12.2.2 Does angular momentum depend on reference point? (Assume that all candidate points are fixed in the same Newtonian reference frame.)

11.2.2

No,

$\vec{L} = \frac{d}{dt}(m\vec{v})$, is the same with respect to any point in the same Newtonian reference frame in which $\vec{v}$ is measured.

12.2.4 What is the relation between the dynamics 'Linear Momentum Balance' equation and the statics 'Force Balance' equation?

11.2.4

$$\vec{F}_{net} = \sum \vec{F} = \frac{d}{dt}(\vec{L}) = \dot{\vec{L}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

12.2.5 What is the relation between the dynamics 'Angular Momentum Balance' equation and the statics 'Moment Balance' equation?

11.2.5

$$\vec{M}_{net/C} = \frac{d}{dt}\left(\vec{H}_{/C}\right) = \dot{\vec{H}}_{/C}$$

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12.2.6 A ball of mass $m = 0.1\,\text{kg}$ is thrown from a height of $h = 10\,\text{m}$ above the ground with velocity $\vec{v} = 120\,\text{km/h}\hat{\imath} - 120\,\text{km/h}\hat{\jmath}$. What is the kinetic energy of the ball at its release?

11.2.6

$m = 0.1$ Kg;

$h = 10$ m

$\vec{V} = 120 \frac{km}{hr} \hat{i} - 120\ Km/hr\ \hat{j}$

$\vec{V} = \frac{100}{3} [\hat{i} - \hat{j}]$ m/sec

$KE = \frac{1}{2} m\vec{V}\cdot\vec{V} = \frac{0.1}{2}\left[\left(\frac{100}{3}\right)^2 + \left(\frac{100}{3}\right)^2\right]$ Joules

$K.E = 111.11$ Joules

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12.2.7 A ball of mass $m = 0.2\,\text{kg}$ is thrown from a height of $h = 20\,\text{m}$ above the ground with velocity $\vec{v} = 120\,\text{km/h}\hat{\imath} - 120\,\text{km/h}\hat{\jmath} - 10\,\text{km/h}\hat{k}$. What is the kinetic energy of the ball at its release?

11·2·7

$m = 0.2$ kg , $h = 20$ m

$\vec{V} = (120\hat{i} - 120\hat{j} - 10\hat{K})$ Km/hr

$KE = \frac{1}{2} m \, \vec{V}\cdot\vec{V} = 0.1 \left[V_x^2 + V_y^2 + V_z^2 \right]$

$= 0.1 \left[\left(\frac{1000}{3}\right)^2 + \left(\frac{100}{3}\right)^2 + (2.77)^2 \right]$

$K.E = 222.989$ Joules.

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12.2.8 How do you calculate P, the power of all external forces acting on a particle, from the forces $\vec{F}_i$ and the velocity $\vec{v}$ of the particle?

11.2.8

$$\text{Power} = \sum_{i=1}^{n_1} \frac{d}{dt}(\vec{F}_i \cdot \vec{dr}) = \sum_{i=1}^{n_1} \vec{F}_i \cdot \vec{V}$$

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12.2.9 A particle A has velocity $\vec{v}_A$ and mass m_A. A particle B has velocity $\vec{v}_B = 2\,\vec{v}_A$ and mass equal to the other $m_B = m_A$. What is the relationship between:

a) $\vec{L}_{\mathrm{A}}$ and $\vec{L}_{\mathrm{B}}$,

b) $\vec{H}_{\mathrm{A/C}}$ and $\vec{H}_{\mathrm{B/C}}$, and

c) E_{KA} and E_{KB}?

$$m_B = m_A \quad , \quad \vec{V}_B = 2\vec{V}_A$$

a) $$\vec{L}_B = m_B\vec{V}_B = 2m_A\vec{V}_A = 2\vec{L}_A$$

$$\vec{L}_B = 2\vec{L}_A$$

b) $$\vec{H}_{B/C} = \vec{r}_{B/C} \times \vec{L}_B = 2\vec{r}_{B/C} \times \vec{L}_A$$

$$\vec{H}_{A/C} = \vec{r}_{A/C} \times \vec{L}_A$$

If $\vec{r}_{B/C} = \vec{r}_{A/C}$

then,

$$\vec{H}_{B/C} = 2\,\vec{H}_{A/C}$$

c) $$E_{KA} = \frac{1}{2} m_A \vec{V}_A \cdot \vec{V}_A \quad , \quad E_{KB} = \frac{1}{2} m_B \vec{V}_B \cdot \vec{V}_B$$

$$E_{KB} = \frac{1}{2} m_B \vec{V}_B \cdot \vec{V}_B = \frac{1}{2} m_A (2\vec{V}_A)\cdot(2\vec{V}_A) = 4E_{KA}$$

$$E_{KB} = 4E_{KA}$$

12.2.10 A bullet of mass 50 g travels with a velocity $\vec{v} = 0.8\,\mathrm{km/s}\hat{\imath} + 0.6\,\mathrm{km/s}\hat{\jmath}$. (a) What is the linear momentum of the bullet? (Answer in consistent units.)

$m = 50$ grams

$\vec{V} = [0.8\hat{i} + 0.6\hat{j}]$ km/sec

Linear momentum of the Bullet

$\vec{L} = m\vec{v} = \frac{50}{1000}\left[0.8\hat{i} + 0.6\hat{j}\right]$ km/sec

$\vec{L} = [40\hat{i} + 30\hat{j}]$ kg m/sec

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12.2.11 A particle has position $\vec{\boldsymbol{r}} = 4\,\text{m}\hat{\boldsymbol{\imath}} + 7\,\text{m}\hat{\boldsymbol{\jmath}}$, velocity $\vec{\boldsymbol{v}} = 6\,\text{m/s}\hat{\boldsymbol{\imath}} - 3\,\text{m/s}\hat{\boldsymbol{\jmath}}$, and acceleration $\vec{\boldsymbol{a}} = -2\,\text{m/s}^2\hat{\boldsymbol{\imath}} + 9\,\text{m/s}^2\hat{\boldsymbol{\jmath}}$. For each position of a point P defined below, find $\vec{\boldsymbol{H}}_P$, the angular momentum of the particle with respect to the point P.

a) $\vec{\boldsymbol{r}}_P = 4\,\text{m}\hat{\boldsymbol{\imath}} + 7\,\text{m}\hat{\boldsymbol{\jmath}}$,

b) $\vec{\boldsymbol{r}}_P = -2\,\text{m}\hat{\boldsymbol{\imath}} + 7\,\text{m}\hat{\boldsymbol{\jmath}}$, and

c) $\vec{\boldsymbol{r}}_P = 0\,\text{m}\hat{\boldsymbol{\imath}} + 7\,\text{m}\hat{\boldsymbol{\jmath}}$,

d) $\vec{\boldsymbol{r}}_P = \vec{\boldsymbol{0}}$

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11.2.11

$\vec{r} = (4\hat{i} + 7\hat{j})\,m$

$\vec{v} = (6\hat{i} - 3\hat{j})\,m/sec$

$\vec{a} = (-2\hat{i} + 9\hat{j})\,m/sec^2$

Say mass = m

$\vec{H}_{/P} = \vec{r}_{/P} \times \vec{L} = m\,\vec{r}_{/P} \times \vec{v}$

a) $\vec{r}_P = (4\hat{i} + 7\hat{j})\,m$

$\vec{H}_{/P} = m(4\hat{i} + 7\hat{j}) \times (6\hat{i} - 3\hat{j})\ kg\,m^2/sec$

$= m(-12 - 42)\hat{k}\ kg\,m^2/sec$

$\vec{H}_{/P} = -54m\,\hat{k}\ kg\,m^2/sec$

b) $\vec{r}_P = (-2\hat{i} + 7\hat{j})\,m$

$\vec{H}_{/P} = m(-2\hat{i} + 7\hat{j}) \times (6\hat{i} - 3\hat{j})\ kg\,m^2/sec$

$= m(6 - 42)\hat{k}\ kg\,m^2/sec$

$\vec{H}_{/P} = -36m\,\hat{k}\ kg\,m^2/sec$

c) $\vec{H}_{/P} = m(7\hat{j}) \times (6\hat{i} - 3\hat{j})\ kg\,m^2/sec$

$\vec{H}_{/P} = -42m\,\hat{k}\ kg\,m^2/sec$

d) $\vec{H}_{/P} = m(\vec{0}) \times (6\hat{i} - 3\hat{j})$

$\vec{H}_{/P} = 0\ kg\,m^2/sec$

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12.2.12 The position vector of a particle of mass 1 kg at an instant t is $\vec{r} = 2\,\text{m}\hat{\imath} - 0.5\,\text{m}\hat{\jmath}$. If the velocity of the particle at this instant is $\vec{v} = -4\,\text{m/s}\hat{\imath} + 3\,\text{m/s}\hat{\jmath}$, compute (a) the linear momentum $\vec{L} = m\vec{v}$ and (b) the angular momentum ($\vec{H}_{/O} = \vec{r}_{/O} \times (m\vec{v})$).

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11.2.12

$m = 1\,kg$, $\vec{r} = (2\hat{i} - 0.5\hat{j})$

$\vec{V} = -4\hat{i} + 3\hat{j}$

a) Linear Momentum,

$\vec{L} = m\vec{v} = (-4\hat{i} + 3\hat{j})$ Kg m/sec

b) $\vec{H}_{/o} = m\vec{r} \times \vec{V}$

$= (2\hat{i} - 0.5\hat{j}) \times (-4\hat{i} + 3\hat{j})$ Kg m²/sec²

$= (6-2)\hat{k}$ Kg m²/sec

$\vec{H}_{/o} = 4\hat{k}$ Kg m²/sec

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12.2.13 The position of a particle of mass $m = 0.5\,\text{kg}$ is $\vec{\boldsymbol{r}}(t) = \ell\,\sin(\omega t)\hat{\boldsymbol{\imath}} + h\,\hat{\boldsymbol{\jmath}}$; where $\omega = 2\,\text{rad/s}, h = 2\,\text{m}, \ell = 2\,\text{m}$, and $\vec{\boldsymbol{r}}$ is measured from the origin.

a) Find the kinetic energy of the particle at $t = 0\,\text{s}$ and $t = 5\,\text{s}$.

b) Find the rate of change of kinetic energy at $t = 0\,\text{s}$ and $t = 5\,\text{s}$.

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11.2.13

$m = 0.5\,\text{kg}$

$\vec{r}(t) = \ell \sin\omega t\,\hat{i} + h\,\hat{j}$

$\vec{V}(t) = \dfrac{d\vec{r}}{dt} = \omega \ell \cos\omega t\,\hat{i}$

$\omega = 2\,\text{rad/sec}$

$h = 2\,\text{m}$

$\ell = 2\,\text{m}$

a) $E_K = \dfrac{1}{2} m\,\vec{V}\cdot\vec{V}$

$= \dfrac{1}{2} m(\omega\ell\cos\omega t)^2 = \dfrac{1}{2} m\omega^2\ell^2\cos^2\omega t = 4\cos^2\omega t$

at $t = 0$

$E_K = 4$ Joules

at $t = 5$ sec

$E_K = 4\left[\cos(2\times5)\right]^2$

$E_K = 2.8161$ Joules

b) $\dot{E}_K = \dfrac{d}{dt}\left[\dfrac{1}{2} m\omega^2\ell^2\cos^2\omega t\right]$

$= 4\dfrac{d}{dt}\cos^2\omega t = -8\cos\omega t\sin\omega t = -4\sin 2\omega t$

at $t = 0$

$\dot{E}_K = 0$ Joules/sec

at $t = 5$ sec

$\dot{E}_K = -4\sin(20)$

$\dot{E}_K = -3.6517$ Joules/sec

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12.2.14 For a particle

$$E_K = \frac{1}{2}\, m\, v^2.$$

Why does it follow that $\dot{E}_K = m\, \vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{a}}$? [hint: write v^2 as $\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{v}}$ and then use the product rule of differentiation.]

$$E_K = \frac{1}{2} m\, \vec{v} \cdot \vec{v}$$

$$\dot{E}_K = \frac{d}{dt}\left(\frac{1}{2} m\, \vec{v} \cdot \vec{v}\right)$$

$$= \frac{1}{2} m \left[\vec{v} \cdot \frac{d\vec{v}}{dt} + \frac{d\vec{v}}{dt} \cdot \vec{v}\right]$$

$$= m \left[\vec{v} \cdot \frac{d\vec{v}}{dt}\right]$$

$$\dot{E}_K = m\vec{v} \cdot \vec{a}$$

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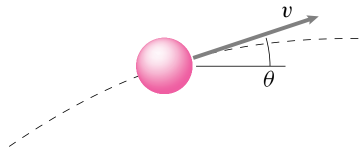
12.2.15 Consider a projectile of mass m at some instant in time t during its flight. Let $\vec{v}$ be the velocity of the projectile at this instant (see the figure). In addition to the force of gravity, a drag force acts on the projectile. The drag force is proportional to the square of the speed (speed$\equiv |\vec{v}| = v$) and acts in the opposite direction. Find an expression for the net power of these forces ($P = \sum \vec{F} \cdot \vec{v}$) on the particle.



Problem 12.15

11·2·15

$\vec{F}_D = -bv^2\hat{e}_t$ $\quad \vec{F}_g = -mg\hat{j}$

Say $\vec{v} = v_x\hat{i} + v_y\hat{j}$

$F_D = -bv^2\hat{e}_t = -bv^2\frac{\vec{v}}{v} = -bv\vec{v}$, $\quad v = \sqrt{v_x^2 + v_y^2}$

Net Power of force

$$P = \sum \vec{F}\cdot\vec{v}$$
$$= -(bv)\,\vec{v}\cdot\vec{v} - mg\hat{j}\cdot\vec{v}$$
$$= -bv^3 - mgv_y$$
$$= -bv^3 - mg(v\sin\theta)$$

$$P = -mgv\sin\theta - bv^3$$

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12.2.16 A 10 gm wad of paper is tossed into the air. At some instant, the position, velocity, and acceleration of its center of mass are $\vec{r} = 3\,\mathrm{m}\hat{\imath} + 3\,\mathrm{m}\hat{\jmath} + 6\,\mathrm{m}\hat{k}$, $\vec{v} = -9\,\mathrm{m/s}\hat{\imath} + 24\,\mathrm{m/s}\hat{\jmath} + 30\,\mathrm{m/s}\hat{k}$, and $\vec{a} = -10\,\mathrm{m/s^2}\hat{\imath} + 24\,\mathrm{m/s^2}\hat{\jmath} + 32\,\mathrm{m/s^2}\hat{k}$, respectively. What is the translational kinetic energy of the wad at the instant of interest?

$m = 10$ gms

$E_k = \frac{1}{2} m \vec{V}\cdot\vec{V} = \frac{1}{2}\left(\frac{10}{1000}\right)\left(9^2 + 24^2 + 30^2\right)$ Joules

$E_k = 7.785$ Joules

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

12.2.17 A 2 kg particle moves so that its position $\vec{r}$ is given by

$$\vec{r}(t) = [5\sin(at)\hat{\imath} + bt^2\hat{\jmath} + ct\hat{k}]\text{ meters}$$

where a $= \pi/\sec$, $b = .25/\sec^2$, $c = 2/\sec$.

a) What is the linear momentum of the particle at $t = 1$ sec?

b) What is the force acting on the particle at $t = 1$ sec?

$m = 2\text{ kg}$

$\vec{r} = [5\sin at\,\hat{i} + bt^2\hat{j} + ct\hat{k}]$

$\vec{V} = 5a\cos at\,\hat{i} + 2bt\,\hat{j} + c\,\hat{k}$

$\vec{a} = -5a^2\sin at\,\hat{i} + 2b\,\hat{j}$

a) $\vec{L}\big|_{t=1} = m\vec{v}\big|_{t=1} = 2[5\pi\cos\pi + \cancel{2\cos}\ 2(0.25)\hat{j} + 2\hat{k}]$

$= 2[-15.708\,\hat{i} + 0.5\,\hat{j} + 2\hat{k}]$

$\vec{L}\big|_{t=1} = -10\pi\hat{i} + \hat{j} + 4\hat{k}$

b) $\vec{F}\big|_{t=1} = m\vec{a}\big|_{t=1}$

$= 2[-5\pi^2\sin\pi\,\hat{i} + 0.5\,\hat{j}]$

$\vec{F} = 1\,\hat{j}\ N$

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12.2.18 A particle A has mass m_A and velocity $\vec{v}_A$. A particle B at the same location has mass $m_B = 2\ m_A$ and velocity equal to the other $\vec{v}_B = \vec{v}_A$. Point C is a reference point. What is the relationship between:

a) $\vec{L}_A$ and $\vec{L}_B$,

b) $\vec{H}_{A/C}$ and $\vec{H}_{B/C}$, and

c) E_{K_A} and E_{K_B}?

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11-2.18

$m_B = 2 m_a$

$\vec{V}_B = \vec{V}_A$

$\vec{r}_B = \vec{r}_A$

(i) $\vec{L}_B = m_B \vec{V}_B = 2 m_A \vec{V}_A = 2\vec{L}_A$

$\vec{L}_B = 2\vec{L}_A$

(ii) $\vec{H}_{B/C} = \vec{r}_{B/C} \times \vec{L}_B = \vec{r}_{A/C} \times 2\vec{L}_A$

$\vec{H}_{B/C} = 2\vec{H}_{A/C}$

(iii) $E_{KB} = \frac{1}{2} m_B \vec{V}_B \cdot \vec{V}_B = 2\left(\frac{1}{2} m_A \cdot \vec{V}_A \cdot \vec{V}_A\right) = 2E_{KA}$

$E_{KB} = 2E_{KA}$

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12.2.19 A particle of mass $m = 3\,\text{kg}$ moves in space. Its position, velocity, and acceleration at a given time are $\vec{r} = 2\,\text{m}\hat{\imath} + 3\,\text{m}\hat{\jmath} + 5\,\text{m}\hat{k}$, $\vec{v} = -3\,\text{m/s}\hat{\imath} + 8\,\text{m/s}\hat{\jmath} + 10\,\text{m/s}\hat{k}$, and $\vec{a} = -5\,\text{m/s}^2\hat{\imath} + 12\,\text{m/s}^2\hat{\jmath} + 16\,\text{m/s}^2\hat{k}$, respectively. For this particle at the instant of interest, find its:

a) linear momentum $\vec{L}$,

b) rate of change of linear momentum $\dot{\vec{L}}$,

c) angular momentum about the origin $\vec{H}_{/O}$,

d) rate of change of angular momentum about the origin $\dot{\vec{H}}_{/O}$,

e) kinetic energy E_K, and

f) rate of change of kinetic energy $\dot{E}_K$.

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11.2.19

$m = 3\,kg$

a) $\vec{L} = m\vec{v}$
$= 3[-3\hat{i} + 8\hat{j} + 10\hat{k}]$ kg m/sec
$\vec{L} = [-9\hat{i} + 24\hat{j} + 30\hat{k}]$ Kgm/sec

b) $\dot{\vec{L}} = m\frac{d\vec{v}}{dt} = m\vec{a}$
$= 3[-5\hat{i} + 12\hat{j} + 16\hat{k}]$ Kg m/sec^2
$\dot{\vec{L}} = [-15\hat{i} + 36\hat{j} + 48\hat{k}]$ Kg m/sec^2

c) $\vec{H}_{/o} = \vec{r}_{/o} \times m\vec{v}$
$= [2\hat{i} + 3\hat{j} + 5\hat{k}] \times [-9\hat{i} + 24\hat{j} + 30\hat{k}]$ Kg m^2/sec
$= [(90-120)\hat{i} + (-60-45)\hat{j} + (48+27)\hat{k}]$ Kg m^2/sec
$\vec{H}_{/o} = [-30\hat{i} - 105\hat{j} + 75\hat{k}]$ Kg m^2/sec

d) $\dot{\vec{H}}_{/o} = \vec{r}_{/o} \times m\vec{a}$
$= [2\hat{i} + 3\hat{j} + 5\hat{k}] \times [-15\hat{i} + 36\hat{j} + 48\hat{k}]$ Kg m^2/sec^2
$\dot{\vec{H}}_{/o} = [-36\hat{i} - 171\hat{j} + 117\hat{k}]$ Kg m^2/sec^2

e) $E_K = \frac{1}{2} m\vec{v}\cdot\vec{v} = \frac{3}{2}[3^2 + 8^2 + 10^2] = 256.6$ Joules

f) $\dot{E}_K = m\vec{a}\cdot\vec{v} = [-15\hat{i} + 36\hat{j} + 48\hat{k}]\cdot[3\hat{i} + 8\hat{k} + 10\hat{j}]$
$= 45 + 288 + 480 = 813$ Joules/sec

#24 Solution Michelle deSouza

Monday, March 8, 2021 1:31 PM

$\boxed{12.2.19}$

Given:

- mass $m = 3\ kg$
- $\vec{r} = 2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}\ m$
- $\vec{v} = -3\hat{\imath} + 8\hat{\jmath} + 10\hat{k}\ m/s$
- $\vec{a} = -5\hat{\imath} + 12\hat{\jmath} + 16\hat{k}\ m/s^2$

Solve at this instant in time for:

a) linear momentum $\vec{L}$

$$\vec{L} = m\vec{v}$$
$$= 3\,kg\,(-3\hat{\imath} + 8\hat{\jmath} + 10\hat{k})\ m/s$$
$$\vec{L} = -9\hat{\imath} + 24\hat{\jmath} + 30\hat{k}\quad kg\cdot m/s$$

b) rate of change of linear momentum $\dot{\vec{L}}$

$$\dot{\vec{L}} = \Sigma\vec{F} = m\vec{a}$$
$$= 3\,kg\,(-5\hat{\imath} + 12\hat{\jmath} + 16\hat{k})\ m/s^2$$
$$\dot{\vec{L}} = -15\hat{\imath} + 36\hat{\jmath} + 48\hat{k}\quad N \qquad 1\ \frac{kg\cdot m}{s^2} = 1\ N$$

c) angular momentum about origin $\vec{H}_{/O}$

$$\vec{H}_{/O} = \vec{r}\times m\vec{v}$$
$$\vec{H}_{/O} = (2\hat{\imath} + 3\hat{\jmath} + 5\hat{k})\,m \times 3\,kg\,(-3\hat{\imath} + 8\hat{\jmath} + 10\hat{k})\ m/s$$
$$\vec{r}\times\vec{v} = \begin{matrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 3 & 5\\ -3 & 8 & 10\end{matrix} \rightarrow \begin{vmatrix}3 & 5\\ 8 & 10\end{vmatrix}\hat{\imath} - \begin{vmatrix}2 & 5\\ -3 & 10\end{vmatrix}\hat{\jmath} + \begin{vmatrix}2 & 3\\ -3 & 8\end{vmatrix}\hat{k}$$
$$= [3(10) - 8(5)]\hat{\imath} - [2(10) - (-3)(5)]\hat{\jmath} + [2(8) - (-3)(3)]\hat{k}$$
$$\vec{r}\times\vec{v} = -10\hat{\imath} - 35\hat{\jmath} + 25\hat{k}$$
$$\vec{H}_{/O} = -30\hat{\imath} - 105\hat{\jmath} + 75\hat{k}\quad kg\cdot m^2/s$$

d) rate of change of angular momentum about origin $\dot{\vec{H}}_{/O}$

$$\dot{\vec{H}}_{/O} = \vec{r}\times m\vec{a}$$
$$= (2\hat{\imath} + 3\hat{\jmath} + 5\hat{k})\,m \times 3\,kg\,(-5\hat{\imath} + 12\hat{\jmath} + 16\hat{k})\ m/s^2$$
$$\dot{\vec{H}}_{/O} = -36\hat{\imath} - 171\hat{\jmath} + 117\hat{k}\quad N\cdot m$$

e) kinetic energy E_K

$$E_K = \tfrac{1}{2}m\,\vec{v}\cdot\vec{v}$$
$$= \tfrac{1}{2}(3\,kg)(-3\hat{\imath} + 8\hat{\jmath} + 10\hat{k}\ m/s)\cdot(-3\hat{\imath} + 8\hat{\jmath} + 10\hat{k}\ m/s)$$
$$E_K = 259.5\ J \qquad 1\ J = 1\ \frac{kg\cdot m}{s^2}\cdot m$$

f) rate of change of kinetic energy $\dot{E}_K$

$$\dot{E}_K = \cancel{\tfrac{1}{2}}m\vec{a}\cdot\vec{v}$$

edit 03/15:
incorrect factor of 1/2 here

$$= \cancel{\tfrac{1}{2}}(3\,kg)(-5\hat{\imath} + 12\hat{\jmath} + 16\hat{k}\ m/s^2)\cdot(-3\hat{\imath} + 8\hat{\jmath} + 10\hat{k}\ m/s)$$
$$= \cancel{406.5}\ W \qquad 1\ W = 1\ J/s$$

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Derivations

$$\vec{F} = m\vec{a} \qquad \vec{a} = \tfrac{d}{dt}(\vec{v}),\ m = \text{const.}$$
$$\vec{F} = \tfrac{d}{dt}(m\vec{v}) \qquad m\vec{v} = \vec{L}$$
$$\vec{F} = \dot{\vec{L}} \qquad \vec{F} = \text{force sum}$$
$$\vec{L} = \text{linear momentum}$$

$$\vec{F} = m\vec{a} \qquad \vec{r}\times\{\}$$
$$\vec{r}\times\vec{F} = \underbrace{\vec{r}\times m\vec{a}} \longrightarrow \text{try } \tfrac{d}{dt}(\vec{r}\times m\vec{v})$$
$$= \tfrac{d\vec{r}}{dt}\times m\vec{v} + \vec{r}\times m\tfrac{d\vec{v}}{dt}$$
$$\vec{r}\times\vec{F} = \tfrac{d}{dt}(\vec{r}\times m\vec{v}) \qquad = \cancel{\vec{v}\times m\vec{v}} + \vec{r}\times m\vec{a}$$
$$\vec{M} = \tfrac{d}{dt}(\vec{H}_{/O}) \qquad \vec{M} = \text{moment sum} \qquad = \vec{r}\times m\vec{a}\ \checkmark$$
$$\vec{H}_{/O} = \text{angular momentum}$$

only works if $\frac{d\vec{r}}{dt} = \vec{v}$ and $\frac{d\vec{v}}{dt} = \vec{a}$ aka $\vec{r}$, $\vec{v}$, and $\vec{a}$ are defined correctly relative to each other — e.g. if $\vec{r}$ defined relative to still point and $\vec{v}$ defined relative to moving point, AMB does not work!

$$\vec{F} = m\vec{a} \qquad \{\}\cdot\vec{v}$$
$$\vec{F}\cdot\vec{v} = \underline{m\vec{a}\cdot\vec{v}} \longrightarrow \text{try } \tfrac{d}{dt}(\tfrac{1}{2}m\vec{v}\cdot\vec{v})$$
$$\vec{F}\cdot\vec{v} = \tfrac{d}{dt}(\tfrac{1}{2}m\vec{v}\cdot\vec{v}) \qquad \vec{v}\cdot\vec{v} = v^2 \qquad = \tfrac{1}{2}m\tfrac{d\vec{v}}{dt}\cdot\vec{v} + \tfrac{1}{2}m\vec{v}\cdot\tfrac{d\vec{v}}{dt}$$
$$P = \tfrac{d}{dt}(\tfrac{1}{2}mv^2) \qquad P = \text{power} \qquad = \tfrac{1}{2}m\vec{a}\cdot\vec{v} + \tfrac{1}{2}m\vec{v}\cdot\vec{a}$$
$$P = \dot{E}_K \qquad E_K = \text{kinetic energy} \qquad = m\vec{a}\cdot\vec{v}\ \checkmark$$

Similar warning applies! E.g. if $\vec{a}$ is defined relative to an accelerating point, this derivation does not work

12.2.20 A particle has position $\vec{r} = 3\,\text{m}\hat{\imath} - 2\,\text{m}\hat{\jmath} + 4\,\text{m}\hat{k}$, velocity $\vec{v} = 2\,\text{m/s}\hat{\imath} - 3\,\text{m/s}\hat{\jmath} + 7\,\text{m/s}\hat{k}$, and acceleration $\vec{a} = 1\,\text{m/s}^2\hat{\imath} - 8\,\text{m/s}^2\hat{\jmath} + 3\,\text{m/s}^2\hat{k}$. For each position of a point P defined below, find the rate of change of angular momentum, $\dot{\vec{H}}_P$, of the particle with respect to the point P.

a) $\vec{r}_P = 3\,\text{m}\hat{\imath} - 2\,\text{m}\hat{\jmath} + 4\,\text{m}\hat{k}$,

b) $\vec{r}_P = 6\,\text{m}\hat{\imath} - 4\,\text{m}\hat{\jmath} + 8\,\text{m}\hat{k}$,

c) $\vec{r}_P = -9\,\text{m}\hat{\imath} + 6\,\text{m}\hat{\jmath} - 12\,\text{m}\hat{k}$, and

d) $\vec{r}_P = \vec{0}$

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11.2.20

$$\dot{\vec{H}}_{/P} = \vec{r}_{/P} \times m\vec{a}$$

$$= (\vec{r} - \vec{r}_P) \times m\vec{a}$$

a) $m\left\{(3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}) - (3\hat{\imath} - 2\hat{\jmath} + 4\hat{k})\right\} \times [\hat{\imath} - 8\hat{\jmath} + 3\hat{k}]$

$$= m\,[0 \times (\hat{\imath} - 8\hat{\jmath} + 3\hat{k})]$$

$$\dot{\vec{H}}_{/P} = 0$$

b) $\dot{\vec{H}}_{/P} = m\,[(3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}) - (6\hat{\imath} - 4\hat{\jmath} + 8\hat{k})] \times [\hat{\imath} - 8\hat{\jmath} + 3\hat{k}]$

$$= -m\,[(3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}) \times (\hat{\imath} - 8\hat{\jmath} + 3\hat{k})]$$

$$\dot{\vec{H}}_{/P} = -m\,[26\hat{\imath} - 5\hat{\jmath} - 22\hat{k}]$$

c) $\dot{\vec{H}}_{/P} = m\,[(3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}) - (-9\hat{\imath} + 6\hat{\jmath} - 12\hat{k})] \times [\hat{\imath} - 8\hat{\jmath} + 3\hat{k}]$

$$= m\,[12\hat{\imath} - 8\hat{\jmath} + 16\hat{k}] \times [\hat{\imath} - 8\hat{\jmath} + 3\hat{k}]$$

$$\dot{\vec{H}}_{/P} = m\,[104\hat{\imath} - 20\hat{\jmath} - 92\hat{k}]$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

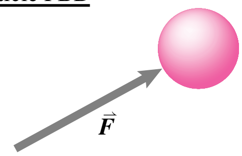
12.2.21 A particle of mass $m = 6\,\text{kg}$ is moving in space. Its position, velocity, and acceleration at some instant are $\vec{r} = 1\,\text{m}\hat{\imath} - 2\,\text{m}\hat{\jmath} + 4\,\text{m}\hat{k}$, $\vec{v} = 3\,\text{m/s}\hat{\imath} + 4\,\text{m/s}\hat{\jmath} - 7\,\text{m/s}\hat{k}$, and $\vec{a} = 5\,\text{m/s}^2\hat{\imath} + 11\,\text{m/s}^2\hat{\jmath} - 9\,\text{m/s}^2\hat{k}$, respectively. At this instant, find:

a) the net force $\sum\vec{F}$ on the particle,

b) the net moment on the particle about the origin $\sum\vec{M}_O$ due to the applied forces, and

c) the power P of the applied forces.

Particle FBD



Problem 12.21: FBD of the particle

11.2.22

a) $\sum\vec{F} = \dot{\vec{L}} = m\vec{a}$

$= 6[5\hat{\imath} + 11\hat{\jmath} - 9\hat{k}]$

$\vec{F} = 30\hat{\imath} + 66\hat{\jmath} - 54\hat{k}$ N

b) $\sum\vec{M}_{/O} = \dot{\vec{H}}_{/O}$

$= m\,\vec{r}_{/O} \times \vec{a}_{/O}$

$= m[\hat{\imath} - 2\hat{\jmath} + 4\hat{k}] \times [5\hat{\imath} + 11\hat{\jmath} - 9\hat{k}]$

$= 6[-26\hat{\imath} + 29\hat{\jmath} + 21\hat{k}]$

$\sum\vec{M}_{/O} = -156\hat{\imath} + 174\hat{\jmath} + 126\hat{k}$ N·m

c) Power,

$P = \sum m\vec{a}\cdot\vec{v}$

$= 6[5\hat{\imath} + 11\hat{\jmath} - 9\hat{k}]\cdot[3\hat{\imath} + 4\hat{\jmath} - 7\hat{k}]$

$= 6[15 + 44 - 63]$

$P = -24$ Joules/sec

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

12.2.22 At a time of interest, a particle with mass $m_1 = 5\,\text{kg}$ has position, velocity, and acceleration $\vec{r}_1 = 3\,\text{m}\hat{\imath}$, $\vec{v}_1 = -4\,\text{m/s}\hat{\jmath}$, and $\vec{a}_1 = 6\,\text{m/s}^2\hat{\jmath}$, respectively. Another particle with mass $m_2 = 5\,\text{kg}$ has position, velocity, and acceleration $\vec{r}_2 = -6\,\text{m}\hat{\imath}$, $\vec{v}_2 = 5\,\text{m/s}\hat{\jmath}$, and $\vec{a}_2 = -4\,\text{m/s}^2\hat{\jmath}$, respectively. For this system of two particles, and at this time, find its

a) linear momentum $\vec{L}$,

b) rate of change of linear momentum $\dot{\vec{L}}$

c) angular momentum about the origin $\vec{H}_{/O}$,

d) rate of change of angular momentum about the origin $\dot{\vec{H}}_{/O}$,

e) kinetic energy E_K, and

f) rate of change of kinetic energy $\dot{E}_K$.

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11-2-21

$m_1 = 5\,\text{Kg}$ | $m_2 = 5\,\text{Kg}$

$\vec{r}_1 = 3\hat{i}\ m$ | $\vec{r}_2 = -6\hat{i}$

$\vec{v}_1 = -4\hat{j}\ m/s$ | $\vec{v}_2 = 5\hat{j}$

$\vec{a}_1 = 6\hat{j}\ m/s^2$ | $\vec{a}_2 = -4\hat{j}$

a) $\vec{L} = \sum m_i \vec{v}_i$

$= 5(-4\hat{j}) + 5(5\hat{j})$

$\vec{L} = 5\hat{j}\ \text{Kg}\,m/\text{sec}$

b) $\dot{\vec{L}} = \sum m_i \vec{a}_i$

$= 5(6\hat{j}) + 5(-4\hat{j})$

$\dot{\vec{L}} = 10\hat{j}\ \text{Kg}\,m/\text{sec}^2$

c) $\vec{H}_{/o} = \sum \vec{r}_{/o} \times m\vec{v}$

$= 5[3\hat{i}\times(-4\hat{j}) + (-6\hat{i})\times(5\hat{j})]$

$\vec{H}_{/o} = -210\hat{k}\ \text{Kg}\,m^2/\text{sec}$

d) $\dot{\vec{H}}_{/o} = \sum \vec{r}_{/o} \times m\vec{a}$

$= 5[3\hat{i}\times 6\hat{j} + (-6\hat{i})\times(-4\hat{j})]$

$= 5[18\hat{k} + 24\hat{k}]$

$\dot{\vec{H}}_{/o} = 210\hat{k}\ \text{Kg}\,m^2/\text{sec}^2$

e) $E_K = \sum \frac{1}{2} m_i \vec{v}_i = \frac{5}{2}[(4)^2 + (5)^2] = 102.5$ Joules.

$E_K = 102.5$ Joules

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f) $\dot{E}_K = \sum m\vec{a}\cdot\vec{v}$

$= 5\left[(6\hat{j})\cdot(-4\hat{j}) + (-4\hat{j})\cdot(5\hat{j})\right]$

$= 5\left[-24 - 20\right]$

$\dot{E}_K = -220$ Joules/sec

12.2.23 A particle of mass $m = 250$ gm is shot straight up (parallel to the y-axis) from the x-axis at a distance $d = 2$ m from the origin. The velocity of the particle is given by $\vec{\boldsymbol{v}} = v\hat{\boldsymbol{\jmath}}$ where $v^2 = v_0^2 - 2ah$, $v_0 = 100\,\mathrm{m/s}$, $a = 10\,\mathrm{m/s^2}$ and h is the height of the particle from the x-axis.

a) Find the linear momentum of the particle at the outset of motion ($h = 0$).

b) Find the angular momentum of the particle about the origin at the outset of motion ($h = 0$).

c) Find the linear momentum of the particle when the particle is 20 m above the x-axis.

d) Find the angular momentum of the particle about the origin when the particle is 20 m above the x-axis.

(42)

$m = 250$ grams $= \frac{1}{4}$ kg

$\vec{r}_o = 2\hat{i}$

$\vec{V} = \sqrt{V_0^2 - 2ah}\,\hat{j}$

$V_0 = 100\,\mathrm{m/sec}$

$a = 10\,\mathrm{m/sec^2}$

$h = y$ if $r_{P/O} = x\hat{i} + y\hat{j}$

a) $\vec{L}\Big|_{t=0} = m\vec{V}\Big|_{t=0}$

$= \frac{1}{4}\left[\sqrt{V_0^2 - 2a(0)}\,\hat{j}\right]$ kg m/sec

$= \frac{V_0}{4}\hat{j}$ kg m/sec

$\vec{L}\Big|_{t=0} = 25\hat{j}$ kg m/sec

b) $\vec{H}\Big|_{t=0} = \vec{r}_{/o} \times m\vec{V}\Big|_{t=0}$

$= 2\hat{i} \times 25\hat{j}$

$\vec{H}\Big|_{t=0} = 50\hat{k}$ kg m²/sec

c) for $h = 20$ m

$\vec{L} = m\vec{V}$

$= \frac{1}{4}\left[\sqrt{V_0^2 - 2ah}\right]\hat{j}$

$\vec{L} = 24.5\hat{j}$ kg m/sec

d) for $h = 20$m

$\vec{r}_{/o} = \hat{i} + 20\hat{j}$

$\vec{H}_{/o} = \vec{r}_{/o} \times m\vec{V}$

$= (2\hat{i} + 20\hat{j}) \times (24.5\hat{j})$

$\vec{H}_{/o} = 49\hat{k}$ kg m²/sec

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*