

11.2.1 Given that $\ddot{\theta} + k^2\theta = \beta \sin \omega t$,
 $\theta(0) = 0$, and $\dot{\theta}(0) = \dot{\theta}_0$, find $\theta(t)$.

Answer:

$$\theta(t) = \frac{1}{k} \left(\dot{\theta}_0 - \frac{\beta \omega}{k^2 - \omega^2} \right) \sin kt + \frac{\beta}{k^2 - \omega^2} \sin kt$$

10.2.1

$$\ddot{\theta} + k^2\theta = \beta \sin \omega t$$

$$\theta(0) = 0 \quad \text{and} \quad \dot{\theta}(0) = \dot{\theta}_0$$

the homogenous solution

$$\theta_h = A_1 \sin kt + A_2 \cos kt$$

The particular solution

$$\theta_p = \phi \sin \omega t$$

$$\ddot{\theta}_p = -\omega^2 \phi \sin \omega t$$

then,

$$-\omega^2 \phi + k^2 \phi = \beta$$

$$\phi (k^2 - \omega^2) = \beta$$

$$\phi = \frac{\beta}{k^2 - \omega^2}$$

$$\text{Soln} = \theta = A_1 \sin kt + A_2 \cos kt + \frac{\beta}{k^2 - \omega^2} \sin \omega t$$

$$\text{when } t=0, \quad \theta = \theta_0$$

$$\theta_0 = A_2 + \frac{\beta}{k^2 - \omega^2} \sin 0$$

Disclaimer: We have lost track of the many people who wrote these solutions.
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$$A_2 = \theta_0 = 0$$

$$\dot{\theta}_0 = k A_1 \cos kt + A_2 k (-\sin kt) + \frac{\beta \omega}{k^2 - \omega^2} \cos \omega t$$

$$\dot{\theta}_0 = k A_1 + \frac{\beta \omega}{k^2 - \omega^2}$$

$$A_1 = \frac{1}{k} \left[\dot{\theta}_0 - \frac{\beta \omega}{k^2 - \omega^2} \right]$$

$$\theta = \frac{1}{k} \left[\dot{\theta}_0 - \frac{\beta \omega}{k^2 - \omega^2} \right] \sin kt + \beta \frac{\sin \omega t}{k^2 - \omega^2}$$

11.2.2 Three SDOF systems, each with the same mass but different stiffnesses, $k_1 = k$, $k_2 = 2k$, and $k_3 = 4k$, and different damping, $c_1 = c$, $c_2 = 2c$, and $c_3 = c/2$, are subjected to the same periodic forcing, $F = F_0 \sin pt$ where p is less than the resonant frequency of each

of the systems. Sketch approximately the response of each of the three oscillators assuming all of them to be under-damped. Clearly mark the transient and steady state part of the response, and indicate the relative values of the response amplitudes.

$$10.2.2 \quad \begin{array}{ll} k_1 = k & c_1 = c \\ k_2 = 2k & c_2 = 2c \\ k_3 = 4k & c_3 = c/2 \end{array} \quad F = F_0 \sin pt$$

$$m \ddot{x} + c \dot{x} + kx = F_0 \sin pt \quad (1)$$

$$x_p(t) = X \cos(pt - \phi) \quad (2)$$

Substituting (2) in (1),

$$X [(k - m p^2) \cos(pt - \phi) - c p \sin(pt - \phi)] = F_0 \sin pt \quad (3)$$

$$\begin{aligned} \cos(pt - \phi) &= \cos pt \cos \phi + \sin pt \sin \phi \\ \sin(pt - \phi) &= \sin pt \cos \phi - \cos pt \sin \phi \end{aligned} \quad \left. \begin{array}{l} \text{Using these and} \\ \text{comparing co-efficients} \\ \text{in (3)} \end{array} \right\}$$

$$X [(k - m p^2) \cos \phi + c p \sin \phi] = F_0 \quad (4)$$

$$X [(k - m p^2) \sin \phi - c p \cos \phi] = 0 \quad (5)$$

$$X = \frac{F_0}{[(k - m p^2)^2 + c^2 p^2]^{1/2}} \quad \phi = \tan^{-1} \left(\frac{c p}{k - m p^2} \right)$$

In terms of frequency,

$$\frac{X}{F_0} = \frac{1}{\left[\left\{ 1 - \left(\frac{\omega}{\omega_n} \right)^2 \right\}^2 + \left\{ 2 \zeta \frac{\omega}{\omega_n} \right\}^2 \right]^{1/2}} \quad \phi = \tan^{-1} \left\{ \frac{2 \zeta \frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n} \right)^2} \right\}$$

11.2.3 A 3 kg mass is suspended by a spring ($k = 10 \text{ N/m}$) and forced by a 5 N sinusoidally oscillating force with a period of 1 s. What is the amplitude of the steady-state oscillations (ignore the “homogeneous” solution)

Answer:

$$A = 0.17 \text{ m}$$

11.2.3

$$m = 3 \text{ kg}$$

$$k = 10 \text{ N/m}$$

$$F_0 = 5 \text{ N}$$

$$T = 1 \text{ s}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1} \text{ rad/sec}$$

The Eqn

$$m\ddot{x} + kx = F_0 \cos \omega t \quad \longrightarrow \quad (1)$$

The homogeneous solution of this differential equation is given by.

$$x_h(t) = C_1 \cos \omega t + C_2 \sin \omega t$$

The Particular solution is given by.

$$x_p(t) = X \cos \omega t$$

On substitution and solving (1) we obtain

$$X = \frac{F_0}{k - m\omega^2} \quad \longrightarrow \quad (2)$$

$$X = \frac{5}{10 - 3(2\pi)^2} =$$

$$X = -0.1696$$

The negative sign is because the ratio of $\left(\frac{\omega}{\omega_n}\right) > 1$.

$$X = \frac{F_0}{k - m\omega^2} = \frac{\delta_{st}}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \rightarrow \textcircled{3}$$

In Equation $\textcircled{3}$ $\left(\frac{\omega}{\omega_n}\right) > 1$.

11.2.6 The transient response of an oscillatory system shows exponential decay of the peak displacements at each cycle. The second peak is found to be twice as big as the fifth peak. Find the damping ratio ξ for the system. How many cycles does it take for the peak displacement to drop below 5% of the first peak displacement?

10-2.6

From logarithmic decrement

$$n = \frac{\sqrt{1-\xi^2}}{2\pi\xi} \ln \left[\frac{x_0}{x_n} \right]$$

$$3 = \frac{\sqrt{1-\xi^2}}{2\pi\xi} \ln [2]$$

$$740.52\xi^2 = 1$$

$$\xi^2 = 1.35 \times 10^{-3}$$

$$\xi = 0.036$$

No of cycles it takes for peak displacement to drop below 5% of the first peak.

$$n = \frac{\sqrt{1-\xi^2}}{2\pi\xi} \ln \left[\frac{x_0}{0.05x_0} \right]$$

$$n = 4.418 \times 2.995$$

$$n = 13.23$$

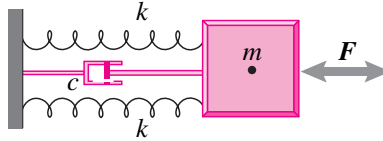
It should excite 14 cycles for its amplitude to fall below 5% of initial Amplitude

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11.2.8 Consider the system shown in the figure. You are given that $m = 10$ kg, $k = 50$ N/m, and $c = 5$ kg/s. A periodic force $F = F_0 \cos pt$ acts on the system as shown where $F_0 = 25$ N and $p = 2.5$ rad/s.

- Find the resonant frequency of the system.
- Find the steady state response of the system, specifying the amplitude and phase of the motion.
- What is the displacement amplification ($G = A/(F_0/k)$)?

- Find the work done by the force on the system in one cycle.
- Find the energy lost to the damper in one cycle.
- Find the quality factor, Q , of the system using the energy calculations.



Problem 11.8

10.2.8

$$m = 10 \text{ kg}$$

$$k = 50 \text{ N/m}$$

$$c = 5 \text{ kg/s}$$

$$\xi = \frac{c}{\sqrt{4km}} = \frac{5}{\sqrt{4 \times 10 \times 50}} = 0.11$$

$$F_0 = 25 \text{ N}$$

$$p = \omega = 2.5 \text{ rad/sec}$$

$$a) \quad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{10}} = \sqrt{5} = 2.23 \text{ rad/sec}$$

b) Steady state amplitude

$$X = \frac{(F_0/k)}{\left\{ \left[1 - \left(\frac{\omega}{\omega_n} \right)^2 \right]^2 + \left[2\xi \frac{\omega}{\omega_n} \right]^2 \right\}^{1/2}}$$

$$X = 1.41 \text{ m}$$

Phase,

$$\phi = \tan^{-1} \left(\frac{2\xi\tau}{1-\tau^2} \right)$$

$$\text{where } \tau = \frac{\omega}{\omega_n} = 1.12$$

$$\phi = \tan^{-1} \left(\frac{2 \times 0.11 \times 1.12}{1 - (1.12)^2} \right)$$

$$\phi = -44.08^\circ$$

c) Displacement amplification is

$$\frac{\bar{X}}{\left(\frac{F_0}{k}\right)} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$\frac{\bar{X}}{\delta_{st}} = 2.82$$

d)

$$W_F = \int_0^T F(t) \dot{x}(t) dt \quad \left| \begin{array}{l} x = \bar{X} \cos(pt - \phi) \\ \dot{x} = -\bar{X}p \sin(pt - \phi) \end{array} \right.$$

$$W_F = \int_0^T F_0 \cos(pt) \cdot -\bar{X}p \sin(pt - \phi) dt$$

on solving

$$W_F = F_0 \pi \bar{X} \sin \phi$$

$$W_F = 25 \times 10 \times 1.41 \times 0.695$$

$$W_F = 77.03 \text{ Joules.}$$

e) Energy lost in damper

$$\begin{aligned} W_d &= \int_0^T c \dot{x}^2 dt = \int_0^T X^2 c p^2 \sin^2(pt - \phi) \cdot -\sin(pt - \phi) \cdot dt \\ &= X^2 \int_0^T p^2 c \sin^2(pt - \phi) dt \end{aligned}$$

$$= \pi c p X^2$$

$$W_d = \pi \times 5 \times 2.5 \times (1.41)^2$$

$$W_d = 78.072$$

11.2.9 A MEMS cantilever beam resonator is used for mass measurement of biological molecules by comparing the shift in the resonant frequency of the beam after the test molecule is attached to the free end of the beam. In a SDOF model of the resonator, it is equivalent to finding the difference in the resonant

frequency of the system with mass m and $m + \Delta m$. If the 'effective mass' of the beam (mass to be used in the SDOF model) is 2.05×10^{-15} kg, the stiffness is 0.625 N/m, and the Q of the resonator is 900 , find the shift in the resonant peak in Hz when a biological molecule of mass 1.36×10^{-21} kg is attached to the end of the beam (equivalently to the mass m).

$$10.2.9 \quad m = 2.05 \times 10^{-15} \text{ kg} \quad k = 0.625 \text{ N/m} \\ Q = 900 \quad \Delta m = 1.36 \times 10^{-21} \text{ kg}$$

$$Q = \frac{1}{2\zeta} \quad \Delta f = \frac{f_0}{Q} \quad Q = \frac{1}{2\zeta} \Rightarrow \zeta = \frac{1}{2Q} \\ \zeta = \frac{c_0}{\sqrt{4km}} \quad \omega_d = \omega \sqrt{1-\zeta^2} \Rightarrow \zeta = \frac{1}{1800}$$

$$\omega_d = \omega \sqrt{1-\zeta^2} = \sqrt{\frac{k}{m}} \sqrt{1-\frac{c_0^2}{4km}} \\ \Rightarrow \Delta \omega_d = \sqrt{\frac{k}{m}} \left[-\frac{1}{2} m^{-3/2} \Delta m \sqrt{1-\frac{c_0^2}{4km}} + \sqrt{\frac{k}{m}} \frac{1}{2} \left(\frac{1-\frac{c_0^2}{4km}}{4km} \right) \right. \\ \left. + \frac{c_0^2}{4km^2} \Delta m \right]$$

$$\Rightarrow \Delta \omega_d = \sqrt{\frac{k}{m}} \left[\frac{-\Delta m}{2m} \sqrt{1-\frac{c_0^2}{4km}} + \frac{1}{2} \frac{(c_0^2/4km)}{(1-\frac{c_0^2}{4km})^{3/2}} \Delta m \right]$$

$$\Delta \omega_d = \sqrt{\frac{k}{m}} \left[-\frac{\sqrt{1-\zeta^2}}{2m} + \frac{\zeta^2}{2m(1-\zeta^2)^{3/2}} \right] \Delta m$$

$$\Delta \omega_d = \sqrt{\frac{k}{m}} \left[-\sqrt{1-\zeta^2} + \frac{\zeta^2}{(1-\zeta^2)^{3/2}} \right] \frac{\Delta m}{2m}$$

$$= \sqrt{\frac{0.625}{2.05 \times 10^{-15}}} \left[-\sqrt{1-\frac{1}{(1800)^2}} + \frac{(1/1800)^2}{(1-(1/1800)^2)^{3/2}} \right] \frac{1.36 \times 10^{-21}}{2 \times 2.05 \times 10^{-15}}$$

$$\Delta \omega_d = 5.792 \text{ rad/sec.}$$

$$\Delta f = \frac{\Delta \omega_d}{2\pi} \quad \Delta \omega_d = 2\pi \Delta f$$

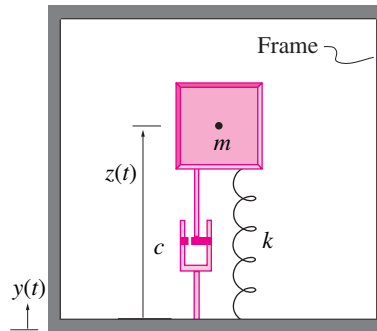
$$\therefore \Delta f = 0.922 \text{ Hz}$$

11.2.11 An accelerometer is a sensor that is used to measure acceleration of a body. It can be modeled as a single degree of freedom spring-mass-dashpot system that is attached to a body frame as shown in the figure. Assume that the body undergoes vertical motion denoted by $y(t)$ and, as a result, the mass of the accelerometer undergoes vertical motion $z(t)$ relative to the frame. From a measurement of $z(t)$ we want to know if we can determine the acceleration $\ddot{y}$ of the frame. Neglect gravity.

- What is the absolute or inertial acceleration of the mass in terms of $z(t)$ and $y(t)$ and their derivatives?
- Write the equation of motion of the mass in terms of z and y .
- Assume that $y(t) = y_0 \sin pt$. What is the magnitude of acceleration of the frame (that is, the peak acceleration of this sine wave)?
- Find the steady state response $z(t)$ of the accelerometer when $y(t) = y_0 \sin pt$ (a big mess). Plot the amplitude of the response vs the am-

plitude of the frame acceleration as the frequency p is varied.

- One would like a signal (z) that is proportional to acceleration $\ddot{y}$ independent of the frequency of shaking. Show that this requires that the frequency of frame motion p must be much smaller than the natural frequency λ_n .
- What is the amplitude of the response (the magnitude of z) of the accelerometer when $p \gg \lambda_n$?



Problem 11.11: A single degree of freedom spring-mass model of an accelerometer.

10.2.11

$$a) \quad x(t) = y(t) + z(t)$$

$$m \ddot{x}(t) = -kz - c\dot{z}$$

$$b) \quad m \ddot{z} + k z + c \dot{z} = -m \ddot{y}$$

$$c) \quad y(t) = y_0 \sin pt$$

$$\ddot{y}(t) = -p^2 y_0 \sin pt$$

The magnitude of acceleration of the frame is $p^2 y_0$.

$$d) \quad \text{From (b) we get}$$

$$m \ddot{z} + c \dot{z} + k z = -m \ddot{y}$$

$$\therefore m \ddot{z} + c \dot{z} + k z = m p^2 y_0 \sin pt$$

Steady state amplitude is

$$= \frac{m p^2 y_0}{\sqrt{(k - m p^2)^2 + c^2 p^2}}$$

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Phase ϕ is given by

$$\tan \phi = \frac{c p}{k - m p^2}$$

The steady state response is given by

$$\frac{m p^2 y_0}{\sqrt{(k - m p^2)^2 + c^2 p^2}} \sin(pt - \phi)$$

e)

$$y = y_0 \sin pt$$

$$\ddot{y} = -p^2 y_0 \sin pt$$

$$\ddot{y} = p^2 y_0 \sin(pt + \pi)$$

$$\ddot{y} = A_y \sin(pt + \pi)$$

$$z = A_z \sin(pt - \phi)$$

$$A_z = \frac{m A_y}{\sqrt{(k - m p^2)^2 + c^2 p^2}}$$

$$A_z = \frac{1}{\sqrt{(\omega_n^2 - p^2)^2 + \frac{c^2 p^2}{m^2}}} A_y$$

$$A_z = \frac{1}{\sqrt{\omega_n^4 \left(1 - \frac{p^2}{\omega_n^2}\right)^2 + (2\omega_n p \xi)^2}} A_y$$

$$A_z = \frac{1}{\omega_n^2} \times \frac{1}{\sqrt{\left(1 - \frac{p^2}{\omega_n^2}\right)^2 + \left(2\xi \frac{p}{\omega_n}\right)^2}} A_y$$

if $\frac{p}{\omega} \rightarrow 0$

$$A_z = \frac{A_y}{\omega_n^2}$$

$$A_z \propto A_y$$

f)
$$Z = \frac{m p^2 y_0}{\sqrt{(k - m p^2)^2 + c^2 p^2}}$$

$$Z = \frac{r^2 y_0}{\sqrt{(1 - r^2)^2 + (2\xi r)^2}}$$

where $r^2 = \left(\frac{p}{\omega_n}\right)^2$

$$Z = \frac{r^2 y_0}{\sqrt{1 + r^4 + 4\xi^2 r^2 - 2r^2}}$$

$$Z = \frac{y_0}{\sqrt{\frac{1}{\delta^4} + 1 + \frac{4\xi^2}{\delta^2} - \frac{2}{\gamma^2}}}$$

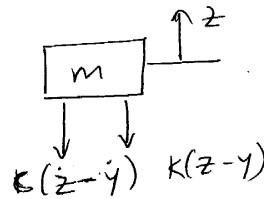
$$\delta \rightarrow \infty$$

$$Z = \frac{y_0}{\sqrt{0 + 1 + 0 - 0}}$$

$$Z = y_0$$

9.109]

- a) Absolute acceleration of mass = $\ddot{z}(t)$
 Relative acceleration of mass wrt. frame = $\ddot{z}(t) - \ddot{y}(t)$

b) FBD

$$\sum F_{\text{ext}} = \dot{L}$$

$$-k(z - y) - c(\dot{z} - \dot{y}) = m\ddot{z}$$

Re-arranging

$$m\ddot{z} + k z + c\dot{z} = k y + c\dot{y} \quad \text{--- (I)}$$

c) $y(t) = y_0 \sin(pt)$

$$\ddot{y}(t) \Big|_{\text{max}} = ?$$

Differentiating twice $\ddot{y}(t) = -y_0 p^2 \sin(pt)$

$$\ddot{y}(t) \Big|_{\text{max}} = y_0 p^2$$

d) ~~Let $x = z - y$~~

d) Let $x = z - y$, $\Rightarrow \dot{x} = \dot{z} - \dot{y}$; $\ddot{x} = \ddot{z} - \ddot{y}$

Putting in (I)

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{y}$$

But $\ddot{y} = -p^2 y_0 \sin(pt)$

Thus $m\ddot{x} + c\dot{x} + kx = mp^2 y_0 \sin(pt)$ — (II)

The solution to equation (II) is

$$x(t) = \underset{\substack{\uparrow \\ \text{homogeneous}}}{x_h(t)} + \underset{\substack{\uparrow \\ \text{particular}}}{x_p(t)}$$

From Box 9.6, page 447 for underdamped case

$$x_h = e^{-\left(\frac{c}{2m}\right)t} \{ A \cos(\lambda_d t) + B \sin(\lambda_d t) \}$$

$$\text{where } \lambda_d = \sqrt{\left(\frac{k}{m}\right) - \left(\frac{c}{2m}\right)^2}$$

From Box 9.10, page 492

$$x_p(t) = A_0 \cos(pt - \phi)$$

$$\text{where } A_0 = \frac{mp^2 y_0 / k}{\sqrt{4\xi^2 r^2 + (1-r^2)^2}}, \quad r = \frac{p}{\lambda_n}, \quad \xi = \frac{c}{2\sqrt{km}}$$

$$\lambda_n^2 = \frac{k}{m}$$

Put $\gamma = \frac{p}{\lambda_n} = \frac{p}{\sqrt{k/m}}$ we have

$$A_0 = \frac{r^2 y_0}{\sqrt{4\xi^2 r^2 + (1-r^2)^2}}$$

Also $\phi = \tan^{-1} \left(\frac{2\xi r}{1-r^2} \right)$

Thus our solution now looks like

$$\begin{aligned} x(t) &= x_h(t) + x_p(t) \\ &= e^{-(\frac{c}{2m})t} \{ A \cosh \lambda_d t + B \sinh \lambda_d t \} + \frac{r^2 y_0}{\sqrt{4\xi^2 r^2 + (1-r^2)^2}} \cos(pt - \phi) \end{aligned}$$

In steady state i.e. approx after $t \geq 5 \left(\frac{2m}{c} \right)$, the first term (or $x_h(t)$) is almost equal to zero. Thus

$$x_{\text{steady}} = \frac{r^2 y_0}{\sqrt{4\xi^2 r^2 + (1-r^2)^2}} \cos(pt - \phi) \quad \text{--- (11)}$$

But

$$z = x + y$$

$$z_{\text{steady}} = x_{\text{steady}} + y$$

$$z_{\text{steady}} = \frac{r^2 y_0}{\sqrt{4\xi^2 r^2 + (1-r^2)^2}} \cos(pt - \phi) + y_0 \sin pt$$

where $\phi = \tan^{-1} \left(\frac{2\xi r}{1-r^2} \right)$; $r = \frac{p}{\lambda_n}$; $\xi = \frac{c}{2\sqrt{km}}$

Amplitude $z_0 = \sqrt{\left(\frac{\gamma^2 \gamma_0}{\sqrt{4\xi^2 \gamma^2 + (1-\gamma^2)^2}} \right)^2 + \gamma_0^2}$

$$z_0 = \gamma_0 \sqrt{\frac{\gamma^4}{4\xi^2 \gamma^2 + (1-\gamma^2)^2} + 1}$$

Thus $\frac{z_0}{\gamma_0} = \sqrt{1 + \frac{\gamma^4}{4\xi^2 \gamma^2 + (1-\gamma^2)^2}}$

See attached figure for plot of $\frac{z_0}{\gamma_0}$ vs $\gamma = \frac{p}{\lambda_n}$ at different damping ($\xi = \text{eps}$)

From the figure where

$$\gamma = \frac{p}{\lambda_n} \ll 1$$

$$\frac{z_0}{\gamma_0} \approx 1$$

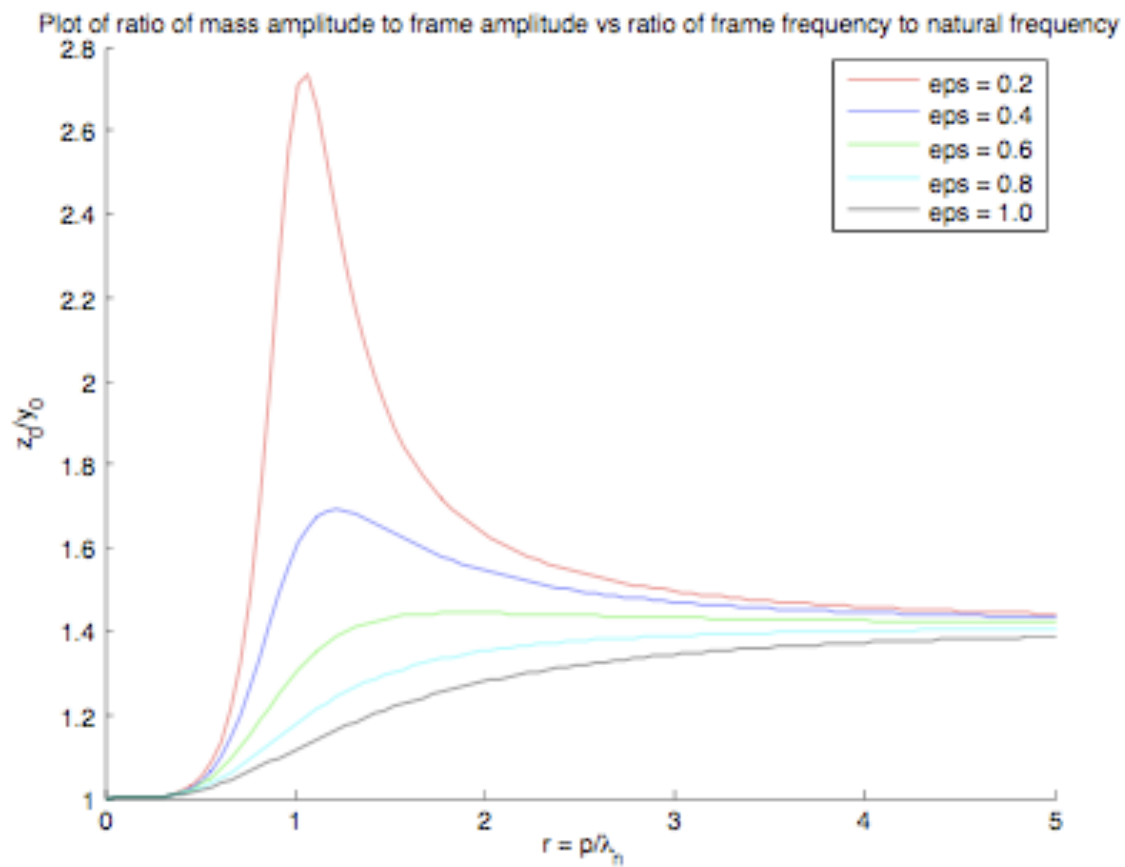
To get accelerator amplitude $\{p^2 \gamma_0\}$, see refer to (II)

$$x_0 = \frac{\gamma^2 \gamma_0}{4\xi^2 \gamma^2 + (1-\gamma^2)^2}$$

when $\gamma = \frac{p}{\lambda_n} \ll 1$ the denominator of above expression is ≈ 1

$$\therefore x_0 = \gamma^2 \gamma_0 = \frac{p^2 \gamma_0}{\lambda_n^2} = \frac{\text{accelerator amplitude}}{\lambda_n^2}$$

Thus $\boxed{\text{accelerator amplitude} = p^2 \gamma_0 = \lambda_n^2 x_0} \quad - \text{(Answer!)}$



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e) When $\gamma = \frac{p}{\lambda_n} \gg 1$, from attached figure we see that

$$\frac{z_0}{y_0} \approx 1.4 \quad (\text{actually } \sqrt{2})$$

thus $\boxed{z_0 = 1.4 y_0}$ when $p \gg \lambda_n$ { In terms of absolute ~~acceleration~~ values }

or referring to (III) when $\gamma \gg 1$

$$4\zeta^2 \gamma^2 + (1 - \gamma^2)^2 \approx \gamma^4$$

$$\text{thus } x_0 \approx \frac{\gamma^2 y_0}{\sqrt{\gamma^4}} \approx y_0$$

$\boxed{x_0 \approx y_0}$ when $p \gg \lambda_n$ { In terms of relative displacement }

Problem 9.109 Correction:

The above solution assumes z to be the absolute coordinates of motion of the mass, however z is the motion of the mass relative to frame as shown in the figure in the book. The solution above still holds, however, you need to swap x for z . The final answers are:

- a) Absolute acceleration of mass: $z''(t) + y''(t)$ (where $' = d/dt$ and so on...)
- b) Equation of motion $mz'' + kz + cz' = -my''$
- c) $y_0 p^2$ (stays same)
- d) accelerator amplitude $= \lambda_n^2 z_0$ (final answer), The graph will look a little different as the plot will be x (original wrong x) vs t
- e) $z_0 = y_0$