13.2 Collisions and explosions

Preparatory Problems

13.2.1 Assuming θ, v_0, and e to be known quantities, write the following equations in matrix form set up to solve for v'_{Ax} and v'_{Ay}:

$$\sin\theta v'_{Ax} + \cos\theta v'_{Ay} = e v_0 \cos\theta$$
$$\cos\theta v'_{Ax} - \sin\theta v'_{Ay} = v_0 \sin\theta.$$

*

13.2.2 The equation $(\vec{v}'_1 - \vec{v}'_2)\cdot\hat{n} = e(\vec{v}_2 - \vec{v}_1)\cdot\hat{n}$ relates relative velocities of two point masses before and after frictionless impact in the normal direction $\hat{n}$ of the impact. If $\vec{v}'_1 = v_{1_x}\hat{\imath} + v_{1_y}\hat{\jmath}$, $\vec{v}'_2 = -v_0\hat{\imath}$, $e = 0.5$, $\vec{v}_2 = \vec{0}$, $\vec{v}_1 = 2\,\text{ft/s}\hat{\imath} - 5\,\text{ft/s}\hat{\jmath}$, and $\hat{n} = \frac{1}{\sqrt{2}}(\hat{\imath} + \hat{\jmath})$, find the scalar equation relating the velocities in the normal direction. *

13.2.3 The following three equations are obtained by applying the principle of conservation of linear momentum on some system.

$$m_0 v_0 = 24\,\text{m/s}\, m_A - .67 m_B v_B - .58 m_C v_C$$
$$0 = 36\,\text{m/s}\, m_A + .33 m_B v_B + 0.3 m_C v_C$$
$$0 = 23\,\text{m/s}\, m_A - .67 m_B v_B - .58 m_C v_C.$$

Assume v_0, v_B, and v_C are the only unknowns. Write the equations in matrix form set up to solve for the unknowns. *

13.2.4 The following three equations are obtained to solve for v'_{Ax}, v'_{Ay}, and v'_{Bx}:

$$(v'_{Bx} - v'_{Ax})\cos\theta = v'_{Ay}\sin\theta - 10\,\text{m/s}$$
$$v'_{Ax}\sin\theta = v'_{Ay}\cos\theta - 36\,\text{m/s}$$
$$m_B v'_{Bx} + m_A v'_{Ax} = (-60\,\text{m/s})\, m_A.$$

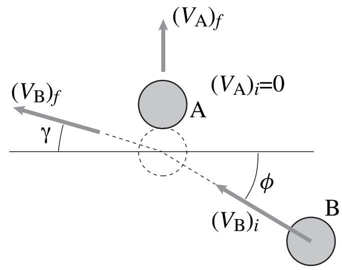
Set up these equations in matrix form. *

13.2.5 Solve for the unknowns v'_{A_x}, v'_{A_y}, and v'_{B_x} in problem 13.2.4 taking $\theta = 50^\circ$, $m_A = 1.5\,m_B$ and $m_B = 0.8\,\text{kg}$. Use any computer program. *

13.2.6 Using the matrix form of equations in Problem 13.2.1, solve for v'_{Ax} and v'_{Ay} if $\theta = 20^\circ$ and $v_0 = 5\,\text{ft/s}$. *

More-Involved Problems

13.2.7 Two frictionless equal-mass pucks sliding on a plane collide as shown below. Puck A is initially at rest. Given that $(V_B)_i = 1.0\,\text{m/s}$, $(V_A)_i = 0$, and $(V_A)_f = 0.5\,\text{m/s}$, find the approach angle ϕ and rebound angle γ. The coefficient of restitution is $e = 0.9$.



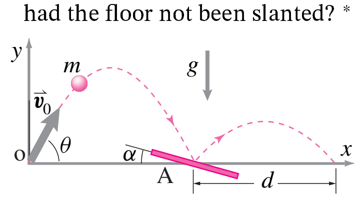
Problem 13.2.7

13.2.8 Reconsider problem 13.2.7. Given instead that $\gamma = 30^\circ$, $(V_A)_i = 0$, and $(V_A)_f = 0.5\,\text{m/s}$, find the initial velocity of puck B.

13.2.9 A ball of mass $m = 0.5\,\text{kg}$ is thrown up in the air with initial speed $v_0 = 50\,\text{m/s}$ at an angle $\theta = 60^\circ$. The ball lands on and bounces off a slanted floor that makes an angle $\alpha = 15^\circ$ with the horizontal. Assume the collision with the floor to be elastic and ignore air drag on the ball.

a) Find the impulse of the collision of the ball, *

b) After bouncing off the slanted floor, how much horizontal distance does the ball travel before landing on the ground again? Is this distance more, less, or the same as it would have travelled had the floor not been slanted? *



Problem 13.2.9

13.2.10 Solve the general two-particle frictionless collision problem. For example, write computer code that has lines like this near the start :

`m1=3; m2=19`	Set values of masses
`v1zero=[10 20]`	Initial velocity of mass 1
`v2zero=[-5 3]`	Initial velocity of mass 2
`e=.5`	Set coefficient of restitution
`theta=pi/4`	Angle that the normal to contact plane makes, measured CCW from +x axis, in radians

Your program (function, code, script) should calculate the impulse of mass 1 on mass 2, and the velocities of the two masses after the collision. Your program should assume consistent units for all quantities.

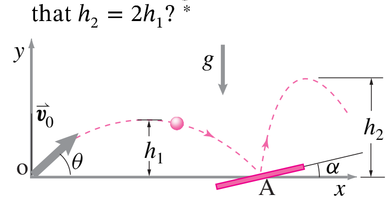
a) You should demonstrate that your program works by solving at least 4 different problems for which you can check your answer by simple pencil-and-paper calculations. These problems should have as much variety as possible. Sketch these problems clearly, show their analytic solution, and show that the computer agrees. *

b) Solve the problem given in the sample text given in the initial problem statement.

13.2.11 A projectile is launched at $\theta = 40^\circ$ with speed $v_0 = 25\,\text{m/s}$. The projectile lands on a steel plate that can be adjusted to make any angle α with the horizontal. The projectile bounces off the steel plate without losing any energy. The projectile is required to reach a height after rebound twice as much as it did during its flight before hitting the plate. Ignore air resistance.

a) Find the required angle α of the plate. *

b) Can you always find some α for any launch angle $\theta < \pi/2$ such that $h_2 = 2h_1$? *

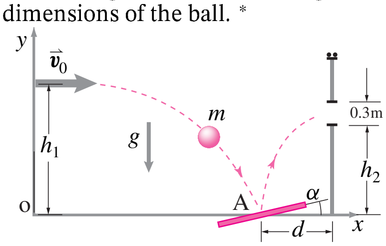


Problem 13.2.11

13.2.12 Two equal mass cars approach an intersection at right angles. They crash and stick together. One of the cars was going at 30 mph before the crash. The other car's path gets deflected by 15°. How fast was it going? *

13.2.13 A ball m is thrown horizontally at height h and speed v_0. It then has a sequence of bounces on the horizontal ground. Treating each collision as frictionless with restitution coefficient e how far has the ball travelled horizontally when it just finishes bouncing? Answer in terms of some or all of m, g, h, v_0 and e. *

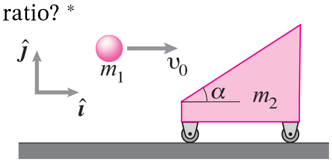
13.2.14 A game involves using a pedal to direct a falling ball into a fixed vertical slot by simply rotating the pedal when the ball hits the pedal. A model of this game is shown in the figure. The ball is thrown horizontally with an initial speed $v = 10\,\mathrm{m/s}$ from a height $h_1 = 3\,\mathrm{m}$. The pedal is located at $d = 2\,\mathrm{m}$ from the wall that houses the slot at height $h_2 = 2\,\mathrm{m}$. The slot itself is 0.3 m in extent. The coefficient of restitution between the pedal and the ball is $e = 0.9$. The air resistance is negligible. Find the angle α or the range of this angle, so that the ball makes it through the slot. You can ignore the dimensions of the ball. *



Problem 13.2.14

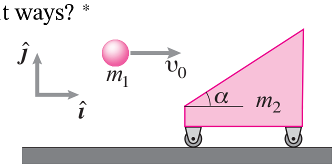
13.2.15 An airplane is flying steadily at an altitude of 30,000 ft at a speed of 500 mph. It explodes into two equal pieces. One piece is found to the right of the airplane's initial trajectory and 8 miles forward of the explosion point. Where should you look for the other piece? Assume the interaction impulse is in the horizontal plane and make the approximation that the two pieces fly in frictionless parabolic trajectories. *

13.2.16 Consider the simultaneous collision of a ball with a ramp and the ramp with the ground. Consider the ball to be much more massive than the cart; $m_1 = 20\,\mathrm{kg}$ and $m_2 = 1\,\mathrm{kg}$. The angle of the inclined face is very shallow, $\alpha = 2°$. The ball hits the cart with the velocity $\vec{v}_1 = 50\,\mathrm{m/s}\hat{\imath}$. The impact of the ball with the ramp is elastic and frictionless. The ramp ends up moving in the $\hat{\imath}$ direction. Find the subsequent velocities of the ball and the cart using the two methods discussed in Sample 13.4 and Sample 13.5. Comment on the answers you get. How will your answers change if you reversed the mass ratio? *



Problem 13.2.16

13.2.17 Consider the simultaneous collisions problem of problem 13.2.16 again. Now assume that $m_1 = 5\,\mathrm{kg}$, $m_2 = 10\,\mathrm{kg}$, $\vec{v}_1 = 50\,\mathrm{m/s}$, $e = 0.75$, and the angle $\alpha = 88°$. What is the net loss of energy in the impacts as calculated the two different ways? *



Problem 13.2.17