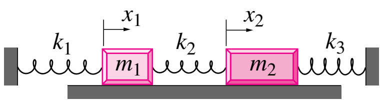
11.3 Normal Modes

11.3.1 A two degree of freedom mass-spring system, made up of two unequal masses m_1 and m_2 and three springs with unequal stiffnesses k_1, k_2 and k_3, is shown in the figure. All three springs are relaxed in the configuration shown. Neglect friction.

a) Derive the equations of motion for the two masses.

b) Does each mass undergo simple harmonic motion? *



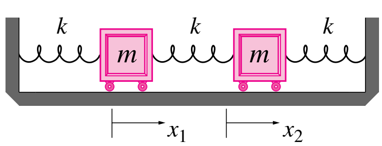
Problem 11.3.1

11.3.2 **Normal Modes.** Three equal springs (k) hold two equal masses (m) in place. There is no friction. x_1 and x_2 are the displacements of the masses from their equilibrium positions.

a) How many independent normal modes of vibration are there for this system? *

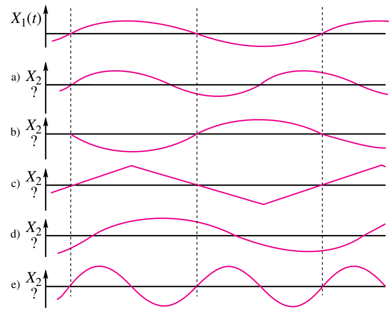
b) Assume the system is in a normal mode of vibration and it is observed that $x_1 = A\sin(ct) + B\cos(ct)$ where A, B, and c are constants. What is $x_2(t)$? (The answer is not unique. You may express your answer in terms of any of A, B, c, m and k.) *

c) Find all of the frequencies of normal-mode-vibration for this system in terms of m and k. *



Problem 11.3.2

11.3.3 $x_1(t)$ and $x_2(t)$ are measured positions on two points of a vibrating structure. $x_1(t)$ is shown. Some candidates for $x_2(t)$ are shown. Which of the $x_2(t)$ could possibly be associated with a normal mode vibration of the structure? Answer “could” or “could not” next to each choice and briefly explain your answer (If a curve looks like it is meant to be a sine/cosine curve, it is.)

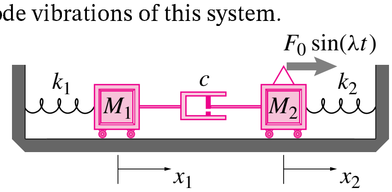


Problem 11.3.3

More-Involved Problems

11.3.4 Two masses are connected to fixed supports and each other with the two springs and dashpot shown. The displacements x_1 and x_2 are defined so that $x_1 = x_2 = 0$ when both springs are unstretched.

For the special case that $C = 0$ and $F_0 = 0$ clearly define two different sets of initial conditions that lead to normal mode vibrations of this system.

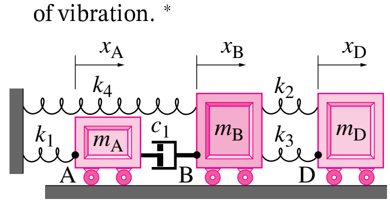


Problem 11.3.4

11.3.5 As in problem 10.4.11, a system of three masses, four springs, and one damper are connected as shown. Assume that all the springs are relaxed when $x_A = x_B = x_D = 0$.

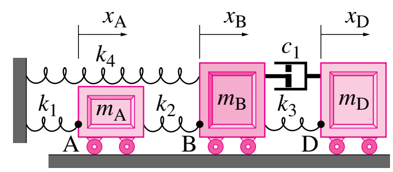
a) In the special case when $k_1 = k_2 = k_3 = k_4 = k$, $c_1 = 0$, and $m_A = m_B = m_D = m$, find a normal mode of vibration. Define it in any clear way and explain or show why it is a normal mode in any clear way. *

b) In the same special case as in (a) above, find another normal mode of vibration. *



Problem 11.3.5

11.3.6 As in problem 10.4.10, a system of three masses, four springs, and one damper are connected as shown. In the special case when $c_1 = 0$, find the normal modes of vibration.

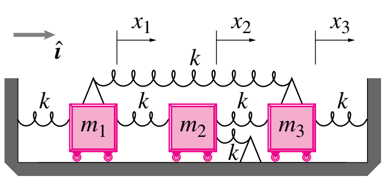


Problem 11.3.6

11.3.7 Normal modes. All three masses have $m = 1\,\text{kg}$ and all 6 springs are $k = 1\,\text{N/m}$. The system is at rest when $x_1 = x_2 = x_3 = 0$.

a) Find as many different initial conditions as you can for which normal mode vibrations result. In each case, find the associated natural frequency. (we will call two initial conditions $[v]$ and $[w]$ different if there is no constant c so that $[v_1\ v_2\ v_3] = c[w_1\ w_2\ w_3]$. Assume the initial velocities are zero.)

b) For the initial condition
$$[x_0] = [\,0.1\,\text{m}\ \ 0\ \ 0\,],$$
$[\dot{x}_0] = [\,0\ \ 2\,\text{m/s}\ \ 0\,]$
what is the initial (immediately after the start) acceleration of mass 2?



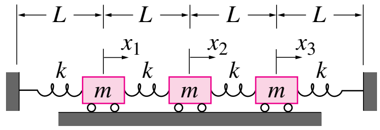
Problem 11.3.7

11.3.8 For the three-mass system shown, assume $x_1 = x_2 = x_3 = 0$ when all the springs are fully relaxed. One of the normal modes is described with the initial condition $(x_{1_0}, x_2, x_3) = (1, 0, -1)$.

a) What is the angular frequency ω for this mode? Answer in terms of L, m, k, and g. (Hint: Note that in this mode of vibration the middle mass does not move.) *

b) Make a neat plot of x_3 versus x_1 for one cycle of vibration with this mode.

c) Make a single plot with both x_1 and x_3 vs t for one cycle of this mode.



Problem 11.3.8