13.2.1 Assuming θ, v_0, and e to be known quantities, write the following equations in matrix form set up to solve for v'_{Ax} and v'_{Ay}:

$$\sin\theta v'_{Ax} + \cos\theta v'_{Ay} = e v_0 \cos\theta$$
$$\cos\theta v'_{Ax} - \sin\theta v'_{Ay} = v_0 \sin\theta.$$

Answer:
$v'_{Ax} = -v_0(1-e)\sin\theta\cos\theta$
$v'_{Ay} = -v_0(e\cos^2\theta + \sin^2\theta)$

12.2.1

$$\begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix} \begin{pmatrix} v'_{Ax} \\ v'_{Ay} \end{pmatrix} = v_0 \begin{pmatrix} e\cos\theta \\ \sin\theta \end{pmatrix}$$

$$\begin{pmatrix} v'_{Ax} \\ v'_{Ay} \end{pmatrix} = v_0 \begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}^{-1} \begin{pmatrix} e\cos\theta \\ \sin\theta \end{pmatrix}$$

$$= \frac{v_0}{-1} \begin{bmatrix} -\sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} \begin{pmatrix} e\cos\theta \\ \sin\theta \end{pmatrix}$$

$$\begin{pmatrix} v'_{Ax} \\ v'_{Ay} \end{pmatrix} = -v_0 \begin{bmatrix} (1-e)\sin\theta\cos\theta \\ e\cos^2\theta + \sin^2\theta \end{bmatrix}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

13.2.2 The equation $(\vec{v}_1' - \vec{v}_2') \cdot \hat{n} = e(\vec{v}_2 - \vec{v}_1) \cdot \hat{n}$ relates relative velocities of two point masses before and after frictionless impact in the normal direction $\hat{n}$ of the impact. If $\vec{v}_1' = v_{1_x}\hat{\imath} + v_{1_y}\hat{\jmath}$, $\vec{v}_2' = -v_0\hat{\imath}$, $e = 0.5$, $\vec{v}_2 = \vec{0}$, $\vec{v}_1 = 2\,\text{ft/s}\hat{\imath} - 5\,\text{ft/s}\hat{\jmath}$, and $\hat{n} = \frac{1}{\sqrt{2}}(\hat{\imath} + \hat{\jmath})$, find the scalar equation relating the velocities in the normal direction.

Answer:
$v_{1_x} + v_{1_y} + v_0 = 1.5$

$$(\vec{V}_1' - \vec{V}_2') \cdot \hat{n} = e(\vec{V}_2 - \vec{V}_1) \cdot \hat{n}$$

$$(v_{1x}\hat{i} + v_{1y}\hat{j} + v_0\hat{i}) \frac{1}{\sqrt{2}}(\hat{i} + \hat{j}) = e(-2\hat{i} + 5\hat{j}) \cdot \frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$$

$$(v_{1x} + v_0)\hat{i} + v_{1y} = e \times (-2 + 5)$$

$$v_{1x} + v_{1y} + v_0 = 0.5 \times (3)$$

$$v_{1x} + v_{1y} + v_0 = 1.5$$

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13.2.3 The following three equations are obtained by applying the principle of conservation of linear momentum on some system.

$$\begin{aligned} m_0 v_0 &= 24\,\mathrm{m/s}\; m_A - .67 m_B v_B - .58 m_C v_C \\ 0 &= 36\,\mathrm{m/s}\; m_A + .33 m_B v_B + 0.3 m_C v_C \\ 0 &= 23\,\mathrm{m/s}\; m_A - .67 m_B v_B - .58 m_C v_C. \end{aligned}$$

Assume v_0, v_B, and v_C are the only unknowns. Write the equations in matrix form set up to solve for the unknowns.

Answer:

$$\begin{pmatrix} 0 & -.67m_B & -.58m_c \\ 0 & 0.33m_B & 0.03m_c \\ -m_0 & -.67m_B & -.58m_c \end{pmatrix} \begin{pmatrix} v_0 \\ v_B \\ v_c \end{pmatrix} = -m_A \begin{pmatrix} 23\,\mathrm{m/s} \\ 36\,\mathrm{m/s} \\ 24\,\mathrm{m/s} \end{pmatrix}$$

v_0, v_B, v_c are unknown

$$\begin{bmatrix} 0 & -.67m_B & -0.58m_c \\ 0 & 0.33m_B & .03m_c \\ -m_0 & -0.67m_B & -.58\,m_c \end{bmatrix} \begin{bmatrix} v_0 \\ v_B \\ v_c \end{bmatrix} = \begin{pmatrix} -23m_A \\ -36m_A \\ -24m_A \end{pmatrix}$$

13.2.4 The following three equations are obtained to solve for v'_{Ax}, v'_{Ay}, and v'_{Bx}:

$$\begin{aligned}(v'_{Bx} - v'_{Ax})\cos\theta &= v'_{Ay}\sin\theta - 10\,\mathrm{m/s}\\ v'_{Ax}\sin\theta &= v'_{Ay}\cos\theta - 36\,\mathrm{m/s}\\ m_B v'_{Bx} + m_A v'_{Ax} &= (-60\,\mathrm{m/s})\, m_A.\end{aligned}$$

Set up these equations in matrix form.

Answer:

$$\begin{pmatrix} -\cos\theta & -\sin\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \\ 1 & 0 & \frac{m_B}{m_A} \end{pmatrix}\begin{pmatrix} v'_{Ax} \\ v'_{Ay} \\ v'_{Bx}\end{pmatrix} = -\begin{pmatrix} 10\,\mathrm{m/s} \\ 36\,\mathrm{m/s} \\ 60\,\mathrm{m/s}\end{pmatrix}$$

12.2.4

$$\begin{bmatrix} -\cos\theta & -\sin\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \\ m_A & 0 & m_B \end{bmatrix}\begin{pmatrix} V'_{Ax} \\ V'_{Ay} \\ V'_{Bx}\end{pmatrix} = \begin{bmatrix} -10 \\ -36 \\ -60m_A \end{bmatrix}$$

13.2.5 Solve for the unknowns v'_{A_x}, v'_{A_y}, and v'_{B_x} in problem 13.2.4 taking $\theta = 50^\circ$, $m_A = 1.5\,\mathrm{m}_B$ and $m_B = 0.8\,\mathrm{kg}$. Use any computer program.

Answer:
$v'_{A_x} = 39.62$, $v'_{A_y} = -8.79$, and $v'_{B_x} = 13.59$

```
clear all;
clc;
theta = 50*pi/180; % Angle in radians
mB_by_mA=1.5;
A = [-cos(theta) -sin(theta) cos(theta);
       sin(theta) -cos(theta)     0;
              1       0     mB_by_mA];

 Y = [- 10 36 60]';
 X = A\Y;
```

Matlab gives:
X = [39.62 - 8.79 13.59]';

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

13.2.6 Using the matrix form of equations in Problem 13.2.1, solve for v'_{Ax} and v'_{Ay} if $\theta = 20^\circ$ and $v_0 = 5\,\text{ft/s}$.

Answer:
$v'_{Ax} = 1.6(e-1)$ and $v'_{Ay} = -(4.4e + 0.58)$

12.2.6

$\theta = 20^\circ$ and $v_0 = 5$ ft/s

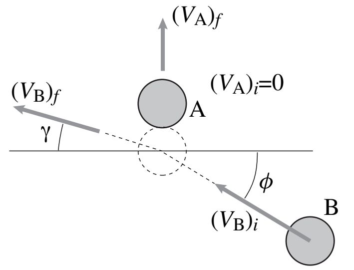
From 12.2.1

$$\begin{pmatrix} v'_{Ax} \\ v'_{Ay} \end{pmatrix} = -v_0 \begin{bmatrix} (1-e)\sin\theta\cos\theta \\ e\cos^2\theta + \sin^2\theta \end{bmatrix}$$

$$\begin{pmatrix} v'_{Ax} \\ v'_{Ay} \end{pmatrix} = -5 \begin{bmatrix} (1-e)\sin 20^\circ \cos 20^\circ \\ e\cos^2 20^\circ + \sin^2 20^\circ \end{bmatrix}$$

$$\begin{pmatrix} v'_{Ax} \\ v'_{Ay} \end{pmatrix} = \begin{pmatrix} (e-1)\times 1.6 \\ -4.4e - 0.58 \end{pmatrix}$$

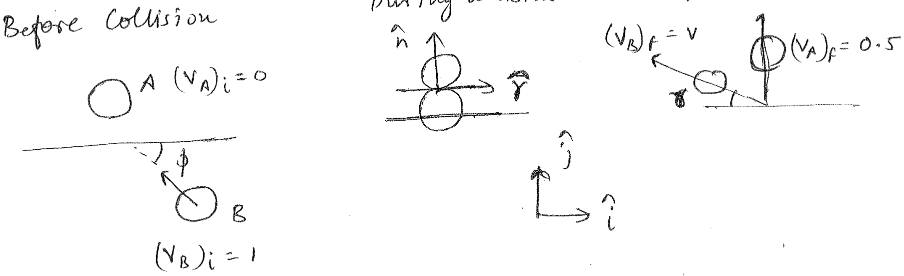
13.2.7 Two frictionless equal-mass pucks sliding on a plane collide as shown below. Puck A is initially at rest. Given that $(V_B)_i = 1.0\,\mathrm{m/s}$, $(V_A)_i = 0$, and $(V_A)_f = 0.5\,\mathrm{m/s}$, find the approach angle ϕ and rebound angle γ. The coefficient of restitution is $e = 0.9$.



Problem 13.7

13.7

Before Collision   During collision   After collision



Given: $(V_A)_i = 0$; $(V_B)_i = 1$; $(V_A)_f = 0.5$; $e = 0.9$

Find ϕ, γ

Let $(V_B)_f = V$

$$(\vec{V}_A)_i = 0$$

$$(\vec{V}_B)_i = -(V_B)_i \cos\phi\,\hat{i} + (V_B)_i \sin\phi\,\hat{j}$$

$$= -\cos\phi\,\hat{i} + \sin\phi\,\hat{j}$$

$$(\vec{V}_A)_f = 0.5\,\hat{j}$$

$$(\vec{V}_B)_f = -(V_B)_f \cos\gamma\,\hat{i} + (V_B)_f \sin\gamma\,\hat{j}$$

$$= -V\cos\gamma\,\hat{i} + V\sin\gamma\,\hat{j}$$

Conservation of linear momentum gives

$$m_A (\vec{V}_A)_i + m_B (\vec{V}_B)_i = m_A (\vec{V}_A)_f + m_B (\vec{V}_B)_f$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Assuming masses are equal $m_A = m_B = m$

$$\Rightarrow \quad m(0) + m(-\cos\phi\,\hat{i} + \sin\phi\,\hat{j}) = m\,0.5\,\hat{j} + m(-v\cos\gamma\,\hat{i} + v\sin\gamma\,\hat{j})$$

$$\Rightarrow \quad -\cos\phi\,\hat{i} + \sin\phi\,\hat{j} = (0.5 + v\sin\gamma)\,\hat{j} - v\cos\gamma\,\hat{i}$$

$$\{\,\}\cdot\hat{i} \qquad -\cos\phi = -v\cos\gamma \qquad \text{— (I)}$$

$$\{\,\}\cdot\hat{j} \qquad \sin\phi = 0.5 + v\sin\gamma \qquad \text{— (II)}$$

We have 2 equation (I), (II) & 3 unknowns, namely, v, ϕ, γ.

Let's use the coefficient of restitution to generate the third equation

$$-e(\vec{v}_A - \vec{v}_B)_i \cdot \hat{n} = (\vec{v}_A - \vec{v}_B)_f \cdot \hat{n}$$

where $\hat{n} = \hat{j}$ {see figure: during collision}

$$\Rightarrow \quad -0.9\left(0 - (-\cos\phi\,\hat{i} + \sin\phi\,\hat{j})\right)\cdot\hat{j} = \left(0.5\,\hat{j} - (-v\cos\gamma\,\hat{i} + v\sin\gamma\,\hat{j})\right)\cdot\hat{j}$$

$$\Rightarrow \quad 0.9\sin\phi = 0.5 - v\sin\gamma \qquad \text{— (III)}$$

~~Subtracting~~ Adding (II) and (III)

$$1.9\sin\phi = 1$$

$\therefore \sin\phi = \frac{1}{1.9} = 0.53 \Rightarrow \phi = 32^\circ$

Putting $\phi = 32^\circ$ in I & II gives

$v\cos\gamma = 0.85$ — IV

$v\sin\gamma = 0.03$ — V

Dividing V / IV $\tan\gamma = \frac{0.03}{0.85} \Rightarrow \gamma = 2.02^\circ$ or $\gamma = 182.02^\circ$

square and add V, IV

$v^2 = 0.7235 \Rightarrow v = \pm 0.85$

Again from IV, V we observe

$v = 0.85 \quad \gamma = 2.02$ is one pair

$v = -0.85 \quad \gamma = 182.02$ is the other

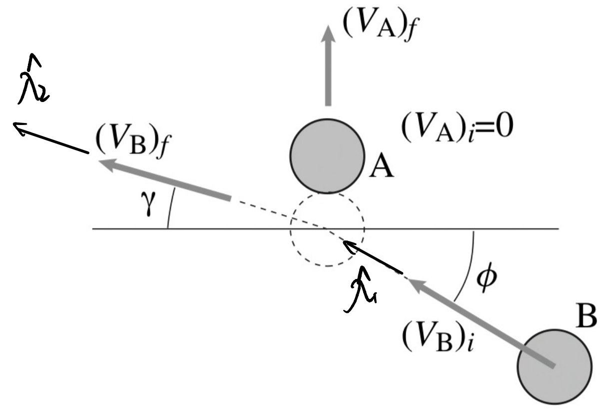
But really both solutions above are equivalent.

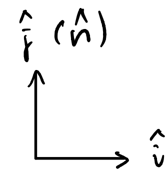
Thus final answer

$$\boxed{\begin{aligned} \gamma &= 2.02^\circ = 0.036 \text{ rad} \\ \phi &= 32^\circ = 0.56 \text{ rad} \end{aligned}}$$

13.2.7 Two frictionless equal-mass pucks sliding on a plane collide as shown below. Puck A is initially at rest. Given that $(V_B)_i = 1.0\,\mathrm{m/s}$, $(V_A)_i = 0$, and $(V_A)_f = 0.5\,\mathrm{m/s}$, find the approach angle ϕ and rebound angle γ. The coefficient of restitution is $e = 0.9$.

Katherine Cao





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Problem 13.2.7

Given: 2 frictionless equal-mass pucks

$m_A = m_B = m$

before collision

$(V_A)_i = 0$

$(V_B)_i = 1.0\,\mathrm{m/s}$ in $\hat{\lambda}_1$, with approach angle ϕ

after collision $(V_A)_f = 0.5\,\mathrm{m/s}$ in $\hat{j}$

coefficient of restitution $e = 0.9$

Find: approach angle ϕ and rebound angle γ

with the problem setup, the normal direction of the plane, the direction approach and separation are measured with respect to, $\hat{n} = \hat{j}$

$\hat{\lambda}_1$ is unit vector in $(V_B)_i$ direction

$\hat{\lambda}_2$ is unit vector in $(V_B)_f$ direction

$\vec{V}_{Ai} = \vec{0}$

$\vec{V}_{Bi} = V_{Bi}\,\hat{\lambda}_1$, $\hat{\lambda}_1 = -\cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath}$

$\vec{V}_{Af} = V_{Af}\,\hat{\jmath}$

$\vec{V}_{Bf} = V_{Bf}\,\hat{\lambda}_2$, $\hat{\lambda}_2 = -\cos\gamma\,\hat{\imath} + \sin\gamma\,\hat{\jmath}$

Conservation of momentum:

$$m_A\vec{V}_{Ai} + m_B\vec{V}_{Bi} = m_A\vec{V}_{Af} + m_B\vec{V}_{Bf}$$

Since $m_A = m_B = m$

$$\vec{V}_{Ai} + \vec{V}_{Bi} = \vec{V}_{Af} + \vec{V}_{Bf}$$

Substitute values

$$\Rightarrow \quad \vec{0} + V_{Bi}(-\cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath}) = V_{Af}\,\hat{\jmath} + V_{Bf}(-\cos\gamma\,\hat{\imath} + \sin\gamma\,\hat{\jmath}) \quad (1)$$

Separate $\hat{\imath}$ $\hat{\jmath}$ direction

$(1)\cdot\hat{\imath}$

$$-V_{Bi}\cos\phi = -V_{Bf}\cos\gamma \quad (2)$$

$(1)\cdot\hat{\jmath}$

$$V_{Bi}\sin\phi = V_{Af} + V_{Bf}\sin\gamma \quad (3)$$

textbook P 663 for Newton's law of collisional restitution

$$(\vec{V}_{Af} - \vec{V}_{Bf}) \cdot \hat{n} = -e(\vec{V}_{Ai} - \vec{V}_{Bi}) \cdot \hat{n}$$

substitute in values

$$\Rightarrow \left(V_{Af}\,\hat{j} - V_{Bf}(-\cos\gamma\,\hat{i} + \sin\gamma\,\hat{j}) \right)\cdot\hat{n}$$

$$= -e\left(\vec{0} - V_{Bi}(-\cos\phi\,\hat{i} + \sin\phi\,\hat{j})\right)\cdot\hat{n} \quad (4)$$

Since $\hat{n} = \hat{j}$, (4) turns into

$$V_{Af} - V_{Bf}\sin\gamma = eV_{Bi}\sin\phi \quad (5)$$

With equations (1), (3), (5)

$$\begin{cases} -V_{Bi}\cos\phi = -V_{Bf}\cos\gamma & (2) \\ V_{Bi}\sin\phi = V_{Af} + V_{Bf}\sin\gamma & (3) \\ V_{Af} - V_{Bf}\sin\gamma = eV_{Bi}\sin\phi & (5) \end{cases}$$

$V_{Bi} = 1.0\ m/s$
$V_{Af} = 0.5\ m/s$
$e = 0.9$

substitute in values

$$\Rightarrow \begin{cases} -(1m/s)\cos\phi = -V_{Bf}\cos\gamma & (6) \\ (1m/s)\sin\phi = 0.5\,m/s + V_{Bf}\sin\gamma & (7) \\ 0.5\,m/s - V_{Bf}\sin\gamma = (0.9\,m/s)\sin\phi & (8) \end{cases}$$

$\boxed{\text{3 unknowns}}$: $\phi,\ \gamma,\ V_{Bf}$

$\boxed{\text{3 equations}}$: (6) (7) (8)

$$\Rightarrow \boxed{\begin{aligned} \phi &= \sin^{-1}\frac{1}{1.9} = 0.554 \text{ rad} \\ \gamma &= \tan^{-1}(0.0306) = 0.0306 \text{ rad} \\ V_B &= 0.851 \text{ m/s} \quad \text{(not required)} \end{aligned}}$$

Check Results:

$$\vec{V}_{Ai} = \vec{0} \text{ m/s}$$

$$\vec{V}_{Bi} = V_{Bi}\,\hat{n}_1 = V_{Bi}\left(-\cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath}\right)$$
$$= (1\text{m/s})(-0.850\,\hat{\imath} + 0.526\,\hat{\jmath})$$

$$\vec{V}_{Af} = V_{Af}\,\hat{\jmath} = (0.5\text{m/s})\,\hat{\jmath}$$

$$\vec{V}_{Bf} = V_{Bf}\,\hat{n}_2 = V_{Bf}\left(-\cos\gamma\,\hat{\imath} + \sin\gamma\,\hat{\jmath}\right)$$
$$= (0.851\text{m/s})(-0.9995\,\hat{\imath} + 0.0306\,\hat{\jmath})$$

Conservation of momentum?

$$\vec{V}_{Ai} + \vec{V}_{Bi} = \vec{V}_{Af} + \vec{V}_{Bf}\ ? \quad \text{Substitute in results}$$

$$\vec{V}_{Ai} + \vec{V}_{Bi} = (1\text{m/s})(-0.850\,\hat{\imath} + 0.526\,\hat{\jmath})$$

$$\vec{V}_{Af} + \vec{V}_{Bf} = (0.5\text{m/s})\,\hat{\jmath} + (0.851\text{m/s})(-0.9995\,\hat{\imath} + 0.0306\,\hat{\jmath})$$
$$= (-0.850 \text{ m/s})\,\hat{\imath} + (0.526 \text{ m/s})\,\hat{\jmath}$$
$$= \vec{V}_{Ai} + \vec{V}_{Bi}$$

Check Newton's law of collisional restitution

$$(\vec{V_{Af}} - \vec{V_{Bf}}) \cdot \hat{n} = -e(\vec{V_{Ai}} - \vec{V_{Bi}}) \cdot \hat{n}$$

Substitute in results (remember $\hat{n} = \hat{j}$)

$$(\vec{V_{Af}} - \vec{V_{Bf}}) \cdot \hat{n} = (\vec{V_{Af}} - \vec{V_{Bf}}) \cdot \hat{j}$$

$$= \left((0.5\,m/s)\hat{j} - (0.851\,m/s)(-0.9995\hat{i} + 0.0306\hat{j})\right) \cdot \hat{j}$$

$$= 0.474\,m/s$$

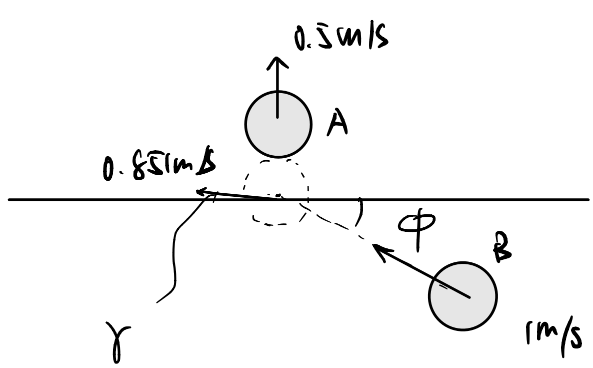
$$-e(\vec{V_{Ai}} - \vec{V_{Bi}}) \cdot \hat{n} = -e(\vec{V_{Ai}} - \vec{V_{Bi}})\hat{j}$$

$$= -0.9\left(\underbrace{0\,m/s\,\hat{i}}_{\text{zero vector pointing any direction}} - (1\,m/s)(-0.850\hat{i} + 0.526\hat{j})\right)\hat{j}$$

$$= 0.473\,m/s$$

Difference of 0.474 m/s and 0.473 m/s due to significant figures throughout calculation

a closer portrait

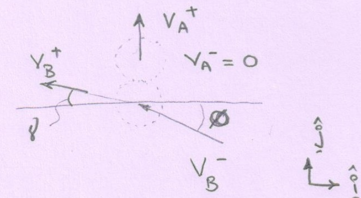


13.2.8 Reconsider problem 13.2.7. Given instead that $\gamma = 30°$, $(V_A)_i = 0$, and $(V_A)_f = 0.5\,\mathrm{m/s}$, find the initial velocity of puck B.

12.2.8:

$V_B^- = ?$ Given $\gamma = 30°$, $V_A^+ = 0.5\,m/s$ & $V_A^- = 0$.

Let $e = 0.9$



Using restitution equation

$$\underline{V}_B^+ - \underline{V}_A^+ = -e\,(\underline{V}_B^- - \underline{V}_A^-) \quad — (1)$$

Using linear momentum conservation

$$\underline{L}^- = \underline{L}^+$$

$$\Rightarrow \quad m_A \underline{V}_A^- + m_B \underline{V}_B^- = m_A \underline{V}_A^+ + m_B \underline{V}_B^+$$

As $m_A = m_B$ & $\underline{V}_A^- = 0$, $\quad \underline{V}_B^- = \underline{V}_A^+ + \underline{V}_B^+ \quad — (2)$

Assuming $\underline{V}_A^+$ is in $\hat{\underline{j}}$ direction (2) results in

$$V_B^- \cos\phi = V_B^+ \cos\gamma \quad — (3)$$

$$V_B^- \sin\phi = V_A^+ + V_B^+ \sin\gamma \quad — (4)$$

and $(1) \cdot \hat{\underline{j}} \Rightarrow \quad V_B^+ \sin\gamma - V_A^+ = -e\, V_B^- \sin\phi \quad — (5)$

From (4) and (5) $\quad V_B^+ = \dfrac{V_A^+}{\sin\gamma}\left[\dfrac{1-e}{1+e}\right] \Rightarrow V_B^+ = \dfrac{0.5\,m/s}{\sin 30°}\left(\dfrac{1-0.9}{1+0.9}\right) = 0.0526\,m/s$

From (3) and (4) $\quad {V_B^-}^2 = {V_B^+}^2 + {V_A^+}^2 + 2 V_A^+ V_B^+ \sin\gamma$

$$\Rightarrow \quad {V_B^-}^2 = \left(0.0526\,\tfrac{m}{s}\right)^2 + \left(0.5\,m/s\right)^2 + 2\,(0.5\,m/s)(0.0526\,m/s)\sin 30$$

$$= \left(0.2791\,(m/s)^2\right)$$

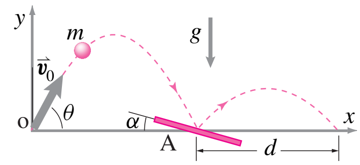
$$\therefore \quad V_B^- = 0.5283\,m/s$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

13.2.9 A ball of mass $m = 0.5\,\mathrm{kg}$ is thrown up in the air with initial speed $v_0 = 50\,\mathrm{m/s}$ at an angle $\theta = 60°$. The ball lands on and bounces off a slanted floor that makes an angle $\alpha = 15°$ with the horizontal. Assume the collision with the floor to be elastic and ignore air drag on the ball.

a) Find the impulse of the collision of the ball,

b) After bouncing off the slanted floor, how much horizontal distance does the ball travel before landing on the ground again? Is this distance more, less, or the same as it would have travelled had the floor not been slanted?

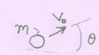


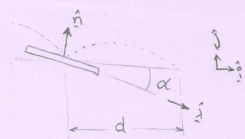
Problem 13.9

Answer:
(a) $2mv_0\sin(\theta-\alpha)(\sin\alpha\,\hat{\imath}+\cos\alpha\,\hat{\jmath})$
(b) $d = \frac{v_0^2}{g}\sin(2\theta-4\alpha)$; The distance after bouncing off a slanted floor is more.

12.2.9.

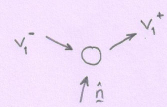
a) Impulse of the collision ?





Given $m = 0.5\,\mathrm{Kg}$
$V_0 = 50\,\mathrm{m/s}$
$\theta = 60°$
$\alpha = 15°$
No drag on the ball.

Impulse of the collision of the ball :



$$P\,\hat{n} = m\underline{v}_1^+ - m\underline{v}_1^- \quad — ①$$

$\hat{n} = \sin\alpha\,\hat{i} + \cos\alpha\,\hat{j}$
$\hat{n} = n_x\hat{i} + n_y\hat{j}$

Let $\underline{v}_1^+ = V_{1x}^+\hat{i} + V_{1y}^+\hat{j}$ and $V_1^- = V_{0x}\hat{i} - V_{0y}\hat{j}$

Using restitution equation $\{\underline{v}_1^+ = -e\,\underline{v}_1^-\}$

$$\{\}\cdot\hat{n} \Rightarrow (V_{1x}^+\hat{i} + V_{1y}^+\hat{j})\cdot(n_x\hat{i}+n_y\hat{j}) = -e\,(V_{0x}\hat{i} - V_{0y}\hat{j})\cdot(n_x\hat{i}+n_y\hat{j})$$

$$\Rightarrow V_{1x}^+ n_x + V_{1y}^+ n_y = -e\,V_{0x}n_x + e\,V_{0y}n_y \quad — ②$$

From ① $P n_x = m(V_{1x}^+ - V_{0x})$ and $P n_y = m(V_{1y}^+ + V_{0y})$

Eliminating P, $V_{1x}^+ = V_{0x} + \frac{n_x}{n_y}(V_{1y}^+ + V_{0y}) \quad — ③$

Substituting ③ in ② $V_{1y}^+ = (-n_x^2 + e\,n_y^2)V_{0y} - n_x n_y V_{0x}(1+e) \quad — ④$

Substituting ④ in ③ $V_{1x}^+ = (n_y^2 - e\,n_x^2)V_{0x} + n_x n_y V_{0y}(1+e) \quad — ⑤$

$$\therefore \text{ Impulse, } P = \frac{m}{n_y}\left[(-n_x^2 + e\,n_y^2)V_{0y} - n_x n_y V_{0x}(1+e) + V_{0y}\right]$$

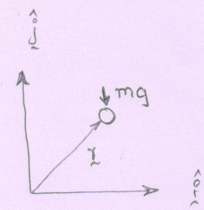
$$\text{(or)} \quad P = m(1+e)\left[n_y V_{0y} - n_x V_{0x}\right] \quad — ⑥$$

$n_x = \sin 15°$ & $n_y = \cos 15°$
$V_{0x} = V_0\cos 60°$ & $V_{0y} = V_0 \sin 60°$ $\Big\}$ $\Rightarrow P = 33.59\ \mathrm{kg}\,\frac{m}{s}$.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

b) Distance travelled after the impulse.

FBD of the ball gives $m\ddot{\underline{r}} = -mg\,\hat{\underline{j}}$



$\Rightarrow \quad \ddot{x} = 0 \quad$ and $\quad \ddot{y} + g = 0$

$\Rightarrow \quad \dot{x} = c_1 \quad$ and $\quad \dot{y} + gt = c_2$

$\Rightarrow \quad x = c_1 t + c_3 \quad$ and $\quad y + \frac{1}{2}gt^2 = c_2 t + c_4$.

At $t = 0$, $x = y = 0$ & $\dot{x} = V_{qx}^{+}$ and $\dot{y} = V_{qy}^{+}$

$\therefore \quad x = V_{qx}^{+} t \quad$ and $\quad y = -\frac{1}{2}gt^2 + V_{qy}^{+} t$

Time taken by the ball to touch the floor again, $t = \dfrac{2V_{qy}^{+}}{g}$

$\therefore$ The distance travelled by the ball, $d = \dfrac{2\,V_{qx}^{+} V_{qy}^{+}}{g} \quad$ — (7)

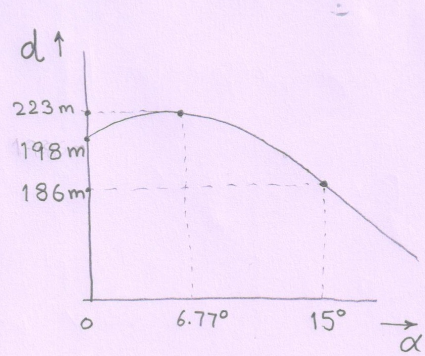
From (4), (5) and (7).

$$d = \frac{2}{g}\left\{\left((\cos^2\alpha - e\sin^2\alpha)\,v_{ox} + \sin\alpha\cos\alpha\,V_{oy}(1+e)\right)\left((-\sin^2\alpha + e\cos^2\alpha)V_{oy} - \sin\alpha\cos\alpha\,V_{ox}(1+e)\right)\right\}$$

If $\alpha = 0$, $\quad d_o = \frac{2}{g}\left\{(V_{ox} + 0)(eV_{oy} - 0)\right\} = \dfrac{2V_{ox}V_{oy}\,e}{g} = 198.6\text{ m}$

If $\alpha = 15°$, $\quad d = 186.5\text{ m}$.

This distance is smaller than that when $\alpha = 0°$. However, the distance is observed to be maximum ($d_{max} = 223.5$ m) when $\alpha = 6.77°$ as shown below.



Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

13.2.10 Solve the general two-particle frictionless collision problem. For example, write computer code that has lines like this near the start :

`m1=3; m2=19`	Set values of masses
`v1zero=[10 20]`	Initial velocity of mass 1
`v2zero=[-5 3]`	Initial velocity of mass 2
`e=.5`	Set coefficient of restitution
`theta=pi/4`	Angle that the normal to contact plane makes, measured CCW from +x axis, in radians

Your program (function, code, script) should calculate the impulse of mass 1 on mass 2, and the velocities of the two masses after the collision. Your program should assume consistent units for all quantities.

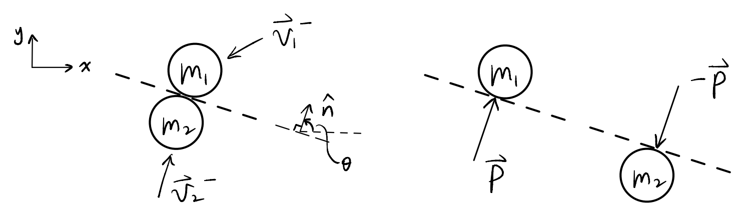
a) You should demonstrate that your program works by solving at least 4 different problems for which you can check your answer by simple pencil-and-paper calculations. These problems should have as much variety as possible. Sketch these problems clearly, show their analytic solution, and show that the computer agrees.

b) Solve the problem given in the sample text given in the initial problem statement.

Answer:
One test problem is this: $\vec{\boldsymbol{v}}_1^- = \hat{\boldsymbol{\imath}}, \vec{\boldsymbol{v}}_2^- = \vec{\boldsymbol{0}}, m_1 = m_2 = 1, e = 0, \theta = 0$. This should have the solution $\vec{\boldsymbol{v}}_1^+ = \vec{\boldsymbol{v}}_2^+ = 0.5\hat{\boldsymbol{\imath}}, \vec{\boldsymbol{P}} = 0.5\hat{\boldsymbol{\imath}}$.

MAE2030 HW P28 (13.2.10) Solution

Duan Li



Given: 2 particles frictionless collision $\rightarrow \vec P \parallel \hat n \rightarrow \vec P = P\hat n$

$m_1, m_2, \vec v_1^-, \vec v_2^-, e, \theta$

Find: $\vec v_1^+$, $\vec v_2^+$, Impulse of m_1 on m_2 $-\vec P$

Solution:

$\hat n = n_x \hat\imath + n_y \hat\jmath$, $n_x = \cos\theta$, $n_y = \sin\theta$

Linear momentum balance

$\vec P = m_1(\vec v_1^+ - \vec v_1^-)$ ① ← 1 vector eqn = 2 scalar eqns

$-\vec P = m_2(\vec v_2^+ - \vec v_2^-)$ ② ← 1 vector eqn = 2 scalar eqns

(more details on next page)

Restitution

$(\vec v_1^+ - \vec v_2^+)\cdot\hat n = -e(\vec v_1^- - \vec v_2^-)\cdot\hat n$ ③ ← 1 scalar eqn

① - ③ in total have 5 scalar eqns & 5 unknown scalars

$$\vec{v}_1^{\,+} = (v_{1x}^+,\ v_{1y}^+)'$$
$$\vec{v}_2^{\,+} = (v_{2x}^+,\ v_{2y}^+)'$$
$$\vec{P} = P\hat{n}$$

Convert from vector eqns ① - ③ to scalar eqns

$$① \Rightarrow P(n_x\hat{i} + n_y\hat{j}) = m_1(v_{1x}^+\hat{i} + v_{1y}^+\hat{j} - v_{1x}^-\hat{i} - v_{1y}^-\hat{j})$$

$$① \cdot \hat{i} \rightarrow Pn_x = m_1(v_{1x}^+ - v_{1x}^-)$$

$$m_1 v_{1x}^+ - n_x P = m_1 v_{1x}^- \qquad ④$$

$$① \cdot \hat{j} \rightarrow Pn_y = m_1(v_{1y}^+ - v_{1y}^-)$$

$$m_1 v_{1y}^+ - n_y P = m_1 v_{1y}^- \qquad ⑤$$

$$② \Rightarrow -P(n_x\hat{i} + n_y\hat{j}) = m_2(v_{2x}^+\hat{i} + v_{2y}^+\hat{j} - v_{2x}^-\hat{i} - v_{2y}^-\hat{j})$$

$$② \cdot \hat{i} \rightarrow -Pn_x = m_2(v_{2x}^+ - v_{2x}^-)$$

$$m_2 v_{2x}^+ + n_x P = m_2 v_{2x}^- \qquad ⑥$$

$$② \cdot \hat{j} \rightarrow -Pn_y = m_2(v_{2y}^+ - v_{2y}^-)$$

$$m_2 v_{2y}^+ + n_y P = m_2 v_{2y}^- \qquad ⑦$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$③ \Rightarrow (v_{1x}^+ - v_{2x}^+)\, n_x + (v_{1y}^+ - v_{2y}^+)\, n_y = -e(\vec{v}_1^- - \vec{v}_2^-) \cdot \hat{n}$$

$$n_x v_{1x}^+ + n_y v_{1y}^+ - n_x v_{2x}^+ - n_y v_{2y}^+ = -e(\vec{v}_1^- - \vec{v}_2^-) \cdot \hat{n} \quad ⑧$$

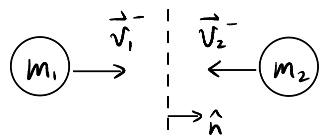
Convert from scalar eqns ④ – ⑧ to a matrix eqn

$$\underbrace{\begin{bmatrix} m_1 & 0 & 0 & 0 & -n_x \\ 0 & m_1 & 0 & 0 & -n_y \\ 0 & 0 & m_2 & 0 & n_x \\ 0 & 0 & 0 & m_2 & n_y \\ n_x & n_y & -n_x & -n_y & 0 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} v_{1x}^+ \\ v_{1y}^+ \\ v_{2x}^+ \\ v_{2y}^+ \\ P \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} m_1 v_{1x}^- \\ m_1 v_{1y}^- \\ m_2 v_{2x}^- \\ m_2 v_{2y}^- \\ -e(\vec{v}_1^- - \vec{v}_2^-) \cdot \hat{n} \end{bmatrix}}_{\vec{b}}$$

$$\hookrightarrow \quad \vec{x} = M \backslash \vec{b}$$

(a) Verify Matlab code (on page 7) with 4 simple cases

Case 1. equal masses head on elastic collision



$m_1 = m_2 = 1$

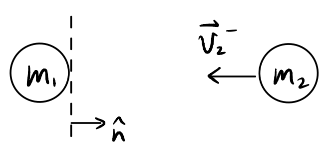
$\vec{v}_1^- = (1, 0)'$, $\vec{v}_2^- = (-1, 0)'$

$e = 1$, $\theta = 0$

Intuitive analytical solution: $\vec{v}_1^+ = \vec{v}_2^-$, $\vec{v}_2^+ = \vec{v}_1^-$

$\vec{v}_1^+ = (-1, 0)'$, $\vec{v}_2^+ = (1, 0)'$, $-\vec{P} = (2, 0)'$

Case 2. m_2 collides onto a "wall"



$m_1 = 10^4$, $m_2 = 1$

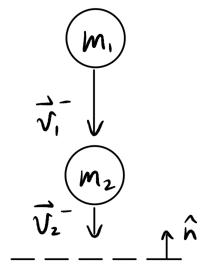
$\vec{v}_1^- = (0, 0)'$, $\vec{v}_2^- = (-2, 0)'$

$e = 1$, $\theta = 0$

Intuitive analytical solution: $\vec{v}_2^+ = -\vec{v}_2^-$

$\vec{v}_1^+ = (0, 0)'$, $\vec{v}_2^+ = (2, 0)'$, $-\vec{P} = (4, 0)'$

Case 3. equal masses stick together after collision



$m_1 = m_2 = 1$

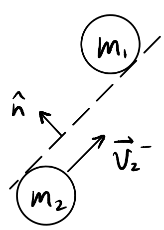
$\vec{v}_1^- = (0, -3)'$, $\vec{v}_2^- = (0, -1)'$

$e = 0$, $\theta = \pi/2$

Intuitive analytical solution: $\vec{v}_1^+ = \vec{v}_2^+ = \frac{1}{2} \sum \vec{v}_i^-$

$\vec{v}_1^+ = (0, -2)'$, $\vec{v}_2^+ = (0, -2)'$, $-\vec{P} = (0, -1)'$

Case 4. barely touching, frictionless



$m_1 = 2$, $m_2 = 3$

$\vec{v}_1^- = (0, 0)'$, $\vec{v}_2^- = (1, 1)'$

$e = 0.8$, $\theta = 3\pi/4$

Intuitive analytical solution: $\vec{v}_1^+ = \vec{v}_1^-$, $\vec{v}_2^+ = \vec{v}_2^-$

$\vec{v}_1^+ = (0, 0)'$, $\vec{v}_2^+ = (1, 1)'$, $-\vec{P} = (0, 0)'$

Analytical solutions for case 1-4 all agree with Matlab outputs.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

(b) Given (assume consistent units)

$m_1 = 3$, $m_2 = 19$

$\vec{v}_1^- = (10, 20)'$, $\vec{v}_2^- = (-5, 3)'$

$e = 0.5$, $\theta = \pi/4$

Output from Matlab

$\vec{v}_1^+ = (-10.7, -0.7)'$

$\vec{v}_2^+ = (-1.7, 6.3)'$

$-\vec{P} = (62.2, 62.2)'$

General two-particle frictionless collision

Author: Duan Li
Mar 18, 2021

```
% This program solves the general two-particle frictionless
% collision problem.

clear all; close all;

% ##### USER INPUT #####
partN = 'b';
caseN = 4; % only for part (a)

% Settings for different parts/cases
if partN == 'a' % part (a)
    switch caseN
        % 4 different cases to verify the code
        case 1
            m1 = 1;
            m2 = 1;
            v1zero = [1, 0]';
            v2zero = [-1, 0]';
            e = 1;
            theta = 0;
        case 2
            m1 = 10000;
            m2 = 1;
            v1zero = [0, 0]';
            v2zero = [-2, 0]';
            e = 1;
            theta = 0;
        case 3
            m1 = 1;
            m2 = 1;
            v1zero = [0, -3]';
            v2zero = [0, -1]';
            e = 0;
            theta = pi/2;
        case 4
            m1 = 2;
            m2 = 3;
            v1zero = [0, 0]';
            v2zero = [1, 1]';
            e = 0.8;
            theta = 3*pi/4;
        otherwise
            error('Invalid case number.')
    end
elseif partN == 'b' % part (b)
    m1 = 3;
    m2 = 19;
```

```
    v1zero = [10, 20]';
    v2zero = [-5, 3]';
    e = 0.5;
    theta = pi/4;
else
    error('Invalid part number.')
end

% pack parameters
p.m1 = m1;
p.m2 = m2;
p.v1i = v1zero;
p.v2i = v2zero;
p.e = e;
p.theta = theta;

% solve the two-particle frictionless collision problem
[I, v1f, v2f] = collision(p);

% print out solutions
disp('Velocity of mass 1 after the collision')
disp(v1f)
disp('Velocity of mass 2 after the collision')
disp(v2f)
disp('Impulse of mass 1 on mass 2')
disp(-I)

function [I, v1f, v2f] = collision(p)
% Input (wrapped in p):
% m1   - mass 1
% m2   - mass 2
% v1i - initial velocity of mass 1, 2x1 vector
% v2i - initial velocity of mass 2, 2x1 vector
% e   - coefficient of restitution
% theta - angle of the normal to contact plane,
%         measured CCW from +x axis, in radians

% Output
% I   - impulse of mass 1 on mass 2, 2x1 vector
% v1f - velocity of mass 1 after the collision, 2x1 vector
% v2f - velocity of mass 2 after the collision, 2x1 vector

% unpack parameters
names = fieldnames(p);
for i = 1:length(names)
    eval([names{i} '= p.' names{i} ';']);
end

% collision plane normal vector
nx = cos(theta);
ny = sin(theta);
n = [nx, ny]';

% Matrix M
```

```
M = [m1,  0,   0,   0, -nx;
    0, m1,   0,   0, -ny;
    0,  0,  m2,   0,  nx;
    0,  0,   0,  m2,  ny;
    nx, ny, -nx, -ny,   0 ];

% right hand side vector b
b = [m1*v1i; m2*v2i; -e*(v1i-v2i)'*n];

% solve for the unknowns
x = M \ b;

% unpack the solution
v1f = x(1:2);
v2f = x(3:4);
I = x(5)*n;
end
```

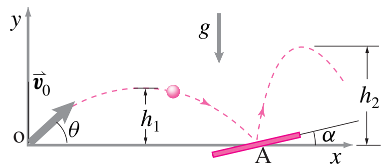
```
Velocity of mass 1 after the collision
  -10.7273
   -0.7273
Velocity of mass 2 after the collision
   -1.7273
    6.2727
Impulse of mass 1 on mass 2
   62.1818
   62.1818
```

Published with MATLAB® R2020b

13.2.11 A projectile is launched at $\theta = 40°$ with speed $v_0 = 25\,\mathrm{m/s}$. The projectile lands on a steel plate that can be adjusted to make any angle α with the horizontal. The projectile bounces off the steel plate without losing any energy. The projectile is required to reach a height after rebound twice as much as it did during its flight before hitting the plate. Ignore air resistance.

a) Find the required angle α of the plate.

b) Can you always find some α for any launch angle $\theta < \pi/2$ such that $h_2 = 2h_1$?



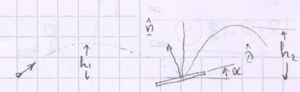
Problem 13.11

Answer:
(a) α is the solution to equ. $\sin(\theta + 2\alpha) = \sqrt{2}\sin\theta$
(b) α can be obtained only for $0 \leq \theta \geq \frac{\pi}{4}$. $\alpha = \frac{\sin^{-1}(\sqrt{2}\sin\theta) - \theta}{2}$

12.2.11

a) $\theta = 40°$ $\quad V_0 = 25\ m/s$

find α for $h_2 = 2h_1$



$$h_1 = y\Big|_{t = \frac{V_{oy}}{g}} = V_{oy}\frac{V_{oy}}{g} - \frac{1}{2}g\frac{V_{oy}^2}{g^{\cancel{2}}} = \frac{V_{oy}^2}{g} - \frac{V_{oy}^2}{2g} = \frac{V_{oy}^2}{2g} \qquad \left[\because y = -\frac{1}{2}gt^2 + V_{oy}t\right]$$

$$h_2 = \frac{V_{1y}^{+2}}{2g} = \frac{\left(-n_x l_x(1+e)V_{0x} + (n_x l_y + e\, n_y l_x)V_{oy}\right)^2}{\cancel{2g}} = 2h_1 = 2\frac{V_{oy}^2}{\cancel{2g}}$$

$$\Rightarrow \left[+s\alpha\, c\alpha(1+e)V_{ox} + (-s^2\alpha + e\,c^2\alpha)V_{oy}\right]^2 = 2V_{oy}^2$$

$\hat{\lambda} = l_x\hat{i} + l_y\hat{j}$

$\hat{\lambda} = c\alpha\,\hat{i} + s\alpha\,\hat{j}$

$\hat{n} = n_x\hat{i} + n_y\hat{j}$

$\hat{n} = -s\alpha\,\hat{i} + c\alpha\,\hat{j}$

$V_{ox} = V_0\cos 40° = 25\,\frac{m}{s}\cos 40°$

$V_{oy} = V_0\sin 40° = 25\,\frac{m}{s}\sin 40°$

$$+\frac{1}{2}\sin 2\alpha(1+e)V_{ox} + (e\,c^2\alpha - s^2\alpha)V_{oy} \pm \sqrt{2}\,V_{oy} = 0$$

(or) $\boxed{\sin\alpha\cos\alpha(1+e)\cos\theta + (e\cos^2\alpha - \sin^2\alpha)\sin\theta \pm \sqrt{2}\sin\theta = 0}$

$180° - \pi$

$40° - ?$ $\frac{40\pi}{180} = \frac{2\pi}{9}$

$\frac{\pi}{2} = 1.57$

Elastic collision $e = 1$

$$\sin 2\alpha\cos\theta + \cos 2\alpha\sin\theta \pm \sqrt{2}\sin\theta = 0$$

$$\sin(2\alpha + \theta) = \pm\sqrt{2}\sin\theta$$

(or) ~~$\sin 2\alpha\cos\theta$~~ $\quad 2\alpha + \theta = \sin^{-1}(\pm\sqrt{2}\sin\theta)$

$$2\alpha = \sin^{-1}(\pm\sqrt{2}\sin\theta) - \theta$$

$$\alpha = \frac{1}{2}\sin^{-1}(\pm\sqrt{2}\sin\theta) - \frac{\theta}{2}$$

$\alpha = \alpha_1\ \&\ \alpha_2 \qquad \theta = 40°$

$\therefore$ Case 1: $\alpha = \alpha_1 \ \rightarrow 2h_1 = h_2 \Rightarrow \alpha_1 = 12.686°$

Case 2: $\alpha = \alpha_2 \ \rightarrow 2h_1 = h_2 \Rightarrow \alpha_2 = -52.686°$ (Infeasible)

$\therefore$ The angle of the steel plate for $h_2 = 2h_1$ is $12.686°$

b) α cannot be found for every $\theta < \pi/2$ as $\sqrt{2}\sin\theta$ can be > 1 if $\theta > 45°$ $\quad\therefore\ \theta_{max} = 45°$

$$\Rightarrow \alpha_{max} = \frac{1}{2}\sin^{-1}(1) - \frac{\pi}{8}$$

$$= \frac{\pi}{4} - \frac{\pi}{8} = \frac{\pi}{8}\ \text{radian}$$

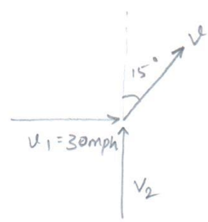
or $\alpha_{max} = 22.5°$

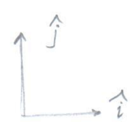
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

13.2.12 Two equal mass cars approach an intersection at right angles. They crash and stick together. One of the cars was going at 30 mph before the crash. The other car's path gets deflected by 15°. How fast was it going?

Answer:
112 mph

12. 2 . 12





$m_1 = m_2 = m$

From conservation of Linear momentum

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\vec{v}$$

$$m\cdot 30\,\hat{i} + m\cdot v_2\,\hat{j} = 2mv(\sin 15^\circ\,\hat{i} + \cos 15^\circ\,\hat{j})$$

$$30\,\hat{i} + v_2\,\hat{j} = 2v\sin 15^\circ\,\hat{i} + 2v\cos 15^\circ\,\hat{j}$$

$$v_2\,\hat{j} = 2v\cos 15^\circ\,\hat{j}$$

$$30\,\hat{i} = 2v\sin 15\,\hat{i}$$

$$v = \frac{30}{2\sin 15}$$

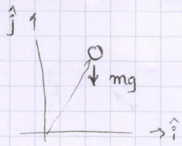
$$\therefore\ v_2 = 30.\cot 15^\circ$$

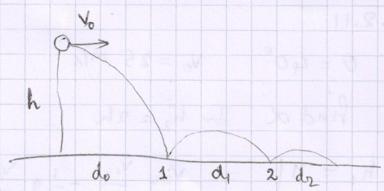
$$v_2 = 112 \text{ mph}$$

13.2.13 A ball m is thrown horizontally at height h and speed v_0. It then has a sequence of bounces on the horizontal ground. Treating each collision as frictionless with restitution coefficient e how far has the ball travelled horizontally when it just finishes bouncing? Answer in terms of some or all of m, g, h, v_0 and e.

Answer:
$v_0\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$

12.2.13





$m\ddot{\underline{r}} = -mg\hat{j}.$

$m\ddot{x} = 0$ & $m\ddot{y} + mg = 0$

$\dot{x} = C_1$ $\qquad \dot{y} + gt = C_3$

$x = C_1 t + C_2$ $\qquad y + \frac{1}{2}gt^2 = C_3 t + C_4$

$x=0,\ t=0,$ $\Rightarrow C_2 = 0$ $\qquad$ At $t=0$, $\dot{y}=0 \Rightarrow C_3 = 0$

At $t=0$, $\dot{x} = V_0$ $\Rightarrow C_1 = V_0$ $\qquad$ At $t=0$, $y=h$ $\Rightarrow h = 0 + C_4$

$\Rightarrow \boxed{x = V_0 t}$ $\qquad \boxed{y = -\frac{1}{2}gt^2 + h}$

$y=0$ at $t = \sqrt{\frac{2h}{g}} \Rightarrow x\big|_{y=0} = V_0\sqrt{\frac{2h}{g}}$

Velocity before touchdown,

$V_{0x}^- = V_0$ & $V_{0y}^- = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh}$

$V_1^- = V_{0x}\hat{i} + V_{0y}\hat{j}$

where $V_{0x} = V_0 = V_{1x}^-$ and $V_{0y} = -\sqrt{2gh} = V_{1y}^-$

$d_0 = V_0\sqrt{\frac{2h}{g}}$, $\quad d_1 = \frac{2V_{1x}^+ V_{1y}^+}{g} = \frac{2V_{1x}^-(-eV_{1y}^-)}{g}$

$\Rightarrow d_1 = \frac{2V_0 e\sqrt{2gh}}{g} = e\,2V_0\sqrt{\frac{2h}{g}}$

$V_{2x}^- = V_{2x}^+$ & $V_{2y}^- = -V_{1y}^+ = -[-eV_{1y}^-] = eV_{1y}^-$
$= -e\sqrt{2gh}$

$V_{2y}^+ = -eV_{2y}^-$
$= e^2\sqrt{2gh}$

$d_2 = \frac{2V_{2x}^+ V_{2y}^+}{g} = \frac{2V_0 e^2\sqrt{2gh}}{g}$
$= e^2 2V_0\sqrt{\frac{2h}{g}}$

$d_3 = \frac{2V_0 e^3}{g}\sqrt{2gh} = e^3 2V_0\sqrt{\frac{2h}{g}}$

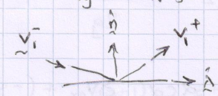
$d = d_0 + d_1 + d_2 + d_3 \cdots = V_0\sqrt{\frac{2h}{g}} + 2eV_0\sqrt{\frac{2h}{g}} + 2e^2V_0\sqrt{\frac{2h}{g}} + 2e^3V_0\sqrt{\frac{2h}{g}} + V_0\sqrt{\frac{2h}{g}} - V_0\sqrt{\frac{2h}{g}} + \cdots$

$= 2V_0\sqrt{\frac{2h}{g}}\left\{\underbrace{1 + e + e^2 + e^3 + \cdots \infty}_{S_\infty = \frac{1}{1-e}}\right\} - V_0\sqrt{\frac{2h}{g}} = V_0\sqrt{\frac{2h}{g}}\left\{2\sum_{i=0}^{\infty} e^i - 1\right\}$

$\therefore d = V_0\sqrt{\frac{2h}{g}}\left\{\frac{2 - 1 + e}{1-e}\right\} \Rightarrow \boxed{d = \left(\frac{1+e}{1-e}\right)V_0\sqrt{\frac{2h}{g}}}$

$V_{1x}^- = V_0$

$V_{1y}^- = -\sqrt{2gh}$



$\underline{V}_1^- = V_{1x}^-\hat{i} + V_{1y}^-\hat{j}$

$\underline{V}_1^+ = V_{1x}^+\hat{i} + V_{1y}^+\hat{j}$ $\qquad \hat{\lambda} = \hat{i}$

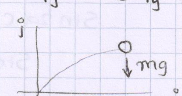
$\underline{V}_1^- \cdot \hat{\lambda} = \underline{V}_1^+ \cdot \hat{\lambda}$ $\qquad \hat{n} = \hat{j}$

$V_{1x}^- = V_{1x}^+$

~~$V_{1y}^- = V_{1y}^+$~~

$\hat{n}\cdot(\underline{V}_1^+ - \cancel{\underline{V}_0^+}) = -e(\underline{V}_1^- - \cancel{\underline{V}_0})\cdot\hat{n}$ $\quad \hat{j}$

$V_{1y}^+ = -eV_{1y}^-$



$m\ddot{\underline{r}} = -mg\hat{j}$

$m\ddot{x} = 0$ & $m\ddot{y} = -mg$

$\dot{x} = C_1$ & $\dot{y} = -gt + C_3$

$x = C_1 t + C_2$ & $y = -\frac{1}{2}gt^2 + C_3 t + C_4$

$x = 0$ @ $t=0 \Rightarrow C_2 = 0$ $\qquad y = 0$ @ $t=0 \Rightarrow C_4 = 0$

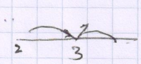
$\dot{x} = C_1 = V_{1x}^+$ $\qquad V_{1y}^+ = -g\cancel{t} + C_3$

$x = V_{1x}^+ t$ $\qquad y = -\frac{1}{2}gt^2 + V_{1y}^+ t$

$y=0$ @ $t=0$ (or) $t = \frac{2V_{1y}^+}{g}$

$x = \frac{2V_{1x}^+ V_{1y}^+}{g}$

$\dot{y} = -\cancel{g}\frac{2V_{1y}^+}{\cancel{g}} + V_{1y}^+ = -V_{1y}^+$

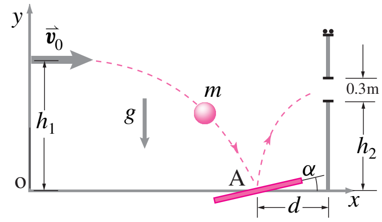


$V_{3x}^- = V_{3x}^+ = V_{2x}^+ = V_{2x}^- = V_{1x}^+ = V_{1x}^- = V_0$

$V_{3y}^+ = -eV_{3y}^- = -e[-V_{2y}^+] = e^3\sqrt{2gh}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

13.2.14 A game involves using a pedal to direct a falling ball into a fixed vertical slot by simply rotating the pedal when the ball hits the pedal. A model of this game is shown in the figure. The ball is thrown horizontally with an initial speed $v = 10\,\mathrm{m/s}$ from a height $h_1 = 3\,\mathrm{m}$. The pedal is located at $d = 2\,\mathrm{m}$ from the wall that houses the slot at height $h_2 = 2\,\mathrm{m}$. The slot itself is $0.3\,\mathrm{m}$ in extent. The coefficient of restitution between the pedal and the ball is $e = 0.9$. The air resistance is negligible. Find the angle α or the range of this angle, so that the ball makes it through the slot. You can ignore the dimensions of the ball.



Problem 13.14

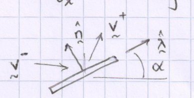
Answer:
$$\alpha = \frac{1}{2}\tan^{-1}\left(\frac{2v_0 - \sqrt{2gh_2}}{v_0 + 2\sqrt{2gh_2}}\right)$$

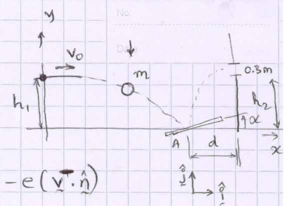
12.2.14

$x = V_0 t \qquad y = -\frac{1}{2} g t^2 + h_1$

$t = \sqrt{\frac{2h_1}{g}}$

$V_{0x}^- = V_0$ & $V_{0y}^- = -g\sqrt{\frac{2h_1}{g}} = -\sqrt{2gh_1}$





$(\underset{\sim}{v}^+ \cdot \hat{n}) = -e(\underset{\sim}{v}^- \cdot \hat{n})$

$\underset{\sim}{v}^- \cdot \hat{\lambda} = \underset{\sim}{v}^+ \cdot \hat{\lambda}$

$\hat{\lambda} = l_x \hat{i} + l_y \hat{j}$

$\hat{n} = n_x \hat{i} + n_y \hat{j}$

$(V_{0x}^+ \hat{i} + V_{0y}^+ \hat{j}) \cdot (n_x \hat{i} + n_y \hat{j}) = -e(V_x^- \hat{i} + V_y^- \hat{j}) \cdot (n_x \hat{i} + n_y \hat{j})$

$V_{0x}^+ n_x + V_{0y}^+ n_y = -e V_x^- n_x - e V_y^- n_y$ —— ①

$V_{0x}^- l_x + V_{0y}^- l_y = V_x^+ l_x + V_y^+ l_y$ —— ②

$\lambda = \underbrace{\cos\alpha}_{l_x}\,\hat{i} + \underbrace{\sin\alpha}_{l_y}\,\hat{j}$

$\hat{n} = \underbrace{-\sin\alpha}_{n_x}\,\hat{i} + \underbrace{\cos\alpha}_{n_y}\,\hat{j}$

$$\begin{bmatrix} n_x & n_y \\ l_x & l_y \end{bmatrix}\begin{bmatrix} V_{0x}^+ \\ V_{0y}^+ \end{bmatrix} = \begin{bmatrix} -en_x & -en_y \\ l_x & l_y \end{bmatrix}\begin{bmatrix} V_x^- \\ V_y^- \end{bmatrix}$$

$V_0 = 10\,\mathrm{m/s}$ & $h_1 = 3\,\mathrm{m}$, $d = h_2 = 2\mathrm{m}$

$$\begin{bmatrix} V_{0x}^+ \\ V_{0y}^+ \end{bmatrix} = \frac{1}{n_x l_y - n_y l_x}\begin{bmatrix} l_y & -n_y \\ -l_x & n_x \end{bmatrix}\begin{bmatrix} -en_x & -en_y \\ l_x & l_y \end{bmatrix}\begin{bmatrix} V_x^- \\ V_y^- \end{bmatrix} = \frac{1}{n_x l_y - n_y l_x}\begin{bmatrix} -en_x l_y - n_y l_x & -l_y e n_y - n_y l_y \\ e l_x n_x + n_x l_x & l_x e n_y + n_x l_y \end{bmatrix}\begin{bmatrix} V_x^- \\ V_y^- \end{bmatrix}$$

$$\begin{bmatrix} V_{0x}^+ \\ V_{0y}^+ \end{bmatrix} = -1\begin{bmatrix} +e s^2\alpha - c^2\alpha & -(1+e)s\alpha c\alpha \\ -(1+e)s\alpha c\alpha & e c^2\alpha - s^2\alpha \end{bmatrix}\begin{bmatrix} V_x^- \\ V_y^- \end{bmatrix} = \begin{bmatrix} c^2\alpha - e s^2\alpha & (1+e)s\alpha c\alpha \\ (1+e)s\alpha c\alpha & s^2\alpha - e c^2\alpha \end{bmatrix}\begin{bmatrix} V_x^- \\ V_y^- \end{bmatrix}$$

$t\big|_{y=y_{max}} = \frac{V_{1y}^+}{g} \qquad x = d = V_{1x}^+ \times \frac{V_{1y}^+}{g}$

$\underset{(2\,m)}{y_{max} = h_2} = -\frac{1}{2} g \left(\frac{V_{1y}^+}{g}\right)^2 + V_{1y}^+ \cdot \frac{V_{1y}^+}{g} = \frac{V_{1y}^{+2}}{2g}$

As $d = h_2 = 2\,m$,

$V_{1x}^+ \approx \frac{V_{1y}^+}{2}$ (or) $2V_{1x}^+ \approx V_{1y}^+$ (ignoring the ball radius)

$\Rightarrow 2(c^2\alpha - e s^2\alpha)\overset{V_0}{V_x^-} + 2(1+e)s\alpha c\alpha \overset{-\sqrt{2gh_1}}{V_y^-} = (1+e)s\alpha c\alpha \overset{V_0}{V_x^-} + (s^2\alpha - e c^2\alpha)\overset{-\sqrt{2gh_1}}{V_y^-}$

Let $e = 1$

$\Rightarrow 2(c\,2\alpha)V_0 - (1+e)\sqrt{2gh_1}\,s2\alpha = \frac{\cancel{(1+e)}}{\cancel{2}} s2\alpha\, V_0 + \sqrt{2gh_1}(c\,2\alpha)$

$c2\alpha(2V_0 - \sqrt{2gh_1}) = s2\alpha(V_0 + 2\sqrt{2gh_1})$

$\tan 2\alpha = \left(\frac{2V_0 - \sqrt{2gh_1}}{V_0 + 2\sqrt{2gh_1}}\right) \qquad \alpha = \frac{1}{2}\tan^{-1}\left[\frac{2V_0 - \sqrt{2gh_1}}{V_0 + 2\sqrt{2gh_1}}\right]$

Given $V_0 = 10\,\mathrm{m/s}$ and $h_1 = 3\,\mathrm{m}$ $\Rightarrow \alpha = 12.97°$

If $e = 0.9$, $\cos^2\alpha(2V_0 - e\sqrt{2gh_1}) + \sin^2\alpha(-2eV_0 + \sqrt{2gh_1}) - \sin\alpha\cos\alpha(1+e)(V_0 + 2\sqrt{2gh_1}) = 0$

$\Rightarrow \alpha = 14.45°$

KOKUYO Campus

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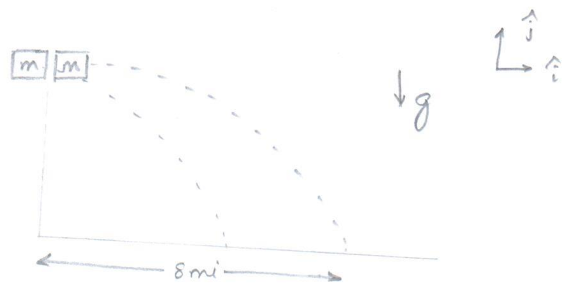
13.2.15 An airplane is flying steadily at an altitude of 30,000 ft at a speed of 500 mph. It explodes into two equal pieces. One piece is found to the right of the airplane's initial trajectory and 8 miles forward of the explosion point. Where should you look for the other piece? Assume the interaction impulse is in the horizontal plane and make the approximation that the two pieces fly in frictionless parabolic trajectories.

Answer:
4 miles forward of the explosion point.

12.2.14

$h = 30{,}000\ ft \equiv 5.5\ mi$

$g = 9.8\ m/s^2 \equiv 79380\ mi/hr^2$



From conservation of linear momentum,

Let mass of the Airplane be $2m$.

$$2m v \hat{i} = m v_1 \hat{i} + m v_2 \hat{i}$$

$$\therefore \quad 2v = v_1 + v_2 \longrightarrow (1)$$

Now, we know that the first half of the aircraft was found at 8 mi from the point of explosion then from the Equation of Projectiles we obtain

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5.5}{79380}} = 0.012\ hr$$

Range $= 8$ mi

$$\therefore \quad R_1 = v_1 t$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$8 = v_1 \times 0.012$

$v_1 = 666.6 \text{ mi/hr}$

We know that $v = 500 \text{ mi/hr}$

substituting in (1) we obtain

$2 \times 500 = 666.6 + v_2$

$v_2 = 333.4 \text{ mi/hr}$

The range of the second half of the airplane

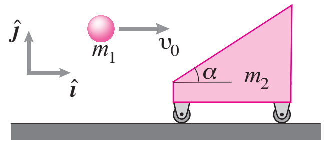
$R_2 = v_2 \times t$

$R_2 = 333.4 \times 0.012 \text{ hr}$

$R_2 = 4 \text{ mi}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

13.2.16 Consider the simultaneous collision of a ball with a ramp and the ramp with the ground. Consider the ball to be much more massive than the cart; $m_1 = 20\,\mathrm{kg}$ and $m_2 = 1\,\mathrm{kg}$. The angle of the inclined face is very shallow, $\alpha = 2°$. The ball hits the cart with the velocity $\vec{v}_1 = 50\,\mathrm{m/s}\hat{\imath}$. The impact of the ball with the ramp is elastic and frictionless. The ramp ends up moving in the $\hat{\imath}$ direction. Find the subsequent velocities of the ball and the cart using the two methods discussed in Sample **??** and Sample **??**. Comment on the answers you get. How will your answers change if you reversed the mass ratio?



Problem 13.16

Answer:
If the collision between the ball and the cart is considered first we get $v_{\text{cart}} = 0.11\,\mathrm{m/s}$ while if the collision between the cart and ground is considered first we get $v_{\text{cart}} = 2.26\,\mathrm{m/s}$. If the mass ratio is reversed we get $v_{\text{cart}} = 0.0055\,\mathrm{m/s}$ and $v_{\text{cart}} = 0.0058\,\mathrm{m/s}$ in the two cases respectively.

12.2.16 Given m_1 (ball mass) $= 20\,\mathrm{Kg}$

(a) m_2 (Cart mass) $= 1\,\mathrm{Kg}$

$\alpha = 2°$

$\underset{\sim}{v}_1^- = \underbrace{50\,\mathrm{m/s}}_{v_0}\,\hat{\imath}$

LMB = Conserved. $\left\{ m_1 \underset{\sim}{v}_1^- + m_2 \cancel{\underset{\sim}{v}_2^-} = m_1 \underset{\sim}{v}_1^+ + m_2 \underset{\sim}{v}_2^+ \right\} \cdot \hat{\imath}, \hat{\jmath}$

$\underset{\sim}{v}_1^+ = v_{1x}\hat{\imath} + v_{1y}\hat{\jmath}$

$\underset{\sim}{v}_2^+ = v_{2x}\hat{\imath} + v_{2y}\hat{\jmath}$

$\Rightarrow$ $m_1 v_0 = m_1 v_{1x} + m_2 v_{2x}$ ①

$0 = m_1 v_{1y} + m_2 v_{2y}$ ②

$\hat{\lambda} = \cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath}$

$\hat{n} = -\sin\alpha\,\hat{\imath} + \cos\alpha\,\hat{\jmath}$

$(\underset{\sim}{v}_2^+ - \underset{\sim}{v}_1^+)\cdot\hat{n} = e(\underset{\sim}{v}_1^- - \underset{\sim}{v}_2^-)\cdot\hat{n} \Rightarrow \left.\begin{array}{l} -(v_{2x}-v_{1x})\sin\alpha \\ +(v_{2y}-v_{1y})\cos\alpha \end{array}\right\} = -e v_0 \sin\alpha.$ —③

$\Rightarrow$ $-P_1\sin\alpha - m_1 v_{1x} + v_0 m_1 = 0$ —④

$P_1\cos\alpha - m_1 v_{1y} = 0.$ —⑤

$\underset{\sim}{P} = m_1(\underset{\sim}{v}_1^+ - \underset{\sim}{v}_1^-)$

$P_1\hat{n} = m_1(v_{1x}\hat{\imath} + v_{1y}\hat{\jmath} - v_0)$

$$\begin{bmatrix} m_1 & 0 & m_2 & 0 & 0 \\ 0 & m_1 & 0 & m_2 & 0 \\ \sin\alpha & -\cos\alpha & -\sin\alpha & \cos\alpha & 0 \\ -m_1 & 0 & 0 & 0 & -\sin\alpha \\ 0 & -m_1 & 0 & 0 & \cos\alpha \end{bmatrix} \begin{Bmatrix} v_{1x} \\ v_{1y} \\ v_{2x} \\ v_{2y} \\ P_1 \end{Bmatrix} = \begin{bmatrix} m_1 v_0 \\ 0 \\ -e v_0 \sin\alpha \\ -m_1 v_0 \\ 0 \end{bmatrix}$$

$\underset{\sim}{P}_2 = m_2(\underset{\sim}{v}_2^{++} - \underset{\sim}{v}_2^+)$

$P_2\hat{\jmath} = m_2\left[(v_{2x}^{++}\hat{\imath} + 0\hat{\jmath}) - v_{2x}\hat{\imath} - v_{2y}\hat{\jmath}\right]$

$\Rightarrow$ $v_{2x}^{++} = v_{2x}$

$v_{2x}^{++} = 0.11\,\mathrm{m/s}$

$P_2 = -m_2 v_{2y}$

Method 2

$\underset{\sim}{P}_2 = m_1 \underset{\sim}{v}_1^+ + m_2 \underset{\sim}{v}_2^+ - m_1 \underset{\sim}{v}_1^-$

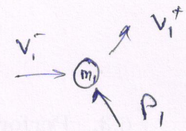
$P_2\hat{\jmath} = m_1 v_{1x}^+\hat{\imath} + m_1 v_{1y}^+\hat{\jmath} + m_2 v_{2x}\hat{\imath} + \cancel{m_2 v_{2y}}\hat{\jmath} - m_1 v_0\hat{\imath}$

(b) $m_1 v_{1x}^+ + m_2 v_{2x}^+ = m_1 v_0$

$m_1 v_{1y}^+ - P_2 = 0$

12.2.16 (b)

$$P_1 \hat{n} = m_1 (v_{1x}^+ \hat{i} + v_{1y}^+ \hat{j}) - m_1 v_0^- \hat{i}$$



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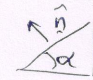
$$m_1 v_{1x}^+ + P_1 \sin\alpha = m_1 v_0$$

$$m_1 v_{1y}^+ - P_1 \cos\alpha = 0.$$



$\hat{n} = -\sin\alpha \hat{i} + \cos\alpha \hat{j}$

Using e $\quad v_0 \hat{i}$

$$(\underline{v}_1^+ - \underline{v}_2^+) \cdot \hat{n} = -e\,(\underline{v}_1^- - \cancel{\underline{v}_2^-}^{\,0}) \cdot \hat{n}$$

$$(v_{1x}^+ \hat{i} + v_{1y}^+ \hat{j} - v_{2x}^+ \hat{i}) \cdot \hat{n} = +e\, v_0 \sin\alpha$$

$$-v_{1x}^+ \sin\alpha + v_{2x}^+ \sin\alpha + v_{1y}^+ \cos\alpha = e\, v_0 \sin\alpha.$$

$$\Rightarrow \begin{bmatrix} m_1 & 0 & m_2 & 0 & 0 \\ 0 & m_1 & 0 & 0 & -1 \\ m_1 & 0 & 0 & \sin\alpha & 0 \\ 0 & m_1 & 0 & -\cos\alpha & 0 \\ -\sin\alpha & \cos\alpha & \sin\alpha & 0 & 0 \end{bmatrix} \begin{Bmatrix} v_{1x}^+ \\ v_{1y}^+ \\ v_{2x}^+ \\ P_1 \\ P_2 \end{Bmatrix} = \begin{bmatrix} m_1 v_0 \\ 0 \\ m_1 v_0 \\ 0 \\ e v_0 \sin\alpha \end{bmatrix}$$

(Heavier Ramp) $\longrightarrow$ Reversed m_r, $v_{2x}^{++} = 0.0055$ m/s & $v_{2x}^+ = 0.0058$ m/s. $\quad v_{2x}^+ = 2.26$ m/s.

$|\underline{v}| = \sqrt{a^2 + b^2}$

$|\underline{v}|^2 = (a^2 + b^2)$

12.2.16

Loss of energy. $\quad E_{K_2} - E_{K_1} = \Delta E_K = ?$

$$E_{K_2}^+ = \frac{1}{2} m_1 |\underline{v}_1^+|^2 + \frac{1}{2} m_2 |\underline{v}_2^+|^2 \quad \& \quad E_{K_1}^- = \frac{1}{2} m_1 |\underline{v}_1^-|^2 + \frac{1}{2} m_2 |\cancel{\underline{v}_2^-}^{\,0}|^2$$

$$E_{K_2}^+ = \frac{1}{2} m_1 (v_{1x}^{+2} + v_{1y}^{+2}) + \frac{1}{2} m_2 (v_{2x}^{+2} + v_{2y}^{+2})$$

$\underline{v}_1^- = 50$ m/s $\hat{i}$

$$E_{K_1}^- = \frac{1}{2} m_1 \times v_0^2 =$$

(or)

$$\Delta E_K = \frac{1}{2} m_{eff} \left[\underline{v}_{rel}^{+2} - \underline{v}_{rel}^{-2} \right]$$

$$\underline{v}_{rel}^+ = |v_1 - v_2| = (v_{1x} - v_{2x}) \hat{i} + v_{1y} \hat{j}$$

$$\underline{v}_{rel}^- = \underline{v}_1^- = v_0 \hat{i}$$

$$|v_{rel}^+|^2 = \{(v_{1x} - v_{2x})^2 + v_{1y}^2\} \qquad |v_{rel}^-|^2 = v_0^2$$

$$\Delta E_K =$$

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```
% Dynamics Solutions
% Chapter 12
% Problem: 12.2.15
% Method 1 Sequence of collisions
clear all
m1 = 1;          % mass of the ball in kg
m2 = 20;         % mass of the cart in kg
alpha = 2*pi/180;   % ramp angle in radian
v0 = 50;         % initial velocity of the ball m/s
e = 0.9;         % coefficient of restitution
M = [m1, 0, m2, 0, 0;...
    0, m1, 0, m2, 0;...
    sin(alpha), -cos(alpha), -sin(alpha), cos(alpha), 0;...
    -m1, 0, 0, 0, -sin(alpha);...
    0, -m1, 0, 0, cos(alpha)]
P = [m1*v0; 0; -e*v0*sin(alpha); -m1*v0; 0]
Vp = M\P
P2 = -m2*Vp(4,1)
% Speed of ramp in x direction after second collision
V2xpp = Vp(3,1)
%
%% Method 2 Simultaneous collisions
Mm = [m1, 0, m2, 0, 0;...
    0, m1, 0, 0, -1;...
    m1, 0, 0, sin(alpha), 0;...
    0, m1, 0, -cos(alpha), 0;...
    -sin(alpha), cos(alpha), sin(alpha), 0, 0]
Pp = [m1*v0; 0; m1*v0; 0; e*v0*sin(alpha)]
Vv = Mm\Pp;
v2vx = Vv(3,1)
Pp2 = Vv(5)
```


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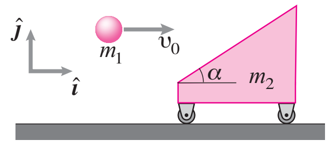
13.2.17 Consider the simultaneous collisions problem of problem 13.2.16 again. Now assume that $m_1 = 5\,\text{kg}$, $m_2 = 10\,\text{kg}$, $\vec{v}_1 = 50\,\text{m/s}$, $e = 0.75$, and the angle $\alpha = 88°$. What is the net loss of energy in the impacts as calculated the two different ways?



Problem 13.17

Answer:
Energy lost is 1829.3 J by one method while it is 1824.9 J by another method.

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```
% Dynamics Solutions
% Chapter 12
% Problem: 12.2.15
% Method 1 Sequence of collisions
clear all
m1 = 5;         % mass of the ball in kg
m2 = 10;        % mass of the cart in kg
alpha = 88*pi/180;   % ramp angle in radian
v0 = 50;        % initial velocity of the ball m/s
e = 0.75;        % coefficient of restitution
M = [m1, 0, m2, 0, 0;...
    0, m1, 0, m2, 0;...
    sin(alpha), -cos(alpha), -sin(alpha), cos(alpha), 0;...
    -m1, 0, 0, 0, -sin(alpha);...
    0, -m1, 0, 0, cos(alpha)]
P = [m1*v0; 0; -e*v0*sin(alpha); -m1*v0; 0]
Vp = M\P
P2 = -m2*Vp(4,1)
% Speed of ramp in x direction after second collision
V2xpp = Vp(3,1)
%
%% Method 2 Simultaneous collisions
Mm = [m1, 0, m2, 0, 0;...
    0, m1, 0, 0, -1;...
    m1, 0, 0, sin(alpha), 0;...
    0, m1, 0, -cos(alpha), 0;...
    -sin(alpha), cos(alpha), sin(alpha), 0, 0]
Pp = [m1*v0; 0; m1*v0; 0; e*v0*sin(alpha)]
Vv = Mm\Pp;
v2vx = Vv(3,1)
Pp2 = Vv(5)
%% Energy loss
Ek11 = 1/2*m1*v0^2 % initial energy in J
Ek21 = 1/2*m1*(Vp(1)^2+Vp(2)^2)+1/2*m2*(Vp(3)^2+Vp(4)^2)
El1 = Ek11-Ek21
Ek12 = 1/2*m1*v0^2 % initial energy in J
Ek22 = 1/2*m1*(Vv(1)^2+Vv(2)^2)+1/2*m2*(Vv(3)^2)
El2 = Ek12-Ek22
meff = m1*m2/(m1+m2);

dEk1 = 1/2*meff*((Vp(1)-Vp(3))^2+Vp(2)^2-v0^2)
dEk2 = 1/2*meff*((Vv(1)-Vv(3))^2+Vv(2)^2-v0^2)
```