# 11.1 Free Vibration of a SDOF System

## Preparatory Problems

**11.1.1** A mass $m$ is connected to a spring $k$ and released from rest with the spring stretched a distance $d$ from its static equilibrium position. It then oscillates back and forth repeatedly crossing the equilibrium. How much time passes from release until the mass moves through the equilibrium position for the second time? Neglect gravity and friction. Answer in terms of some or all of $m$, $k$, and $d$. *

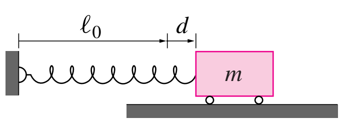
**11.1.2** A spring $k$ with rest length $\ell_0$ is attached to a mass $m$ which slides frictionlessly on a horizontal ground as shown. At time $t = 0$ the mass is released from rest with the spring stretched a distance $d$. Measure the mass position $x$ relative to the wall.

a) What is the acceleration of the mass just after release?

b) Find a differential equation which describes the horizontal motion $x$ of the mass.

c) What is the position of the mass at an arbitrary time $t$?

d) What is the speed of the mass when it passes through $x = \ell_0$ (the position where the spring is relaxed)?




Problem 11.1.2

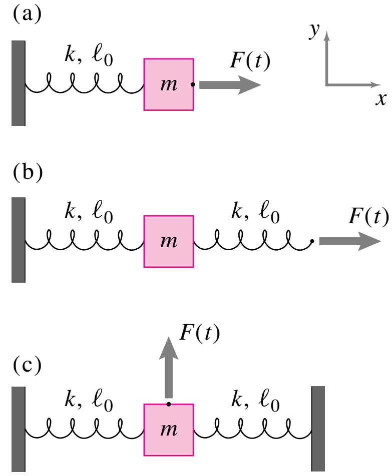
**11.1.3** Reconsider the spring-mass system from problem 11.1.2.

a) Find the potential and kinetic energy of the spring mass system as functions of time.

b) Assigning numerical values to the various variables, use a computer to make a plot of the potential and kinetic energy as a function of time for several periods of oscillation. Are the potential and kinetic energy ever equal at the same time? If so, at what position $x(t)$?

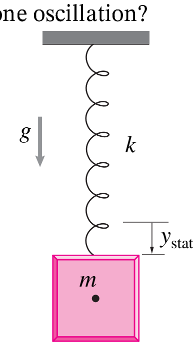
c) Make a plot of kinetic energy versus potential energy. What is the phase relationship between the kinetic and potential energy?

**11.1.4** For the three spring-mass systems shown in the figure, find the equation of motion of the mass in each case. All springs are massless and are shown in their relaxed states. Ignore gravity. (In problem (c) assume vertical motion.) *




Problem 11.1.4

**11.1.5** A mass-spring oscillator hangs vertically under gravity. The mass rests in static equilibrium by stretching the spring by an amount $y_{\text{static}} = 0.025\,\text{m}$. Take your favorite value of $g$ and find the natural frequency of the oscillator. How much time does the oscillator take to complete one oscillation?



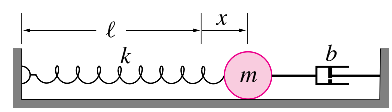

Problem 11.1.5

**11.1.6** A mass moves on a frictionless surface. It is connected to a dashpot with damping coefficient $b$ to its right and a spring with constant $k$ and rest length $\ell$ to its left. At the instant of interest, the mass is moving to the right and the spring is stretched a distance $x$ from its position where the spring is unstretched. There is gravity.

a) Draw a free-body diagram of the mass at the instant of interest.

b) Derive the equation of motion of the mass. *

.




Problem 11.1.6

**11.1.7** A mass-spring-dashpot system has $m = 1\,\text{kg}$, $k = 10\,\text{kN/m}$, and $c = 5\,\text{kg/s}$. Find the natural frequency, damping ratio, and the damped frequency of the system. Specify whether the system is underdamped, critically damped or overdamped. *

**11.1.8** The natural frequency, $\lambda_n$, of a SDOF system is 150 rad/s. Find the minimum damping ($\xi$) that the system must have for the resonant frequency to occur below 100 rad/s? *

## More-Involved Problems

**11.1.9** The equation of motion of an unforced mass-spring-dashpot system is, $m\ddot{x} + c\dot{x} + kx = 0$, as discussed in the text. For a system with $m = 0.4\,\text{kg}, c = 10\,\text{kg/s}$, and $k = 5\,\text{N/m}$,

a) Find whether the system is underdamped, critically damped, or overdamped. *

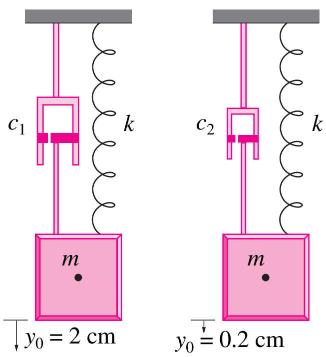
b) Sketch a typical solution of the system.

c) Make an accurate plot of the response of the system (displacement vs time) for the initial conditions $x(0) = 0.1\,\text{m}$ and $\dot{x}(0) = 0$.

.

**11.1.10** You are given to design a SDOF damped oscillator that should show no oscillations at all when disturbed from the equilibrium (*i.e.,* it should return to equilibrium without overshooting on the other side). You are given a spring with stiffness $k = 500\,\mathrm{N/m}$, a hydraulic damper with $c = 10\,\mathrm{kg/s}$, and you have a choice of masses from $m = 1\,\mathrm{kg}$ to $m = 10\,\mathrm{kg}$ in increments of half kg. Find the appropriate mass.

**11.1.11** Two SDOF oscillators with the same $k$ and $m$ but different $c$'s are hung from the ceiling as shown in the figure. The one on the left is pulled down 2 cm and let go. The other is pulled down by 0.2 cm and let go. Which oscillator undergoes more oscillations before reaching the steady state. Find the steady state displacement of each mass.




Problem 11.1.11

**11.1.12** Experiments conducted on free oscillations of a damped oscillator reveal that the amplitude of oscillations drops to 25% of its peak value in just 3 periods of oscillations. The period of oscillation is measured to be 0.6 s and the mass of the system is known to be 1.2 kg. Find the damping coefficient and the spring stiffness of the system. *

**11.1.13** You are required to design a mass-spring-dashpot system that, if disturbed, returns to its equilibrium position the quickest. You are given a mass, $m = 1\,\mathrm{kg}$, and a damper with $c = 10\,\mathrm{kg/s}$. What should be the stiffness of the spring? Your solution needs to include your definition of "'quickest". *