

Experts note that these problems do not use polar coordinates or any other fancy coordinate systems. Such descriptions come later in the text. At this point we want to lay out the basic equations and the qualitative features that can be found by numerical integration of the equations using Cartesian (xyz) coordinates.

12.3.5 An intercontinental missile, modelled as a particle, is launched on a ballistic trajectory from the surface of the earth. The force on the missile from the earth's gravity is $F = mgR^2/r^2$ and is directed towards the center of the earth. When it is launched from the equator it has speed v_0 and in the direction shown, 45° from horizontal (both measured relative to a Newtonian reference frame). For the purposes of this calculation ignore the earth's rotation. You can think of this problem as two-dimensional in the plane shown. If you need numbers, use the following values:

$m = 1000 \text{ kg} = \text{missile mass}$
 $g = 10 \text{ m/s}^2 \text{ at the earth's surface,}$
 $R = 6,400,000 \text{ m} = \text{earth's radius,}$
 and

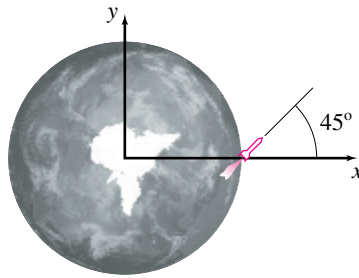
$v_0 = 9000 \text{ m/s.}$

The distance of the missile from the center of the earth is $r(t)$.

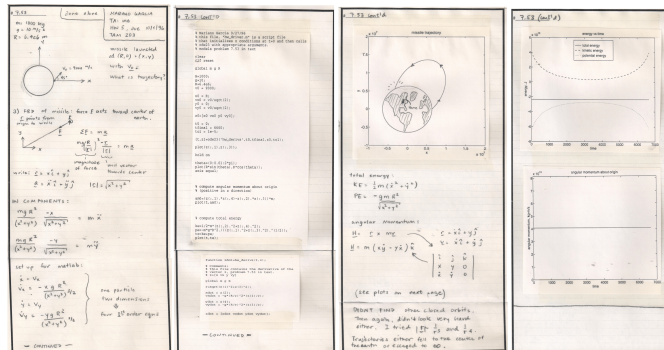
- a) Draw a free-body diagram of the missile. Write the linear momentum balance equation. Break this equation into x and y components.

ponents. Rewrite these equations as a system of 4 first order ODE's suitable for computer solution. Write appropriate initial conditions for the ODE's.

- b) Using the computer (or any other means) plot the trajectory of the rocket after it is launched for a time of 6670 seconds. [Hint: use a much shorter time when debugging your program.] On the same plot draw a (round) circle for the earth.



Problem 12.5: An intercontinental ballistic missile launch.



Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

Problem 12.3.5

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12.3.5 An Intercontinental Missile

An intercontinental missile, modelled as a particle, is launched on a ballistic trajectory from the surface of the earth. The force on the missile from the earth's gravity is $F = mgR^2/r^2$ and is directed towards the center of the earth. When it is launched from the equator it has speed v_0 and in the direction shown in Fig. 12.3.1(a), 45° from horizontal (both measured relative to a Newtonian reference frame). For the purposes of this calculation ignore the earth's rotation. You can think of this problem as two-dimensional in the plane shown. If you need numbers, use the following values:

- $m = 1000$ kg as the missile mass,
- $g = 10$ m/s² at the earth's surface,
- $R = 6,400,000$ m as the earth's radius, and
- $v_0 = 9000$ m/s.

The distance of the missile from the center of the earth is $r(t)$.

- (a) Draw a free-body diagram of the missile. Write the linear momentum balance equation. Break this equation into x and y components. Rewrite these equations as a system of 4 first-order ODEs suitable for computer solution. Write appropriate initial conditions for the ODEs.
- (b) Using the computer (or any other means) plot the trajectory of the rocket after it is launched for a time of 6670 seconds. [Hint: use a much shorter time when debugging your program.] On the same plot draw a (round) circle for the earth.

Part a

The free body diagram of the rocket is in Fig. 12.3.1(b). The direction of the gravitational force $\mathbf{F}$ is always $-\hat{\mathbf{r}}$, the unit vector pointing towards the origin (earth's center). The linear momentum balance equation is then

$$m\mathbf{a} = -F(r)\hat{\mathbf{r}} = -\frac{mgR^2}{r^2}\hat{\mathbf{r}}. \quad (12.3.1)$$

See that m cancels – our results will now be independent of the rocket's mass. Using the definition of the vector

$$r\hat{\mathbf{r}} = \mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix}, \quad (12.3.2)$$

¹Problem 12.3.5 from: Introduction to Statics and Dynamics, Andy Ruina and Rudra Pratap, 2019.

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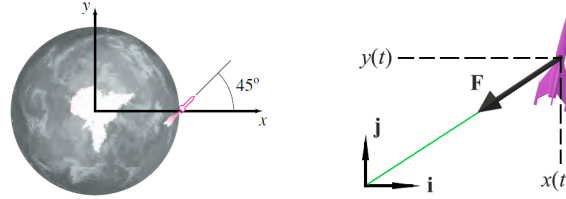


Figure 12.3.1: (a) Intercontinental ballistic missile launch. (b) Free body diagram.

we can then write the linear momentum balance equations as

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = -\frac{mgR^2}{r^3} \begin{pmatrix} x \\ y \end{pmatrix}. \quad (12.3.3)$$

Note that $r = \sqrt{x^2 + y^2}$. Using $v_x \equiv \dot{x}$, and $v_y \equiv \dot{y}$, our first-order form is

$$\frac{d}{dt} \begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ -mgR^2 x / (x^2 + y^2)^{(3/2)} \\ -mgR^2 y / (x^2 + y^2)^{(3/2)} \end{pmatrix}. \quad (12.3.4)$$

The initial conditions given are summed up as (where $\theta = 45^\circ$)

$$x(0) = R, \quad y(0) = 0, \quad v_x(0) = v_0 \cos \theta, \quad v_y(0) = v_0 \sin \theta. \quad (12.3.5)$$

Part b

The trajectory, $x(t)$ versus $y(t)$, of the rocket is shown in Fig. [12.3.2](#). The requested span of time $t \in [0, 6670]$ was utilized, and the earth is shown. The results are scaled by the radius of the earth $R = 6,400,000$ m, so the trajectory shown effectively has units of earth-radii. The MATLAB code used is as follows:

```
% Parameters
p.g = 1e1; % m/s^2
p.R = 6.4e6; % m
v_0 = 9e3; % m/s
theta_0 = deg2rad(45); % radians
t_max = 6.67e3; % s

% Initial conditions
u_0 = [p.R, 0, v_0*cos(theta_0), v_0*sin(theta_0)];

% Solve for the trajectory in time, unpack results
```

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Problem 12.3.5

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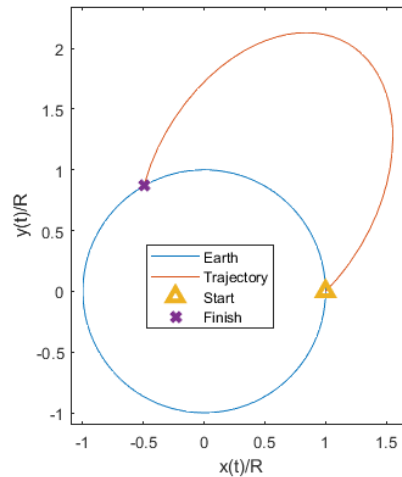


Figure 12.3.2: The trajectory of the rocket after it is launched for a time of 6670 seconds.

```

[t, state] = ode45(@(t, state) ode_fun(t, state, p), [0, t_max], u_0);
x = state(:,1)/p.R;
y = state(:,2)/p.R;

% Plot the results
figure
phi = linspace(0, 2*pi, 100);
plot(cos(phi), sin(phi))
axis equal
hold on
plot(x, y)
plot(x(1), y(1), '^', 'markersize', 8, 'linewidth', 3)
plot(x(end), y(end), 'x', 'markersize', 8, 'linewidth', 3)
axis(1.1*[min([-1; x]), max([1; x]), min([-1; y]), max([1; y])])
]
xlabel('x(t)/R')
ylabel('y(t)/R')
legend('Earth', 'Trajectory', 'Start', 'Finish')

% System of first-order differential equations
function f_dot = ode_fun(~, f, p)

```

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```
% Unpack the state variables
x = f(1);
y = f(2);
v_x = f(3);
v_y = f(4);

% Equations of motion
x_dot = v_x;
y_dot = v_y;
v_x_dot = -p.g*p.R^2*x/(x^2 + y^2)^(3/2);
v_y_dot = -p.g*p.R^2*y/(x^2 + y^2)^(3/2);

% Output evolution of the state
f_dot = [x_dot; y_dot; v_x_dot; v_y_dot];

end
```

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TAM 203
Homework Solutions

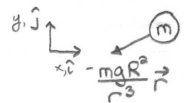
Due 3/27/08

10.61

Given: $F = mgR^2/r^2$, V_0 , 45° from horizontal (θ)
 $m = 1000 \text{ kg}$, $R = 6.4 \times 10^6 \text{ m}$, $V_0 = 9000 \text{ m/s}$

a) Free body diagram:

Assume $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$



$$\Sigma \vec{F} = m\vec{a}, \quad -\frac{mgR^2}{r^3} \vec{r} = m\vec{a}$$

$$-\frac{mgR^2}{(x^2+y^2)^{3/2}} (x\hat{i} + y\hat{j}) = m\ddot{x}\hat{i} + m\ddot{y}\hat{j}$$

In component form,

$$\ddot{x} + \frac{gR^2}{(x^2+y^2)^{3/2}} x = 0 \quad \text{AND} \quad \ddot{y} + \frac{gR^2}{(x^2+y^2)^{3/2}} y = 0$$

Differential equations:

$$\begin{aligned} (1) \quad \dot{x} &= v_x & (3) \quad \dot{y} &= v_y \\ (2) \quad \dot{v}_x &= \frac{-gR^2}{(x^2+y^2)^{3/2}} x & (4) \quad \dot{v}_y &= \frac{-gR^2}{(x^2+y^2)^{3/2}} y \end{aligned}$$

Initial conditions:

$$\begin{aligned} (1) \quad x(0) &= R & (3) \quad y(0) &= 0 \\ (2) \quad v_x(0) &= V_0 \cos \theta & (4) \quad v_y(0) &= V_0 \sin \theta \end{aligned}$$

b) See Matlab code and plot on pages 2-3.

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10.61b – Matlab code

```

function Probl061()
% Problem 10.61 Solution
% March 27, 2008

% VARIABLES (Assume consistent units)
% r = displacement vector [x,y]
% v = velocity vector = dr/dt [vx,vy]

m= 1000;      % Mass of satellite (kg)
R= 6400000;   % Radius of Earth (m)
g= 9.81;      % Gravity acceleration (m/s^2)
v0= 9000;     % Initial velocity (m/s)
theta= 45;    % Launch angle (degrees)

% INITIAL CONDITIONS
x0= R;
y0= 0;
vx0= v0*cosd(theta);
vy0= v0*sind(theta);
z0= [x0 y0 vx0 vy0]'; % pack variables

tspan= [0 6670]; % seconds

[t zarray]= ode45(@rhs,tspan,z0,[],m,R,g);

% Unpack Variables
x= zarray(:,1);
y= zarray(:,2);

plot(x,y,'r--');
title('Plot of Earth and Satellite Orbit')
xlabel('x [m]')
ylabel('y [m]')
axis(1000000*[-8 15 -8 15])
hold on;

% Draw the Earth
t= 0:pi/100:2*pi;
ex= R*cos(t);
ey= R*sin(t);
plot(ex,ey,'b');

end

% THE DIFFERENTIAL EQUATION 'The Right Hand Side'
function zdot = rhs(t,z,m,R,g)

% Unpack variables
x= z(1);
y= z(2);
vx= z(3);
vy= z(4);

```

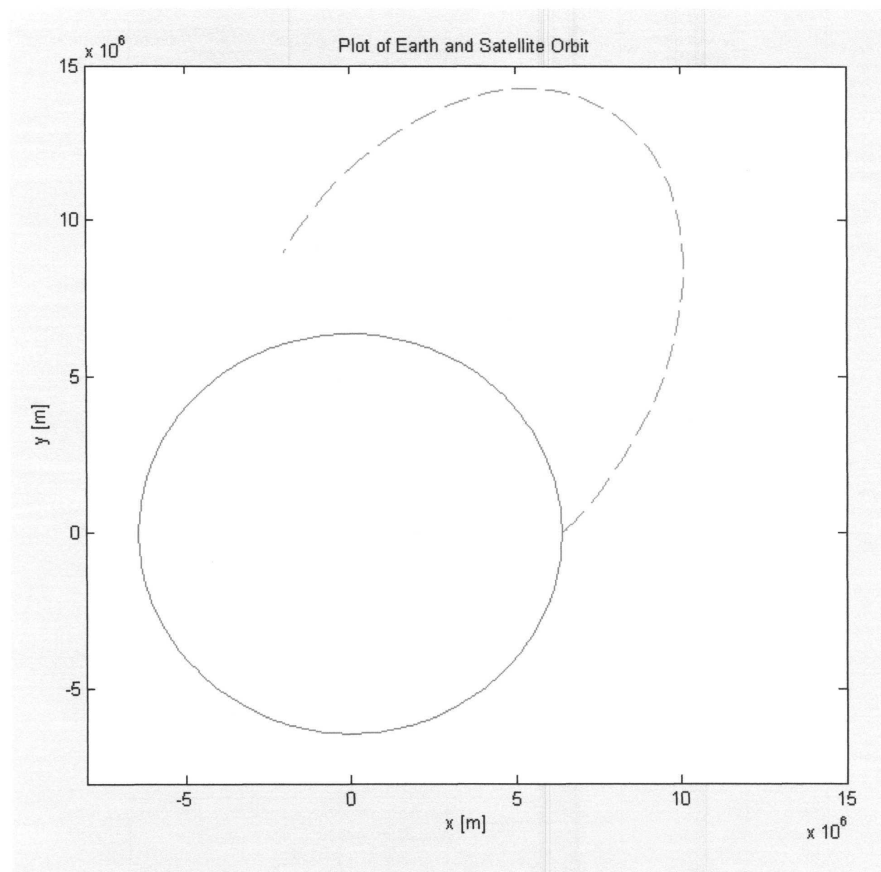
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```
% The equations
xdot= vx;
vxdot= -g*R^2/(x^2+y^2)^(3/2)*x;
ydot= vy;
vydot= -g*R^2/(x^2+y^2)^(3/2)*y;

% Pack the rate of change of x,y,vx and vy
zdot= [xdot ydot vxdot vydot]';

end
```

10.61b – Satellite Orbit Plot



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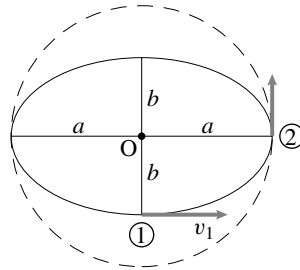
12.3.6 A particle of mass 2 kg moves in the horizontal xy -plane under the influence of a central force $\vec{F} = -k\vec{r}$ (attractive force proportional to distance from the origin), where $k = 200 \text{ N/m}$ and $\vec{r}$ is the position of the particle relative to the force center. Neglect all other forces.

- a) Show that circular trajectories are possible, and determine the relation between speed v and circular radius r_0 which must hold on a circular trajectory. [hint: Write $\vec{F} = m\vec{a}$, break into x and y components, solve the separate scalar equations, pick fortuitous values for the free constants in your solutions.]
- b) It turns out that trajectories are in general elliptical, as depicted in the diagram.

For a particular elliptical trajectory with $a = 1 \text{ m}$ and $b = 0.8 \text{ m}$, the velocity of the particle at point

1 is observed to be perpendicular to the radial direction, with magnitude v_1 , as shown. When the particle reaches point 2, its velocity is again perpendicular to the radial direction.

Determine the speed increment Δv which would have to be added (instantaneously) to the particle's speed at point 2 to transfer it to the circular trajectory through point 2 (the dotted curve).



Problem 12.6

11.3.6

$$\sum \vec{F} = m \vec{a}$$

$$-K\vec{r} = m \ddot{\vec{r}}$$

$$\text{Let } \vec{r} = x\hat{i} + y\hat{j}$$

$$\ddot{\vec{r}} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

$$-K(x\hat{i} + y\hat{j}) = m(\ddot{x}\hat{i} + \ddot{y}\hat{j})$$

$$m\ddot{x} = -Kx \quad \& \quad m\ddot{y} = -Ky$$

$$\text{If } |\vec{r}| = \text{constant} \quad \text{Let } r = a \cos \omega t$$

$$\text{Let } x = A \sin \lambda t + B \cos \lambda t = C_1 \sin(\lambda t + C_2)$$

$$y = C \sin \lambda t + D \cos \lambda t = C_3 \sin(\lambda t + C_4)$$

$$\begin{cases} \ddot{x} = -\omega^2 x \sin \omega t \\ \ddot{y} = -\omega^2 y \cos \omega t = -\omega^2 r \\ \ddot{\vec{r}} = -\left(\frac{K}{m}\right) \vec{r} \end{cases}$$

$$\text{If initial conditions are chosen such that } C_2 = 0, C_4 = \frac{\pi}{2} \quad \& \quad C_1 = \pm C_3$$

Then x, y trace a circle.

$$x = r_0 \sin \lambda t \quad \& \quad y = r_0 \cos \lambda t$$

$$r_0 = \sqrt{x^2 + y^2} = r$$

$$\begin{aligned} \dot{x} &= r_0 \lambda \cos \lambda t \\ \dot{y} &= -r_0 \lambda \sin \lambda t \end{aligned}$$

$$\text{speed, } v = \sqrt{\dot{x}^2 + \dot{y}^2} = r_0 \lambda \Rightarrow v = r_0 \sqrt{K/m}$$

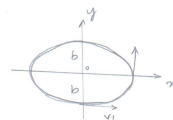
$$\text{b) } a = 1 \text{ m}, \quad b = 0.8 \text{ m}$$

$$x = A \sin \lambda t + B \cos \lambda t$$

$$v|_{x=0} = v_1$$

$$\text{At } t=0 \text{ say } x=0 \quad \& \quad y=-b$$

$$B=0 \quad \& \quad y = C \sin \lambda t + D \cos \lambda t \Rightarrow D = -b$$



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(2)

$$\dot{x} = A\lambda \cos \lambda t - B\lambda \sin \lambda t, \quad \dot{y} = C\lambda \cos \lambda t - D\lambda \sin \lambda t$$

$$t=0$$

$$\dot{x}=v_1, \quad v_1 = A\lambda \quad \& \quad 0 = C\lambda \Rightarrow C=0, D=-b; \quad A = \frac{v_1}{\lambda} \quad \& \quad B=0$$

$$\therefore x = \frac{v_1}{\lambda} \sin \lambda t \quad \& \quad y = -b \cos \lambda t \quad \} \quad \dot{x} = v_1 \cos \lambda t \quad \& \quad \dot{y} = b\lambda \sin \lambda t$$

$$\lambda t = \frac{\pi}{2} \Rightarrow v = \dot{y} = b\lambda$$

$$\Delta V_{\text{required}} = (a-b)\lambda \quad \text{or} \quad \Delta V = (a-b) \sqrt{\frac{k}{m}}$$

12.3.7 Circular motion. Generally when people talk about central force motion they not only mean that the only force is directed at the origin but that the magnitude of the force only depends on the distance from the origin. Thus in 2D

$$\vec{F} = \underbrace{-\frac{x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}}}_{\text{Unit vector}} \cdot \underbrace{F(\sqrt{x^2 + y^2})}_{\text{Magnitude of force}}$$

where the scalar function $F(r)$ expresses the dependence of the central attractive force on distance $r = \sqrt{x^2 + y^2}$. Consider a particle with mass m on a candidate circular orbit

$$\vec{r} = R \cos \lambda t \hat{i} + R \sin \lambda t \hat{j}$$

with constant speed $v = |\vec{v}| = |\dot{\vec{r}}| = R\lambda$. For each of the cases below find the

speed v for circular motion at radius R . Find this by plugging the circular motion equation into $\vec{F} = m\vec{a}$ using the form of $F(r)$ given. Answer in terms of other constants given (e.g., k, m, M, G)

- $F(r) = kr$ (zero-rest-length attractive spring)
- $F(r) = GMm/r^2$ (inverse-square gravitational attraction)
- $F(r) = r^n$ (arbitrary power law attraction)
- $F(r) = F(r)$ (arbitrary function). In this case you need to find how the speed depends on F in general.
- Can you find a function $F(r)$ for which there are two or more circular orbits at the same speed v ?

11.3.7

$$\left. \begin{aligned} \underline{x} &= R[\cos \lambda t \hat{i} + \sin \lambda t \hat{j}] \\ \dot{\underline{x}} &= R\lambda[-\sin \lambda t \hat{i} + \cos \lambda t \hat{j}] \\ \ddot{\underline{x}} &= R\lambda^2[-\cos \lambda t \hat{i} - \sin \lambda t \hat{j}] \end{aligned} \right\} \ddot{\underline{x}} = -\lambda^2 \underline{x}$$

$$\underline{F} = m\ddot{\underline{x}}$$

$$a) \underline{F} = \frac{-\underline{x}}{|\underline{x}|} \cdot F(r) = \frac{-\underline{x}}{|\underline{x}|} \cdot Kx = -K\frac{\underline{x}}{|\underline{x}|} = m\ddot{\underline{x}} = -m\lambda^2 \underline{x}$$

$$\Rightarrow \lambda^2 = \frac{K}{m} \quad \therefore \text{velocity } v = r\sqrt{\frac{K}{m}}$$

$$b) F(r) = \frac{GMm}{r^2} \Rightarrow \underline{F} = \frac{-\underline{x}}{|\underline{x}|} \cdot \frac{GMm}{r^2} = -\frac{GMm}{r^3} \cdot \underline{x} = -m\lambda^2 \underline{x}$$

$$\text{As } r = R$$

$$\Rightarrow \lambda^2 = \frac{GM}{R^3} \quad \therefore v = r\sqrt{\frac{GM}{R^3}}$$

$$v = \sqrt{\frac{GM}{R}} \quad [\because GM = gR^2]$$

$$v = \sqrt{\frac{gR^2}{R}}$$

$$c) F(r) = r^n \Rightarrow \underline{F} = \frac{-\underline{x}}{|\underline{x}|} = -\underline{x} r^{n-1} = -m\lambda^2 \underline{x} \Rightarrow \lambda^2 = \frac{r^{n-1}}{m}$$

$$\therefore v = r\sqrt{\frac{r^{n-1}}{m}} \Rightarrow v = \sqrt{\frac{r^{n+1}}{m}}$$

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$$d) \quad F(r) = -\frac{\gamma}{|r|} \cdot F(r) = -m\lambda^2 r \Rightarrow \lambda^2 = \frac{F(r)}{mR}$$

$$\therefore V = R \sqrt{\frac{F(r)}{mR}} \Rightarrow V = \sqrt{\frac{F(r) \cdot R}{m}}$$

$$e) \quad F(r) = \frac{1}{R} \quad \text{If } F(r) = \frac{1}{r} \quad V = \sqrt{\frac{1}{m}}$$

12.3.8 Circular motion, numerical solution. For each of the cases in problem 12.3.7 pick values for the physical constants. Then pick initial conditions which, according to theory, should give circular orbits. Then numerically solve

the 4 coupled first order ODEs that describe planar motion, make a plot, and show that you do indeed get circular orbits. How big is the discrepancy between your numerical solution and an exact circle?

12.3.8

a) $\underline{F} = m \underline{a}$

$$\frac{-\underline{r}}{|\underline{r}|} \cdot F(r) = m \ddot{\underline{r}}$$

$$m \ddot{\underline{r}} = -k \underline{r}$$

$$m \ddot{x} = -kx \quad \& \quad m \ddot{y} = -ky$$

Let

$$z_1 = x$$

$$z_2 = \dot{x}$$

$$z_3 = y$$

$$z_4 = \dot{y}$$

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = -\frac{k}{m} z_1$$

$$\dot{z}_3 = z_4$$

$$\dot{z}_4 = -\frac{k}{m} z_3$$

b) $\underline{F} = m \underline{a}$

$$\frac{-\underline{r}}{|\underline{r}|} \cdot F(r) = m \ddot{\underline{r}}$$

$$\frac{-\underline{r}}{r^3} \cdot G M m = m \ddot{\underline{r}}$$

$$m \ddot{x} = -\frac{G M m}{r^3} x$$

$$m \ddot{y} = -\frac{G M m}{r^3} y$$

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = -\frac{G M}{(\sqrt{z_1^2 + z_3^2})^3} z_1$$

$$\dot{z}_3 = z_4$$

$$\dot{z}_4 = -\frac{G M}{(\sqrt{z_1^2 + z_3^2})^3} z_3$$

c) $F(r) = r^n$

$$-\underline{r} r^{n+1} = m \ddot{\underline{r}}$$

$$-(x \hat{i} + y \hat{j}) (\sqrt{x^2 + y^2})^{n+1} = m (\ddot{x} \hat{i} + \ddot{y} \hat{j})$$

$$m \ddot{x} = -(\sqrt{x^2 + y^2})^{n+1} x$$

$$m \ddot{y} = -(\sqrt{x^2 + y^2})^{n+1} y$$

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = -(\sqrt{z_1^2 + z_3^2})^{n+1} \frac{z_1}{m}$$

$$\dot{z}_3 = z_4$$

$$\dot{z}_4 = -(\sqrt{z_1^2 + z_3^2})^{n+1} \frac{z_3}{m}$$

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d) $F(r) = \text{Arbitrary}$

Let $F(r) = e^{-r}$

$$\frac{-r}{r} \cdot e^{-r} = m\ddot{r} = -m\lambda^2 r \Rightarrow \lambda^2 = \frac{e^{-r}}{mr}$$

$$V = r\lambda \Rightarrow V = r \sqrt{\frac{e^{-r}}{mr}} \Rightarrow V = \sqrt{\frac{r^2 e^{-r}}{m}}$$

$$-(x\hat{i} + y\hat{j}) \cdot \frac{e^{-(\sqrt{x^2+y^2})}}{\sqrt{x^2+y^2}} = m\ddot{x}\hat{i} + m\ddot{y}\hat{j}$$

$$\ddot{x} = - \frac{e^{-\sqrt{x^2+y^2}}}{m\sqrt{x^2+y^2}} \quad \left| \quad \ddot{y} = - \frac{e^{-\sqrt{x^2+y^2}}}{m\sqrt{x^2+y^2}} \cdot y$$

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = - \frac{e^{-\sqrt{z_1^2+z_3^2}}}{m\sqrt{z_1^2+z_3^2}} \cdot z_1$$

$$\dot{z}_3 = z_4$$

$$\dot{z}_4 = - \frac{e^{-\sqrt{z_1^2+z_3^2}}}{m\sqrt{z_1^2+z_3^2}} \cdot z_3$$

e) $F(r) = 1/r$

$$\frac{-r}{r} \cdot \frac{1}{r} = m\ddot{r} \Rightarrow -m\lambda^2 r \Rightarrow \lambda^2 = \frac{1}{mr^2}$$

$$V = r \sqrt{\frac{1}{mr^2}} = \frac{1}{\sqrt{m}}$$

$$-\frac{(x\hat{i} + y\hat{j})}{x^2+y^2} = m\ddot{x}\hat{i} + m\ddot{y}\hat{j}$$

$$\ddot{x} = - \frac{x}{(x^2+y^2)m}, \quad \ddot{y} = - \frac{y}{(x^2+y^2)m}$$

12.3.9 Two equal mass satellites have circular orbits at two different radii. The one that is closer to the earth has smaller potential energy and bigger kinetic energy. Which satellite has bigger total energy?

③

11.3.9

$$V_1 = \sqrt{\frac{GM}{R_1}} \quad , \quad V_2 = \sqrt{\frac{GM}{R_2}}$$

$$TE_1 = \frac{1}{2} m \frac{GM}{R_1} - \frac{GMm}{R_1} = -\frac{GMm}{2R_1}$$

$$TE_2 = -\frac{GMm}{2R_2}$$

$$\frac{TE_2}{TE_1} = \frac{R_1}{R_2}$$

$$\therefore TE_1 = \frac{R_2}{R_1} TE_2$$

$$TE_1 > TE_2$$

12.3.10 Find initial conditions corresponding to circular motion for a central force problem and simulate this motion on the computer. Use any central force attraction law you like (e.g., zero-length spring, inverse square,...) Check that you get closed circular orbits by plotting several revolutions. Now, in your simulation, apply a slight drag force opposing motion $\vec{F} = -c\vec{v}$. Pick a value for c so that the orbit slowly spirals in (say, less

than 10% per orbit).

- Make a plot of the spiraling orbit.
- Plot the speed $|\vec{v}|$ vs time as it spirals in.
- How is it that a drag force causes the satellite to speed up? Is that numerical error? An approximation in our formulation of the governing equations? A relativistic effect? What?

11.3.10

$$\text{Let } F(r) = Kr$$

$$\sum \vec{F} = m\vec{a}$$

$$-\frac{r}{|r|} \cdot K|r| = -c\vec{v} = m\vec{a}$$

$$-K(x\hat{i} + y\hat{j}) - c(\dot{x}\hat{i} + \dot{y}\hat{j}) = m(\ddot{x}\hat{i} + \ddot{y}\hat{j})$$

$$\ddot{x} = -\frac{c}{m}\dot{x} - \frac{K}{m}x \quad ; \quad \ddot{y} = -\frac{c}{m}\dot{y} - \frac{K}{m}y$$

$$\dot{z}_1 = z_2 \quad , \quad \dot{z}_3 = z_4$$

$$\dot{z}_2 = -\frac{c}{m}z_2 - \frac{K}{m}z_1 \quad ; \quad \dot{z}_4 = -\frac{c}{m}z_4 - \frac{K}{m}z_3$$