11.2 Forced Vibration and resonance

Preparatory Problems

11.2.1 Given that $\ddot{\theta} + k^2\theta = \beta \sin \omega t$, $\theta(0) = 0$, and $\dot{\theta}(0) = \dot{\theta}_0$, find $\theta(t)$. *

11.2.2 Three SDOF systems, each with the same mass but different stiffnesses, $k_1 = k$, $k_2 = 2k$, and $k_3 = 4k$, and different damping, $c_1 = c$, $c_2 = 2c$, and $c_3 = c/2$, are subjected to the same periodic forcing, $F = F_0 \sin pt$ where p is less than the resonant frequency of each of the systems. Sketch approximately the response of each of the three oscillators assuming all of them to be underdamped. Clearly mark the transient and steady state part of the response, and indicate the relative values of the response amplitudes.

11.2.3 A 3 kg mass is suspended by a spring ($k = 10\,\mathrm{N/m}$) and forced by a 5 N sinusoidally oscillating force with a period of 1 s. What is the amplitude of the steady-state oscillations (ignore the "homogeneous" solution) *

More-Involved Problems

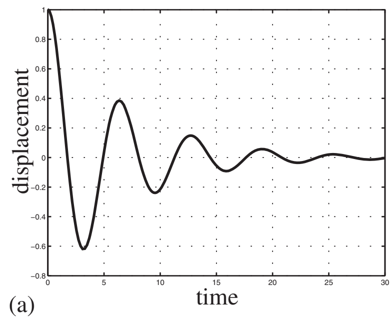
11.2.4 A machine that can be modeled as a SDOF system is put under vibration test for estimating the system parameters m, k, and c. First, a transient test is conducted by disturbing the machine from its equilibrium and letting it settle down to equilibrium again. The transient response is recorded as a displacement versus time plot and is shown in fig. 11.2.4(a). Next, a sinusoidal forcing of amplitude F_0 and angular frequency p is applied on the machine and its steady state response is recorded along with the forcing function. This response is shown in fig. 11.2.4(b).

a) Mark the relevant points on the transient response plot and explain, with equations, which system parameters can be determined using what information from this plot.

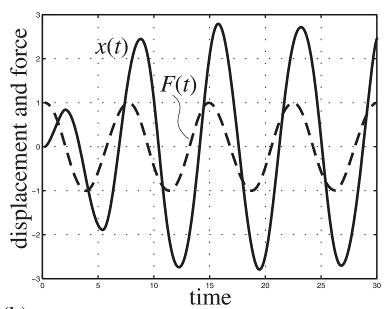
b) On the steady state plot, mark the phase difference between the response and the forcing function. From the given phase, can you find out whether $p > \lambda_n$ or $p < \lambda_n$?

c) From the phase difference of the steady state response and the information obtained from the transient response, can you determine the frequency ratio r? Explain with appropriate equations.

d) From the amplitude of the steady state response, and the rest of the information obtained above, find the rest of the system parameters.



(a)



(b)

Problem 11.2.4

11.2.5 A machine produces a steady-state vibration due to a forcing function described by $Q(t) = Q_0 \sin \omega t$, where $Q_0 = 5000N$. The machine rests on a circular concrete foundation. The foundation rests on an isotropic, elastic half-space. The equivalent spring constant of the half-space is $k = 2{,}000{,}000\,\mathrm{N/m}$ and has a damping ratio $d = c/c_c = 0.125$. The machine operates at a frequency of $\omega = 4$ Hz.

1. What is the natural frequency of the system?
2. If the system were undamped, what would the steady-state displacement be?
3. What is the steady-state displacement given that $d = 0.125$?
4. How much additional thickness of concrete should be added to the footing to reduce the damped steady-state amplitude by 50%? (The diameter must be held constant.)

11.2.6 The transient response of an oscillatory system shows exponential decay of the peak displacements at each cycle. The second peak is found to be twice as big as the fifth peak. Find the damping ratio xi for the system. How many cycles does it take for the peak displacement to drop below 5% of the first peak displacement?

11.2.7 A 50 kg engine is mounted on springs with an equivalent single spring stiffness of 1200 N/m. Using various means, enough damping needs to be provided so that any unwanted vibration dies quickly. Assume that this objective is met by dissipating 80% of the available energy in a single cycle of vibration. Find the damping coefficient of the system.

11.2.8 Consider the system shown in the figure. You are given that $m = 10\,\mathrm{kg}$, $k = 50\,\mathrm{N/m}$, and $c = 5\,\mathrm{kg/s}$. A periodic force $F = F_0 \cos pt$ acts on the system as shown where $F_0 = 25\,\mathrm{N}$ and $p = 2.5\,\mathrm{rad/s}$.

a) Find the resonant frequency of the system.

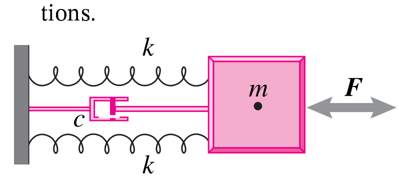
b) Find the steady state response of the system, specifying the amplitude and phase of the motion.

c) What is the displacement amplification ($G = A/(F_0/k)$)?

d) Find the work done by the force on the system in one cycle.

e) Find the energy lost to the damper in one cycle.

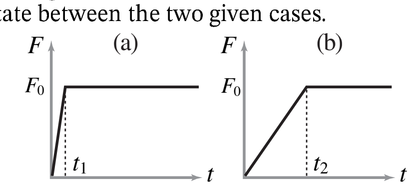
f) Find the quality factor, Q, of the system using the energy calculations.



Problem 11.2.8

11.2.9 A MEMS cantilever beam resonator is used for mass measurement of biological molecules by comparing the shift in the resonant frequency of the beam after the test molecule is attached to the free end of the beam. In a SDOF model of the resonator, it is equivalent to finding the difference in the resonant frequency of the system with mass m and $m + \Delta m$. If the 'effective mass' of the beam (mass to be used in the SDOF model) is 2.05×10^{-15} kg, the stiffness is 0.625 N/m, and the Q of the resonator is 900, find the shift in the resonant peak in Hz when a biological molecule of mass 1.36×10^{-21} kg is attached to the end of the beam (equivalently to the mass m).

11.2.10 A damped mass-spring system is subjected to a constant load $F_0 = 50$ N by ramping the load to the constant level in (a) $t_1 = 2$ s and (b) $t_2 = 10$ s. If the mass of the system $m = 1$ kg, the natural frequency $\lambda_n = 62$ rad/s, and the damping ratio $\xi = 0.2$, find the difference in the settling time of the system to the steady state between the two given cases.



Problem 11.2.10

11.2.11 An accelerometer is a sensor that is used to measure acceleration of a body. It can be modeled as a single degree of freedom spring-mass-dashpot system that is attached to a body frame as shown in the figure. Assume that the body undergoes vertical motion denoted by $y(t)$ and, as a result, the mass of the accelerometer undergoes vertical motion $z(t)$ relative to the frame. From a measurement of $z(t)$ we want to know if we can determine the acceleration $\ddot{y}$ of the frame. Neglect gravity.

a) What is the absolute or inertial acceleration of the mass in terms of $z(t)$ and $y(t)$ and their derivatives?

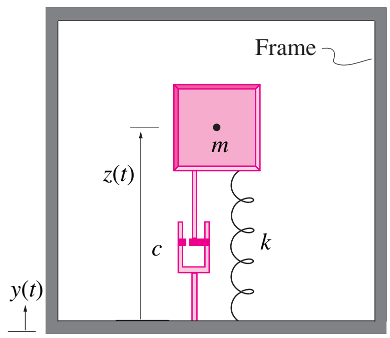
b) Write the equation of motion of the mass in terms of z and y.

c) Assume that $y(t) = y_0 \sin pt$. What is the magnitude of acceleration of the frame (that is, the peak acceleration of this sine wave)?

d) Find the steady state response $z(t)$ of the accelerometer when $y(t) = y_0 \sin pt$ (a big mess). Plot the amplitude of the response vs the amplitude of the frame acceleration as the frequency p is varied.

e) One would like a signal (z) that is proportional to acceleration $\ddot{y}$ independent of the frequency of shaking. Show that this requires that the frequency of frame motion p must be much smaller than the natural frequency λ_n.

f) What is the amplitude of the response (the magnitude of z) of the accelerometer when $p \gg \lambda_n$?



Problem 11.2.11: A single degree of freedom spring-mass model of an accelerometer.

11.2.12 Consider the accelerometer described in Problem 11.2.11. Assume that the frame undergoes a sinusoidal motion given by $y(t) = y_0 \sin pt$.

a) Find the response $z(t)$ of the accelerometer.

b) Given that $m = 0.5$ kg, $k = 5$ kN/m, and $c = 10$ kg/s, find the maximum acceleration that the accelerometer can sense, assuming the accelerometer to work in the frequency range much below its natural frequency (*i.e.*, $p/\lambda_n \ll 1$). Express your answer in terms of the gravitational acceleration g (it is customary to talk about acceleration of various things in terms of 'so many g's').