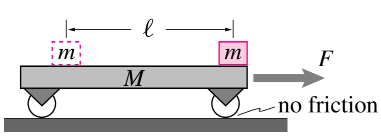
14.1.13 A cart of mass M, initially at rest, can move horizontally along a frictionless track. When $t = 0$, a force F is applied as shown to the cart. During the acceleration of M by the force F, a small box of mass m slides along the cart from the front to the rear. The coefficient of friction between the cart and the box is μ, and it is assumed that the acceleration of the cart is sufficient to cause sliding.

a) Draw free-body diagrams of the cart, the box, and the cart and box together.

b) Write the equation of linear momentum balance for the cart, the box, and the system of cart and box.

c) Show that the equations of motion for the cart and box can be combined to give the equation of motion of the mass center of the system of two bodies.

d) Find the displacement of the cart at the time when the box has moved a distance ℓ along the cart. *



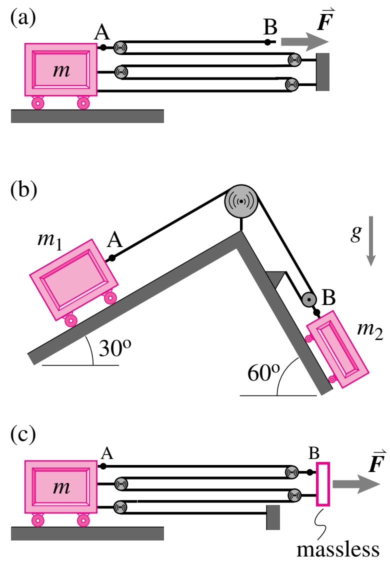
Problem 14.1.13

14.1.14 For the situations pictured, find the accelerations of mass A and of point B. Clearly define any variables, coordinates or sign conventions that you use.

a) A single mass and four pulleys. *

b) Two masses and two pulleys. *

c) A single mass and four pulleys. *



Problem 14.1.14: Various pulley arrangements.

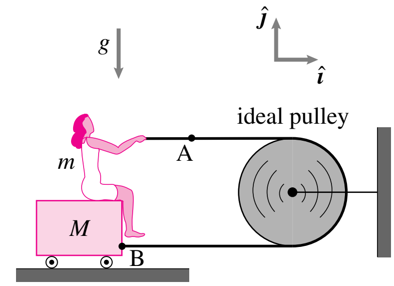
14.1.15 For the situations pictured in problem 14.1.14, find the acceleration of the mass using energy balance ($P = \dot{E}_K$).

14.1.16 A person of mass m, modeled as a rigid object, is sitting on a cart of mass $M > m$ and pulling the string towards herself. The coefficient of friction between her seat and the cart is μ. Point B is attached to the cart and point A is attached to the rope.

a) If you are given that she is pulling rope in with acceleration a_0 relative to herself (that is, $\vec{\boldsymbol{a}}_{A/B} \equiv \vec{\boldsymbol{a}}_A - \vec{\boldsymbol{a}}_B = -a_0\hat{\boldsymbol{\imath}}$) and that she is not slipping relative to the cart, find $\vec{\boldsymbol{a}}_A$. (Answer in terms of some or all of $m, M, g, \mu, \hat{\boldsymbol{\imath}}$ and a_0.) *

b) Find the largest possible value of a_0 without the person slipping off the cart? (Answer in terms of some or all of m, M, g and μ. You may assume her legs get out of the way if she slips backwards.)

c) If instead, $M < m$, what is the largest possible value of a_0 without the person slipping off the cart? (Answer in terms of some or all of m, M, g and μ. You may assume her legs get out of the way if she slips backwards.)

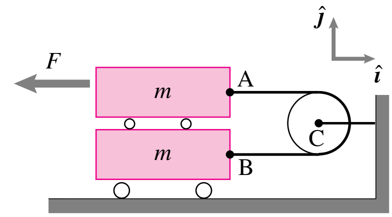


Problem 14.1.16: **Pulley.**

14.1.17 Two blocks and a pulley. Two identical blocks are stacked and tied together by the pulley as shown. Find

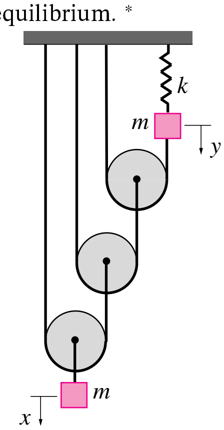
a) the acceleration of point A, and

b) the tension in the line.



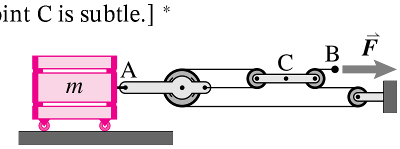
Problem 14.1.17

14.1.18 What is the natural frequency of vibration of this system? Include gravity. x measures the vertical position of the lower mass from equilibrium. y measures the vertical position of the upper mass from equilibrium. *



Problem 14.1.18

14.1.19 For the situation pictured, find the acceleration of mass A and points B and C shown. [Hint: the situation with point C is subtle.] *



Problem 14.1.19

14.1.20 For the situation pictured in problem 14.1.19, find the acceleration of point A using energy balance ($P = \dot{E}_K$). Define any variables, coordinates, or sign conventions that you need to do your calculations and to define your solution.

14.1.21 Design a pulley system. You are to design a pulley system to move a mass. There is no gravity. Point A has a force $\vec{F} = F\hat{\imath}$ pulling it to the right. Mass B has mass m_B. You can connect point A to the mass with any number of ideal strings and ideal pulleys. You can make use of rigid walls or supports anywhere you like (say, to the right or left of the mass). You must design the system so that mass B accelerates to the left with $\frac{F}{2m_B}$ (i.e., $\vec{a}_B = -\frac{F}{2m_B}\hat{\imath}$).

a) Draw the system clearly. Justify your answer with enough words or equations so that a reasonable person, say a grader, can tell that you understand your solution.

b) Find the acceleration of point A.

14.1.22 Design a pulley system. You are to design a pulley system to move a mass. There is no gravity. Point A has a force $\vec{F} = F\hat{\imath}$ pulling it to the right. Mass B has mass m_B. You can connect the point A to the mass with any number of ideal strings and ideal pulleys. You can make use of rigid walls or supports anywhere you like (say, to the right or left of the mass). Draw the system clearly. Justify your answer with enough words or equations to convince a skeptical person that your solution is correct. You must design the system so that the mass B accelerates .

a) to the left with $\frac{F}{m_B}$ (i.e., $\vec{a}_B = -\frac{F}{m_B}\hat{\imath}$)

b) to the left with $\frac{2F}{m_B}$

c) to the left with $\frac{F}{2m_B}$

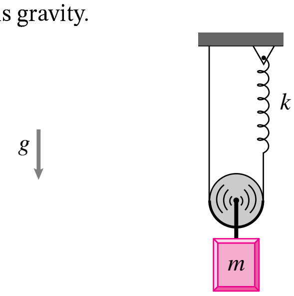
d) to the right with $\frac{2F}{m_B}$

e) to the right with $\frac{F}{2m_B}$

f) to the left with $\frac{8F}{m_B}$

g) to the right with $\frac{F}{5m_B}$

14.1.23 Pulley and spring. For the hanging mass find the period of oscillation. Only vertical motion is of interest. There is gravity.

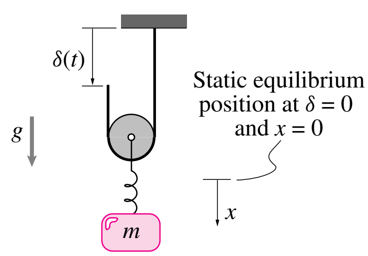


Problem 14.1.23

14.1.24 The spring-mass system shown ($m = 10$ slugs ($\equiv$ lb $\cdot$ sec^2/ft), $k = 10\,\text{lb}/ft$) is excited by moving the free end of the cable vertically according to $\delta(t) = 4\sin(\omega t) \in$, as shown in the figure.

a) Derive the equation of motion for the block in terms of the displacement x from the static equilibrium position, as shown in the figure.

b) If $\omega = 0.9\,\text{rad/s}$, check to see if the pulley is always in contact with the cable (ignore the transient solution).

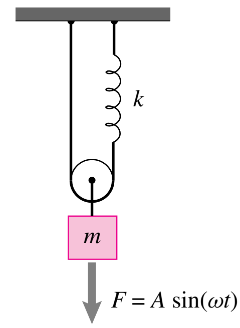


Problem 14.1.24

14.1.25 The block of mass m hanging on the spring with constant k and a string shown in the figure is forced by $F = A\sin(\omega t)$. Do not neglect gravity. The pulley has negligible mass.

a) What is the differential equation governing the motion of the block? You may assume that the only motion is vertical motion. *

b) Given A, m and k, for what values of ω would the string go slack at some point in the cyclical motion? (The common assumption in such problems, which you can use, is to neglect the homogeneous solution to the differential equation. It is assumed that the damping, small enough to be neglected in the governing equations is large enough so that the particular solution will have damped out at the time of observation.) *

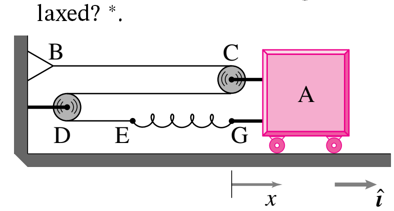


Problem 14.1.25

14.1.26 Block A, with mass m_A, is pulled to the right a distance d from the position it would have if the spring were relaxed. It is then released from rest. Assume ideal string, pulleys and wheels. The spring has constant k.

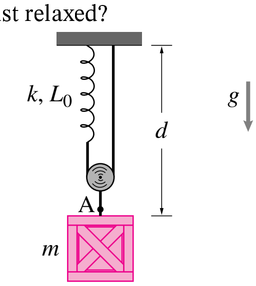
a) What is the acceleration of block A just after it is released (in terms of k, m_A, and d)? *

b) What is the speed of the mass when the mass passes through the position where the spring is relaxed? *.



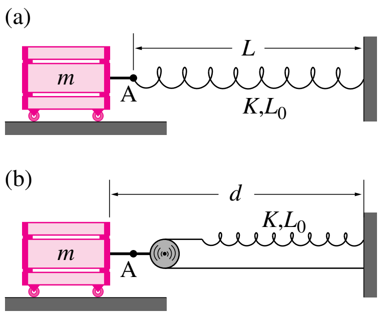
Problem 14.1.26

14.1.27 What is the static displacement of the mass from the position where the spring is just relaxed?



Problem 14.1.27

14.1.28 For the two situations pictured, find the acceleration of point A shown using balance of linear momentum ($\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$). Assuming both masses are deflected an equal distance from the position where the spring is just relaxed, how much smaller or bigger is the acceleration of block (b) than that of block (a). Define any variables, coordinate system origins, coordinates or sign conventions that you need to do your calculations and to define your solution.



Problem 14.1.28

14.1.29 For each of the situations pictured in problem 14.1.28, find the acceleration of the mass using energy balance ($P = \dot{E}_K$). Define any variables, coordinates, or sign conventions that you need to do your calculations and to define your solution.