**10.5.1** Before a collision two particles, $m_A = 1\,\text{kg}$ and $m_B = 2\,\text{kg}$, have velocities of $v_A^- = 10\,\text{m/s}$ and $v_B^- = 5\,\text{m/s}$. After the collision the velocity of A is $v_A^+ = 6\,\text{m/s}$.

a) What is the momentum of A before the collision?

b) What is the momentum of B before the collision?

c) What is the system momentum before the collision?

d) What is the momentum of A after the collision?

e) What is the system momentum after the collision?

f) What is the momentum of B after the collision?

g) What is the impulse that A applies to B during the collision?

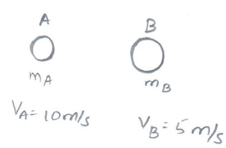
h) What is the impulse that B applies to A during the collision?

i) What is the kinetic energy of the system before the collision?

j) What is the kinetic energy of the system after the collision?

k) What is the coefficient of restitution?

9.5.1




a) $L_A = m_A V_A = 10\ \text{kg m/s}$

b) $L_B = m_B V_B = 10\ \text{kg m/s}$

c) $L_A + L_B = 20\ \text{kg m/s}$

d) $L_A^+ = m_A V_A^+ = 1 \times 6 = 6\ \text{kg m/s}$

e) $L^+ = L^- = L^+ = L_A^- + L_B^- = 20\ \text{kg m/s}$

f) $L^+ = m_A V_A^+ + m_B V_B^+ = m_A V_A^- + m_B V_B^- = 20\ \text{kg m/s}$

$m_B V_B^+ = L_B^+ = 20 - 6 = 14\ \text{kg m/s}$

g) $P^B = m_B(V_B^+ - V_B^-) = 2(7-5) = 4\ \text{kg m/s}$

h) $P^A = m_A(\cancel{M}\,{}_A^+ - V_A^-) = 4\ \text{kg m/s}$

i) $KE^- = \frac{1}{2} m_A V_A^{-2} + \frac{1}{2} m_B V_B^{-2}$

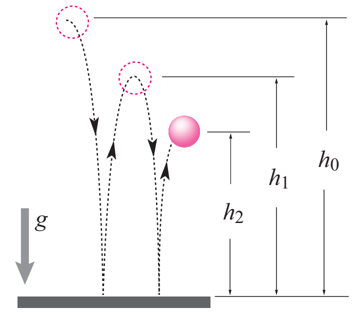
$= \frac{1}{2} \times 100 + \frac{1}{2} \times 2 \times 25 = 75$ Joules

j) $KE^+ = \frac{1}{2} m_A V_A^{+2} + \frac{1}{2} m_B V_B^{+2} = \frac{1}{2} \times 36 + \frac{1}{2} \times 2 \times 49 = 67$ Joules

k) $e = -\dfrac{(V_B^+ - V_A^+)}{(V_B^- - V_A^-)} = -\dfrac{(7-6)}{(5-10)} = \dfrac{1}{5} = 0.2$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**10.5.2** A ball is dropped from a height of $h_0 = 10\,\text{m}$ onto a hard stationary surface. After the first bounce, it reaches a height of $h_1 = 6.4\,\text{m}$. What is the coefficient of restitution between the ball and ground? What is the height of the second bounce, $h_2$?



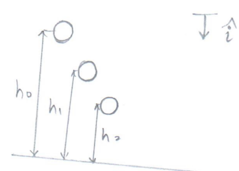

Problem 10.2

9.5.2

$h_0 = 10\,m$

$h_1 = 6.4\,m$

$e = ?$




$\frac{1}{2} m v^2 = m g h_0$

$v^- = \sqrt{2gh_0}$

$\frac{1}{2} m v^{+2} = m g h_1$

$v^+ = \pm \sqrt{2gh_1}$

$$e = \frac{-(v^+ - 0)}{(v^- - 0)} = \frac{-(-\sqrt{2gh_1})}{\sqrt{2gh_0}} = \sqrt{\frac{h_1}{h_0}} = \sqrt{\frac{6.4}{10}} = 0.8$$

$\frac{1}{2} m v_1^2 = m g h_1$

$v_1^- = \sqrt{2gh_1}$

$$e = -\frac{(v_1^+ - 0)}{(v_1^- - 0)} \Rightarrow v_1^+ = -0.8 \times \sqrt{2gh_1} \longrightarrow (1)$$

$\frac{1}{2} m v_1^{+2} = m g h_2$ ( Apply (1))

$\frac{1}{2}(0.8^2 \times 2gh_1) = gh_2$

$h_2 = 0.8^2 h_1$

$h_2 = \frac{8^2}{1000} \times 64 = 4.096\,m$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**10.5.3** A 20 gram, 500 m/s bullet embeds in an initially-stationary 50 kg rigid block. What is the coefficient of restitution? What is the velocity of the block after this collision?

**Answer:**
Note: the mass ratio is so high that there is no real loss of accuracy if you take the post-collision mass $m_1 + m_2$ as 50 kg (the 0.004% error is surely *much* smaller than the inaccuracy in any of the other numbers in the problem).

9.5.3

$m = 0.02\ \text{kg}, \quad V = 500\ \text{m/s}$


$m V_m^- + M V_M^- = m V_m^+ + M V_M^+$

$0.02 \times 500 + 50 \times 0 = (0.02 + 50)\, V^+$

$V^+ = \dfrac{10}{50.02} = 0.2\ \text{m/s}$

$e = 0$

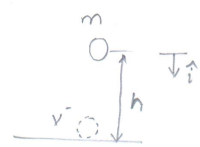
**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**10.5.4** A ball of mass $m$ is dropped vertically from a height $h$. The only force acting on the ball in its flight is gravity. The ball strikes the ground with speed $v^-$ and after collision it rebounds vertically with reduced speed $v^+$ directly proportional to the incoming speed, $v^+ = ev^-$, where $0 < e < 1$. What is the maximum height the ball reaches after one bounce, in terms of $h$, $e$, and $g$.

**Answer:**
$h_{max} = e^2 h$.

9.5.4




$$e = \frac{v^+}{v^-}$$

$$\frac{1}{2} m v^{-2} = mgh$$

$$v^{-2} = 2gh$$

$$\frac{1}{2} m v^{+2} = mgh_1$$

$$\frac{1}{2} e^2 v^{-2} = gh_1$$

$$\frac{1}{2} e^2 2gh = gh_1$$

$$\Rightarrow \quad h_1 = e^2 h$$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**10.5.5** Set up the following equations in matrix form and solve for $v_A$ and $v_B$, if $v_0 = 2.6\,\text{m/s}$, $e = 0.8$, $m_A = 2\,\text{kg}$, and $m_B = 500\text{g}$:

$$m_A v_0 = m_A v_A + m_B v_B$$
$$-e v_0 = v_A - v_B.$$

9.5.5

$$m_A V_0 = m_A V_A + m_B V_B$$

$$-eV_0 = V_A - V_B$$

$$\begin{bmatrix} m_A \\ -e \end{bmatrix} V_0 = \begin{bmatrix} m_A & m_B \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix}$$

$$\begin{bmatrix} V_A \\ V_B \end{bmatrix} = -\frac{1}{m_A + m_B} \begin{bmatrix} -1 & -m_B \\ -1 & m_A \end{bmatrix} \begin{bmatrix} m_A \\ -e \end{bmatrix} V_0$$

$$\Rightarrow \quad V_A = -\frac{(em_B - m_A) V_0}{m_A + m_B}$$

$$V_B = \frac{(m_A + em_A) V_0}{m_A + m_B}$$

$m_A = 2\,\text{kg}$, $m_B = 500\,\text{kg}$, $V_0 = 2.6\,\text{m/s}$, $e = 0.8$

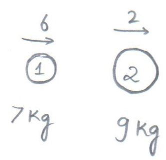
$$V_A = -\frac{(0.8 \times 500 - 2)\, 2.6}{502} = -2.06\,\text{m/s}$$

$$V_B = \frac{(2 \times 1.8) \times 2.6}{502} = 0.018\,\text{m/s}$$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**10.5.6** Before a collision two particles, $m_A = 7\,\text{kg}$ and $m_B = 9\,\text{kg}$, have velocities of $v_A^- = 6\,\text{m/s}$ and $v_B^- = 2\,\text{m/s}$. The coefficient of restitution is $e = .5$. Find the impulse of mass A on mass B and the velocities of the two masses after the collision.

9.5.6




$$P^{on\,2} = m_2(v_2^+ - v_2^-)$$

$$= 9(v_2^+ - 2)$$

$$-P^{on\,1} = m_1(v_1^+ - v_1^-) = 7(v_1^+ - 6) = -9(v_2^+ - 2)$$

$$9v_2^+ + 7v_1^+ = 42 + 18 = 60$$

$$9v_2^+ + 7v_1^+ = 60 \longrightarrow ①$$

$$e = -\frac{(v_2^+ - v_1^+)}{(v_2^- - v_1^-)} \Rightarrow 0.5(2-6) = -(v_2^+ - v_1^+)$$

$$v_2^+ - v_1^+ = 2 \Rightarrow v_2^+ = 2 + v_1^+ \longrightarrow ②.$$

Apply ② in ①.

$$18 + 9v_1^+ + 7v_1^+ = 60$$

$$16v_1^+ = 42$$

$$v_1^+ = \frac{21}{8}\ \text{m/s}$$

$$v_2^+ = \frac{37}{8}\ \text{m/s}$$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$p^{on\,2} = 9\left(\frac{37}{8} - 2\right) = 23.625 \text{ Kg m/s}$$

$$p^{on\,1} = 7\left(\frac{21}{8} - 6\right) = -23.625 \text{ Kg m/s}$$

$$\therefore\ p^{on\,2} = -p^{on\,1}$$

# HW5-P18-10.5.6

Tuesday, March 9, 2021 5:32 PM

**10.5.6** Before a collision two particles, $m_A = 7\,\text{kg}$ and $m_B = 9\,\text{kg}$, have velocities of $v_A^- = 6\,\text{m/s}$ and $v_B^- = 2\,\text{m/s}$. The coefficient of restitution is $e = .5$. Find the impulse of mass A on mass B and the velocities of the two masses after the collision.

Solution by
Michael Zakavorotny

Given: a collision between two particles with

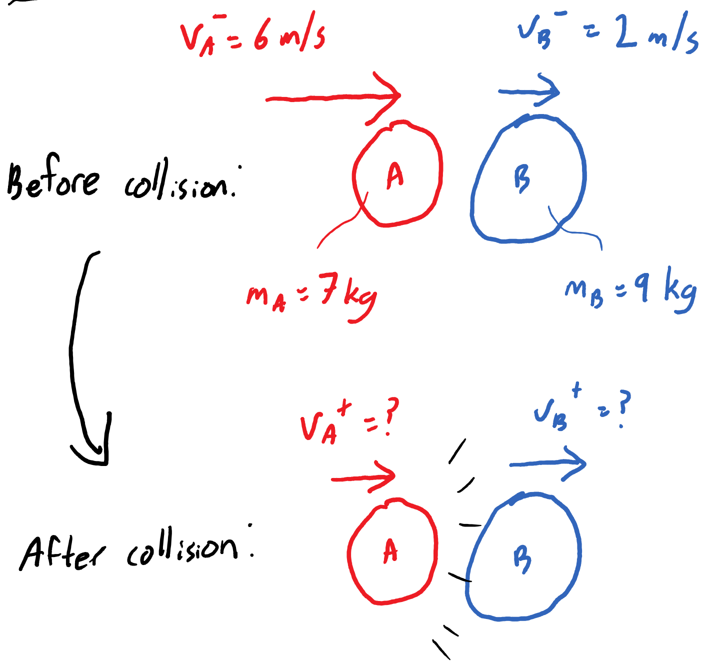
$m_A = 7\text{ kg}$, $m_B = 9\text{kg}$, and $v_A^- = 6\text{ m/s}$,

$v_B^- = 2\text{ m/s}$, where $e = 0.5$,

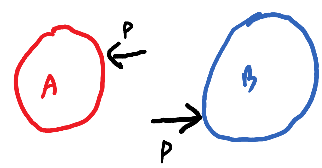
find: - impulse of mass A on mass B

- velocities of each mass after collision

## Problem Diagram




## FBD




## Solution:

Momentum conservation:

$$m_A v_A^- + m_B v_B^- = m_A v_A^+ + m_B v_B^+ \qquad (1)$$

Approach 1 for finding impulse : Restitution equation

$$(v_B^+ - v_A^+) = e(v_A^- - v_B^-)$$
$$v_A^+ = v_B^+ + e(v_B^- - v_A^-) \quad (2)$$

(2) -> (1)

$$m_A v_A^- + m_B v_B^- = m_A v_B^+ + m_A e(v_B^- - v_A^-) + m_B v_B^+$$
$$m_A v_A^-(1+e) + v_B^-(m_B - m_A e) = (m_A + m_B) v_B^+$$
$$v_B^+ = \frac{v_A^-(1+e) + v_B^-(m_B/m_A - e)}{1 + m_B/m_A}$$

$$P_B = \Delta L_B = m_B(v_B^+ - v_B^-)$$
$$= m_B\left[\frac{v_A^-(1+e) + v_B^-(\cancel{m_B/m_A} - e) - v_B^-(1 + \cancel{m_B/m_A})}{1 + m_B/m_A}\right]$$
$$= m_B \cdot \frac{(v_A^- - v_B^-)(1+e)}{1 + m_B/m_A}$$

$$\boxed{P_B = \frac{m_A m_B (v_A^- - v_B^-)(1+e)}{m_A + m_B}}$$

<u>Approach 2</u>: Equations from p. 541-542

Impulse varies between

$P_{inelastic} = -m_{coll} v_{rel}^-$ for completely inelastic ($e=0$)

and $P_{elastic} = -2 m_{coll} v_{rel}^-$ for elastic collision ($e=1$)

$P = -(1+e)\, m_{coll} v_{rel}^-$, where

$m_{coll} = \dfrac{m_1 m_2}{m_1 + m_2}$, $v_{rel}^- = v_2^- - v_1^-$

$$P = \frac{-m_1 m_2 (v_2^- - v_1^-)(1+e)}{m_1 + m_2}$$ same as Approach 1

$$P = -(1+0.5)\cdot \frac{(7\text{ kg})(9\text{ kg})}{7\text{ kg} + 9\text{ kg}} \cdot (2\text{ m/s} - 6\text{ m/s})$$

$$\boxed{P = 23.625 \text{ kg-m/s}}$$

Final velocities:

$$P = m_B \Delta v_B$$

$$P = m_B (v_B^+ - v_B^-)$$

$$\Downarrow$$

$$m_B v_B^+ = m_B v_B^- + P$$

$$v_B^+ = v_B^- + \frac{P}{m_B} = 2 \text{ m/s} + \frac{(23.625 \text{ kg-m/s})}{9 \text{ kg}}$$

$$\boxed{v_B^+ = 4.625 \text{ m/s}}$$

$$m_A v_A^+ = m_A v_A^- - P$$

$$v_A^+ = v_A^- - \frac{P}{m_A} = 6 \text{ m/s} - \frac{23.625 \text{ kg-m/s}}{7 \text{ kg}}$$

$$\boxed{v_A^+ = 2.625 \text{ m/s}}$$

```
% MAE 2030
% HW 5, Q18 Solution
% Solution by Michael Zakoworotny
%

clear all; clc; close all;

% Define symbolic variables
% e - coefficient of resititution
% ma, mb - masses of particles A and B
% va, vb - velocities of particles A and B before collision
% vap, vbp - velocities of particles A and B after collision
syms e ma mb va vb vap vbp real; % use 'real' attribute to force real numbers

% Define symbolic equations for momentum conservation and restitution
momentum = ma*va + mb*vb == ma*vap + mb*vbp;
restitution = (vbp - vap) == e*(va - vb);

% Solve system of equations for the final velocities, in terms of initial
% velocities and parameters
[vap_sol, vbp_sol] = solve([momentum,restitution], [vap,vbp]);

% Solve for impulse on mass B, using the change in momentum of mass B
P_B = mb*vbp_sol - mb*vb;
P_B = simplify(P_B); % get into factored form
disp('The symbolic expression for P_B is:');
pretty(P_B);

% Substitute numerical values:
% e = 0.5, ma = 7 kg, mb = 9 kg, va = 6 m/s, vb = 2 m/s
% Evaluate symbolic expression to obtain numerical value
vap = eval(subs(vap_sol,[e,ma,mb,va,vb],[0.5,7,9,6,2]));
vbp = eval(subs(vbp_sol,[e,ma,mb,va,vb],[0.5,7,9,6,2]));
P_B = eval(subs(P_B,[e,ma,mb,va,vb],[0.5,7,9,6,2]));
disp(['The numerical value of v_A after collision is: ', num2str(vap), ' m/s']);
disp(['The numerical value of v_B after collision is: ', num2str(vbp), ' m/s']);
disp(['The numerical value of P_B after collision is: ', num2str(P_B), ' kg-m/s']);
```

```
The symbolic expression for P_B is:
ma mb (va - vb) (e + 1)
-----------------------
        ma + mb

The numerical value of v_A after collision is: 2.625 m/s
The numerical value of v_B after collision is: 4.625 m/s
The numerical value of P_B after collision is: 23.625 kg-m/s
```

*Published with MATLAB® R2019b*

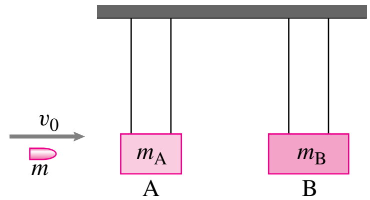
**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

**10.5.9** A bullet of mass $m$ with initial speed $v_0$ is fired in the horizontal direction through block A of mass $m_A$ and becomes embedded in block B of mass $m_B$. Each block is suspended by thin wires. The bullet causes A and B to start moving with speed of $v_A$ and $v_B$ respectively. Determine

a) the initial speed $v_0$ of the bullet in terms of $v_A$ and $v_B$,

b) the velocity of the bullet as it travels from block A to block B,and

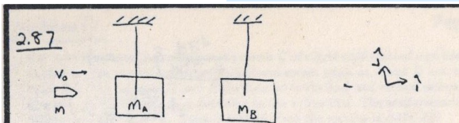
c) the energy loss due to friction as the bullet (1) moves through block A and (2) penetrates block B.



Problem 10.9

**Answer:**
(a) $v_0 = \frac{1}{m}(mv_B + m_Bv_B + m_Av_A)$.
(b) $v_1 = \frac{(m+m_B)}{m}v_B$.
(c) (1) $E_{loss} = \frac{1}{2}m\left[v_0^2 - \left(\frac{m+m_B}{m}\right)^2 v_B^2\right] - \frac{1}{2}m_Av_A^2$. (2) $E_{loss} = \frac{1}{2}\left[\frac{(m+m_B)^2}{m}v_B^2 - (m+m_B)v_B^2\right]$.

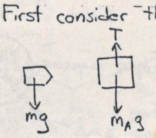
2.87 Page 2




Bullet goes through A, becomes embedded in B.
Resulting velocities of A and B are $V_A$ and $V_B$.

a) find $V_0$ in terms of $V_A$ and $V_B$
b) find bullets velocity as it travels between A and B
c) find $E_{lost}$ as bullet moves through A, and as it penetrates B.

a) First consider the bullet and mass A system:

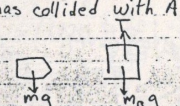



$(\Sigma \underline{F} = \dot{\underline{L}})\cdot \hat{\imath}$
$0 = \dot{L}_x \Rightarrow L_x$ is constant (momentum conserved in x-direction)

$(L_x)_{\text{before collision}} = (L_x)_{\text{after collision}}$

$mV_0 = mV_1 + m_AV_A$ ($V_1$ = bullet's velocity after collision)
$V_1 = V_0 - \frac{m_AV_A}{m}$ ①

Now consider the bullet and $m_B$, after the bullet has collided with A




$(\Sigma \underline{F} = \dot{\underline{L}})\cdot \hat{\imath}$
$0 = \dot{L}_x \Rightarrow$ momentum conserved

$mV_1 = (m+m_B)V_B$
$V_1 = \frac{m+m_B}{m}V_B$ ②

① and ② are 2 eqns with 2 unknowns, solve them for $V_0$:
$V_B\frac{m+m_B}{m} = V_0 - \frac{m_AV_A}{m}$
$V_0 = V_B\left(1+\frac{m_B}{m}\right) + V_A\frac{m_A}{m}$
$V_0 = \frac{1}{m}(mV_B + m_BV_B + m_AV_A)$

b) $V_1 = \frac{m+m_B}{m}V_B$ (eqn 2)

c) $E_{LA}$ = energy lost in moving through A
$E_{LA} = \frac{1}{2}mV_0^2 - \frac{1}{2}m_AV_A^2 - \frac{1}{2}mV_1^2$
$E_{LA} = \frac{1}{2}m\left(V_0^2 - \left(\frac{m+m_B}{m}\right)V_B^2\right) - \frac{1}{2}m_AV_A^2$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

9.5.9

a) $m v_0 = m_A v_A + (m + m_B) v_B$

b) $m v_0 = (m_B + m) v_B$

$$v = \frac{(m+m_B)}{m} v_B$$

c) 1) $E_{loss} = -\frac{1}{2} m \left[\left(\frac{m+m_B}{m}\right) v_B\right]^2 + \frac{1}{2} m v_0^2 - \frac{1}{2} m_A v_A^2$

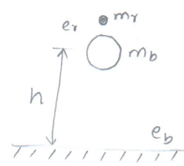
2) $E_{loss} = \frac{1}{2} m \left[\left(\frac{m+m_B}{m}\right) v_B\right]^2 - \frac{1}{2} (m+m_B) v_B^2$

**10.5.10** A basketball with mass $m_b$ is dropped from height $h$ onto the hard solid ground on which it has coefficient of restitution $e_b$. Just on top of the basketball, falling with it and then bouncing against it after the basketball hits the ground, is a small rubber ball with mass $m_r$ that has a coefficient of restitution $e_r$ with the basketball.

a) In terms of some or all of $m_b$, $m_r$, $h$, $g$, $e_b$ and $e_r$ how high does the rubber ball bounce (measure height relative to the collision point)?

b) Assuming the coefficients of restitution are less than or equal to one, for given $h$, what mass and restitution parameters maximize the height of the bounce of the rubber ball and what is that height?

9.5.10




$$h_f = \frac{h}{\left(1 + \frac{m_b}{m_r}\right)^2} \left[ \frac{m_b}{m_r} (e_r + e_b + e_r e_b) - 1 \right]^2$$

if $m_b >> m_r$

$$h_f = h\,(e_r + e_b + e_r e_b)^2$$

**10.5.10** A basketball with mass $m_b$ is dropped from height $h$ onto the hard solid ground on which it has coefficient of restitution $e_b$. Just on top of the basketball, falling with it and then bouncing against it after the basketball hits the ground, is a small rubber ball with mass $m_r$ that has a coefficient of restitution $e_r$ with the basketball.

a) In terms of some or all of $m_b$, $m_r$, $h$, $g$, $e_b$ and $e_r$ how high does the rubber ball bounce (measure height relative to the collision point)?

b) Assuming the coefficients of restitution are less than or equal to one, for given $h$, what mass and restitution parameters maximize the height of the bounce of the rubber ball and what is that height?

Katherine Cao

(a)

Given: collision between basketball - ground, basketball - rubber ball

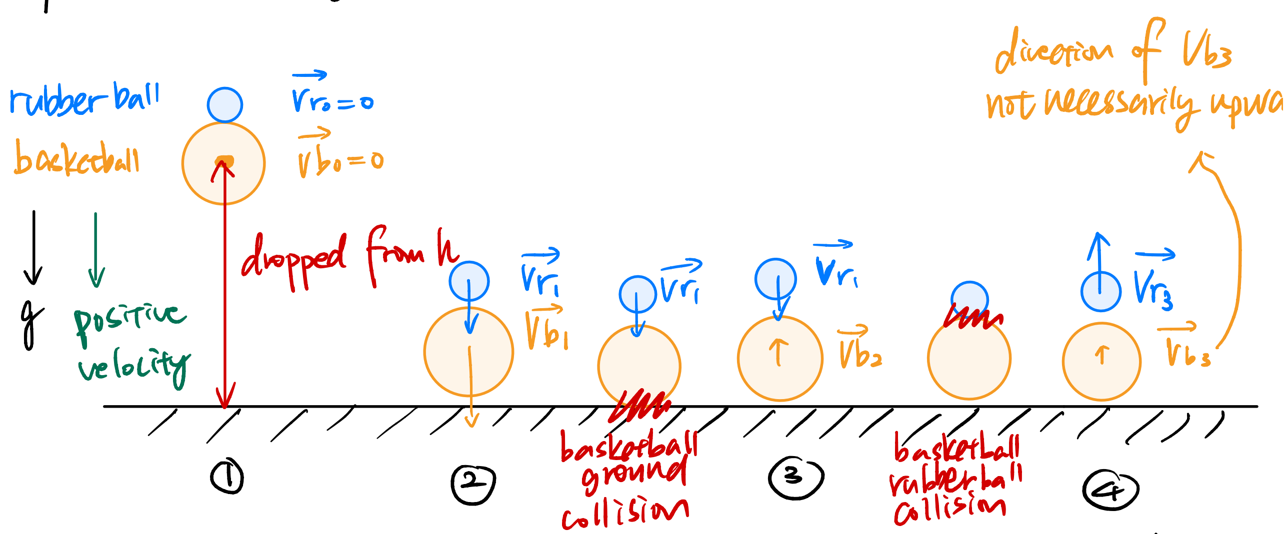
mass of basketball $m_b$

mass of rubber ball $m_r$

drop from height $h$

coefficient of restitution $e_b$, $e_r$

find: the height rubber ball bounce, $h_{max}$




1. rubber ball and basketball dropped from rest from height $h$
2. right before basketball collide with ground at $V_{b1}$
3. right after basketball collide with ground. immediately collide with rubber ball at $V_{b2}$
4. right after basketball collide with rubberball

From ① to ②

First, calculate $V_{r1}$ and $V_{b1}$

(velocity of rubber ball and basketball right before basketball hits the ground)

No external force other than gravity doing work

Conservation of Energy

(gravitational potential energy → kinetic energy)

$$\begin{cases} m_b g h = \frac{1}{2} m_b V_{b1}^2 \\ m_r g h = \frac{1}{2} m_r V_{r1}^2 \end{cases} \Rightarrow \boxed{\begin{cases} V_{b1} = \sqrt{2gh} \\ V_{r1} = \sqrt{2gh} \end{cases}}$$

② to ③

basketball collide with ground with $V_{b1}$,

results in velocity $V_{b2}$

ground remains static, $V_{ground} = 0$

Using eqn (10.56) on p542

coefficient of restitution between basketball and ground

$$V_{ground} - V_{b2} = e_b (V_{b1} - V_{ground})$$

$$\Rightarrow \quad 0 - V_{b2} = e_b (V_{b1} - 0)$$

$$\boxed{V_{b2} = -e_b V_{b1} = -e_b \sqrt{2gh}}$$

(Intuition: if $e_b = 1$, no energy loss during collision, $V_{b2} = -V_{b1}$, same velocity magnitude, direction reversed.)

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

③ to ④

basketball has velocity $V_{b2}$,
collides with rubberball with velocity $V_{r1}$,
resulting in basketball velocity $V_{b3}$ rubberball velocity $V_{r2}$

also using eqn (10.56) coefficient of restitution eqn

$$V_{r2} - V_{b3} = e_r(V_{b2} - V_{r1}) \qquad (i)$$
$$= e_r(-e_b\sqrt{2gh} - \sqrt{2gh})$$

Conservation of momentum:

$$m_r V_{r2} + m_b V_{b3} = m_b V_{b2} + m_r V_{r1}$$
$$= m_b(-e_b\sqrt{2gh}) + m_r\sqrt{2gh} \qquad (ii)$$
$$= \sqrt{2gh}\,(m_r - e_b m_b)$$

eqn (ii) + $m_b \times$ eqn (i)

$$\Rightarrow m_r V_{r2} + m_b V_{b3} + (m_b V_{r2} - m_b V_{b3})$$
$$= \sqrt{2gh}\,(m_r - e_b m_b) + m_b e_r(-e_b\sqrt{2gh} - \sqrt{2gh})$$

$$\Rightarrow (m_r + m_b)V_{r2} = \sqrt{2gh}\,(m_r - e_b m_b - e_r e_b m_b - e_r m_b)$$

$$\boxed{V_{r2} = \frac{m_r - e_b m_b - e_r e_b m_b - e_r m_b}{m_r + m_b}\sqrt{2gh}}$$

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

after (t)

rubber ball gets launched up in the air
reaching maximum height
no external forces doing work
kinetic energy converted to gravitational potential energy

$$m_r g h_{max} = \frac{1}{2} m_r V_{r_2}^2$$

$$h_{max} = \frac{V_{r_2}^2}{2g}$$

$$= \left( \frac{m_r - e_b m_b - e_r e_b m_b - e_r m_b}{m_r + m_b} \sqrt{2gh} \right)^2 / 2g$$

$$\Rightarrow \boxed{h_{max} = \left( \frac{m_r - e_b m_b - e_r e_b m_b - e_r m_b}{m_r + m_b} \right)^2 h}$$

(b)

Having Solved

$$V_{r2} = \frac{m_r - e_b m_b - e_r e_b m_b - e_r m_b}{m_r + m_b}\sqrt{2gh}$$

$$h_{max} = \frac{V_{r2}^2}{2g} = h_{max} = \left(\frac{m_r - e_b m_b - e_r e_b m_b - e_r m_b}{m_r + m_b}\right)^2 h$$

Finding the $e_r$, $e_b$ that give largest $h_{max}$ is equivalent of finding largest $\left(\frac{m_r - e_b m_b - e_r e_b m_b - e_r m_b}{m_r + m_b}\right)^2$

Within the Context of this problem, $V_{r2}$, the velocity of rubber ball after collision with basketball, goes upwards.

Since positive velocity direction is defined downwards, $V_{r2}$ is negative, meaning

$$\frac{m_r - e_b m_b - e_r e_b m_b - e_r m_b}{m_r + m_b} = \frac{m_r - (e_b + e_r e_b + e_r) m_b}{m_r + m_b}$$

is negative

thus, the larger $e_b + e_r e_b + e_r$ becomes,
the more negative $\frac{m_r - (e_b + e_r e_b + e_r) m_b}{m_r + m_b}$ becomes,

the larger $h_{max} = \left(\frac{m_r - e_b m_b - e_r e_b m_b - e_r m_b}{m_r + m_b}\right)^2 h$ becomes.

$h_{max}$ increases monotonically as $e_b + e_r e_b + e_r$ increases

since $e_b \leq 1$, $e_r \leq 1$

$e_b = e_r = 1$ gives $h_{max} = \left(\frac{m_r - 3m_b}{m_r + m_b}\right)^2 h$

and if $m_b >> m_r$

$h_{max} \longrightarrow 9h$

**10.5.12** According to the problem above, unless energy is created in the collision (as in an explosion), $-1 \leq e \leq 1$. Show that, for given masses and given initial velocities, that the loss of system kinetic energy is maximized by $e = 0$.

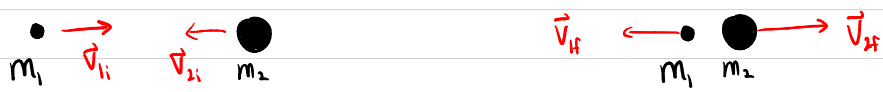
MAE 2030 HW5 Problem 20 Solution

10.5.12

Depei Yu

**10.5.12** According to the problem above, unless energy is created in the collision (as in an explosion), $-1 \leq e \leq 1$. Show that, for given masses and given initial velocities, that the loss of system kinetic energy is maximized by $e = 0$.

Consider two masses, $m_1$ and $m_2$, that undergo a collision:




Before collision After collision

Given: $m_1$, $m_2$, $\vec{v}_{1i}$, $\vec{v}_{2i}$

Find: value of e that maximizes loss of kinetic energy of system

Method: Use Koenig's Theorem and conservation of linear momentum

Define system as both $m_1$ and $m_2$.

Conservation of Linear Momentum: $m_{tot}\vec{v}_G = m_1\vec{v}_1 + m_2\vec{v}_2$

G is the COM of the entire system (both masses)

Since no external forces act on the system of both masses, linear momentum is conserved for the system: $\underbrace{\vec{v}_G^-}_{\text{before collision}} = \underbrace{\vec{v}_G^+}_{\text{after collision}} = \vec{v}_G = \text{constant}$

Use Koenig's Theorem to decompose kinetic energy of system ($E_k$) after collision into two terms: kinetic energy of the system's COM and kinetic energy of particles relative to system COM.

$$E_{kf} = \underbrace{\frac{1}{2} m_{tot} \vec{v}_G^{\,2}}_{\text{Kinetic energy of System COM}} + \underbrace{\frac{1}{2} m_1 \left(\vec{v}_{1f} - \vec{v}_G\right)^2}_{\text{Kinetic energy of } m_1 \text{ relative to system COM}} + \underbrace{\frac{1}{2} m_2 \left(\vec{v}_{2f} - \vec{v}_G\right)^2}_{\text{Kinetic energy of } m_2 \text{ relative to system COM}}$$

Let $E_{ki}$ = total kinetic energy of system before collision and $E_{kf}$ be total kinetic energy after collision.

Regardless of what $E_{ki}$ is, $E_{kf}$ is minimized if both the kinetic energy of $m_1$ relative to the system COM and the kinetic energy of $m_2$ relative to the system COM are 0. This occurs when $\vec{v}_{1f} = \vec{v}_G$ and $\vec{v}_{2f} = \vec{v}_G$.

Looking at the restitution equation, plug in $\vec{v}_{1f} = \vec{v}_G$ and $\vec{v}_{2f} = \vec{v}_G$:

$$e = \frac{\vec{v}_{2f} - \vec{v}_{1f}}{\vec{v}_{1i} - \vec{v}_{2i}} = \frac{\vec{v}_G - \vec{v}_G}{\vec{v}_{1i} - \vec{v}_{2i}} = \frac{0}{\vec{v}_{1i} - \vec{v}_{2i}}$$

$e = 0$ when the kinetic energy of the system after the collision is minimized (and thus the loss of kinetic energy is maximized).

**Disclaimer:** *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*