SAMPLE 10.19 **Collision without energy loss:** A block of mass $m_1 = 2\,\text{kg}$ moves with speed $v_1 = 0.5\,\text{m/s}$ along the x-axis on a frictionless level ground behind another block of mass $m_2 = 10\,\text{kg}$ moving at a speed $v_2 = 0.2\,\text{m/s}$ in the same direction. The first block collides with the second block. Given that there is no loss of energy in this collision, find the speeds of the two blocks immediately after the collision.

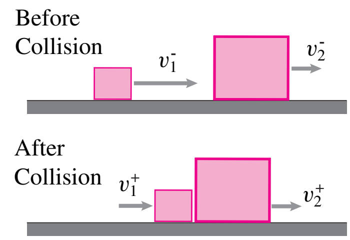

Figure 10.54

Solution We are given the speeds of two blocks (of known masses) just before the collision. It is also given that there is no loss of energy in the collision. We have to find the speed of the two masses immediately after collision.

We know that the linear momentum of the system consisting of the two blocks is conserved during the collision. Thus, if v_1^- and v_2^- are the speeds of the two masses just before the collision and v_1^+ and v_2^+ are their respective speeds immediately after the collision, then we have

$$m_1 v_1^+ + m_2 v_2^+ = m_1 v_1^- + m_2 v_2^- \tag{10.57}$$

Since there is no loss of energy in the collision, the energy of the system is conserved. Thus, $E^- = E^+$, or

$$\frac{1}{2}m_1(v_1^+)^2 + \frac{1}{2}m_2(v_2^+)^2 = \frac{1}{2}m_1(v_1^-)^2 + \frac{1}{2}m_2(v_2^-)^2. \tag{10.58}$$

Thus, we have two equations (eqn. (10.57) and eqn. (10.58)) in two unknowns, v_1^+ and v_2^+, and hence we can solve for them. It is now only a question in algebra. From eqn. (10.58), we have

$$\begin{aligned} m_1\left[(v_1^+)^2 - (v_1^-)^2\right] &= m_2\left[(v_2^-)^2 - (v_2^+)^2\right] \\ \Rightarrow \quad m_1(v_1^+ + v_1^-)(v_1^+ - v_1^-) &= m_2(v_2^- + v_2^+)(v_2^- - v_2^+) \end{aligned} \tag{10.59}$$

But, from eqn. (10.57), $m_1(v_1^+ - v_1^-) = m_2(v_2^- - v_2^+)$. Hence,eqn. (10.59) simplifies to

$$\begin{aligned} v_1^+ + v_1^- &= v_2^+ + v_2^- \\ \Rightarrow \quad v_1^+ - v_2^+ &= v_2^- - v_1^-. \end{aligned} \tag{10.60}$$

Multiplying the above equation by m_1 and subtracting from eqn. (10.57), we get

$$\begin{aligned} (m_1 + m_2)v_2^+ &= 2m_1 v_1^- + v_2^-(m_2 - m_1) \\ \Rightarrow \quad v_2^+ &= \frac{2m_1}{m_1 + m_2}v_1^- + \frac{m_2 - m_1}{m_1 + m_2}v_2^-. \end{aligned}$$

Now substituting the given values, $m_1 = 2\,\text{kg}$, $m_2 = 10\,\text{kg}$, $v_1^- = 0.5\,\text{m/s}$ and $v_2^- = 0.2\,\text{m/s}$ above, we get $v_2^+ = 0.3\,\text{m/s}$. Further, substituting the values of v_2^+ in eqn. (10.60), we get $v_1^+ = 0$, *i.e.,* the first mass comes to a halt!

$$\boxed{v_1^+ = 0 \text{ and } v_2^+ = 0.3\,\text{m/s}}$$

Comments: : Note that rather than using energy conservation equation directly as we did above, we could have used the given energy information t o set $e = 1$ (perfectly elastic collision) in eqn. (10.56) to get $v_2^+ - v_1^+ = -v_2^- + v_1^-$ (rather than deriving it as we did above). We can then solve this equation along with eqn. (10.57) to solve for v_1^+ and v_2^+.

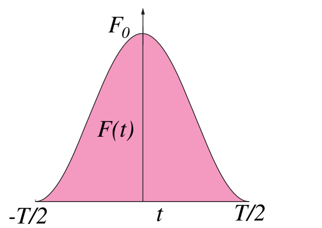

Figure 10.55

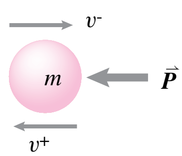

Figure 10.56: Free-body diagram of the ball during collision with collisional impulse $\vec{\boldsymbol{P}}$ acting on it.

SAMPLE 10.20 **Estimating peak force in a collision:** A metal ball of mass $m = 0.5\,\mathrm{kg}$ strikes a stationary surface S_1 with velocity $\vec{\boldsymbol{v}} = 10\,\mathrm{m/s}\hat{\boldsymbol{\imath}}$ and rebounds with velocity $\vec{\boldsymbol{v}} = -9\,\mathrm{m/s}\hat{\boldsymbol{\imath}}$. In a different experiment the same ball strikes another stationary surface S_2 with the same initial velocity and has the same rebound velocity. The contact times during the two experiments were different: 0.1 s and 0.001 s respectively. Assuming that the collisional force between the ball and the two surfaces can be modeled as $F(t) = \frac{F_0}{2}(1+\cos\frac{2\pi t}{T})$ (see fig. 10.55) where $-T/2 \le t \le T/2$ and T is the contact time, find the peak force F_0 in each case.

Solution Let the collisional impulse acting on the ball be $\vec{\boldsymbol{P}}$ (see fig. 10.56) given by

$$\vec{\boldsymbol{P}} = \int_{-T/2}^{T/2} \vec{\boldsymbol{F}}(t)dt.$$

From impulse-momentum relationship, we have

$$\vec{\boldsymbol{P}} = \Delta\vec{\boldsymbol{L}} = m\Delta\vec{\boldsymbol{v}}.$$

Since in the case of each surface, $\Delta\vec{\boldsymbol{v}}$ is the same ($\vec{\boldsymbol{v}}^+ - \vec{\boldsymbol{v}}^- = -19\,\mathrm{m/s}\hat{\boldsymbol{\imath}}$), the change in linear momentum $\Delta L = m\Delta\vec{\boldsymbol{v}}$ is also the same. Hence, the impulse acting on the ball in each case has to be the same. Now, let $\vec{\boldsymbol{P}}_1$ and $\vec{\boldsymbol{P}}_2$ be the impulses acting on the ball during the collision with surface S_1 and S_2 respectively. Then,

$$\begin{aligned}
\vec{\boldsymbol{P}}_1 &= -\int_{-T_1/2}^{T_1/2} \vec{\boldsymbol{F}}_1(t)dt\,\hat{\boldsymbol{\imath}} \\
&= -\int_{-T_1/2}^{T_1/2} \frac{(F_0)_1}{2}\left(1+\cos\frac{2\pi t}{T_1}\right)dt\hat{\boldsymbol{\imath}} \\
&= -\frac{(F_0)_1 T_1}{2}\hat{\boldsymbol{\imath}}
\end{aligned}$$

Similarly,

$$\vec{\boldsymbol{P}}_2 = -\frac{(F_0)_2 T_2}{2}\hat{\boldsymbol{\imath}}.$$

Now, setting $\vec{\boldsymbol{P}}_1 = \Delta L$, we get

$$\begin{aligned}
-\frac{(F_0)_1 T_1}{2}\hat{\boldsymbol{\imath}} &= -m\Delta v\,\hat{\boldsymbol{\imath}} \\
\Rightarrow \quad (F_0)_1 &= \frac{2m\Delta v}{T_1} \\
&= \frac{2\cdot 0.5\,\mathrm{kg}\cdot 19\,\mathrm{m/s}}{0.1\,\mathrm{s}} \\
&= 190\,\mathrm{N}.
\end{aligned}$$

Similarly,

$$\begin{aligned}
(F_0)_2 &= \frac{2m\Delta v}{T_2} \\
&= \frac{2\cdot 0.5\,\mathrm{kg}\cdot 19\,\mathrm{m/s}}{0.001\,\mathrm{s}} \\
&= 19000\,\mathrm{N}.
\end{aligned}$$

Clearly, the peak force is inversely proportional to the collision time. In fact, it is easy to see that for the given model of the impulsive force, the peak force $F_0 = \frac{2m\Delta v}{T}$. Thus if the change in momentum is constant, then the peak force varies as $1/T$.

$$\boxed{(F_0)_1 = 190\,\mathrm{N} \quad \text{and} \quad (F_0)_2 = 19000\,\mathrm{N}}$$

SAMPLE 10.21 A two-ball multiple collision experiment: A tennis ball of approximate mass $m_1 = 60\,\mathrm{gm}$ and a basketball of approximate mass $m_2 = 600\,\mathrm{gm}$ are used in a fun collision experiment. The two balls are held in air, one on top of the other with a tiny gap between them, at a height h from the ground as shown in the figure. The two balls are released simultaneously from rest. The coefficient of restitution between the tennis ball and the basketball is $e_1 = 0.6$ and that between the basketball and the floor is $e_2 = 0.9$. Assume that the collision between the two balls takes place immediately after the basketball rebounds from the floor. Find the height of the tennis ball flight in terms of h as a result of the collision.

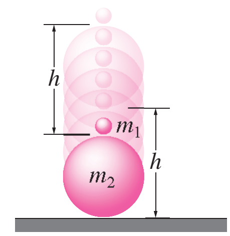

Figure 10.57

Solution We need to track two separate collisions here — one between the basketball and the floor, and second, between the tennis ball and the basketball. We can find the relevant vertical velocities before and after the collisions to determine the velocity of the tennis ball's flight which we can use to find the height of the flight. We will assume upward velocities to be positive.

Collision-1: Just before the basketball hits the floor, let its vertical velocity be v_2^- and let the tennis ball's speed at the same instant be v_1^-. Since both balls undergo free fall from height h before attaining these speeds, we have $v_1^- = v_2^- = -\sqrt{2gh}$. Now let v_2^+ be the speed of the basketball immediately after the collision with the ground (see fig. 10.58). Then, $v_2^+ = -e_2 v_2^- = e_2\sqrt{2gh}$.

Collision-2: We assume that the second collision, the collision between the tennis ball and the basketball, takes place immediately after the first collision. Hence, the velocity of the tennis ball just before the collision with the basketball can be assumed to be $v_1^- = -\sqrt{2gh}$. The second collision is shown in fig. 10.59. The after impact velocities of the two balls are v_1^+ and v_2^{++}. Now, from collision law, we have

$$v_1^+ - v_2^{++} = -e_1(v_1^- - v_2^+) = -e_1(-\sqrt{2gh} - e_2\sqrt{2gh}) = \sqrt{2gh}e_1(1+e_2). \qquad (10.61)$$

The conservation of linear momentum for the two-ball system gives

$$\begin{aligned} m_1 v_1^+ + m_2 v_2^{++} &= m_1 v_1^- + m_2 v_2^+) \\ \Rightarrow \quad v_1^+ + \frac{m_2}{m_1} v_2^{++} &= v_1^- + \frac{m_2}{m_1} v_2^+ \end{aligned}$$

Taking $M = m_2/m_1$, and substituting the values of v_1^- and v_2^+, we get

$$v_1^+ + M v_2^{++} = \sqrt{2gh}(Me_2 - 1). \qquad (10.62)$$

Now solving eqn. (10.61) and eqn. (10.62) simultaneously, we get

$$v_1^+ = \frac{\sqrt{2gh}}{1+M}\left[M(e_1 + e_2 + e_1 e_2) - 1\right].$$

This is the velocity with which the tennis ball takes off on its vertical flight. Let the height of this flight be h_f. Then, from constant acceleration motion formula, we get $(v_1^+)^2 = 2gh_f$, or $h_f = (v_1^+)^2/2g$. Thus, from the derived expression for v_1^+ above, we get

$$h_f = \frac{h}{(1+M)^2}\left[M(e_1 + e_2 + e_1 e_2) - 1\right]^2.$$

Substituting $M = m_2/m_1 = 10$, $e_1 = 0.6$, and $e_2 = 0.9$ above, we get $h_f = 3.11h$. Thus the tennis ball flies off to three times its original height.

$$\boxed{h_f = 3.11h}$$

Note: From the expression obtained for v_1^+, we see that if M is very large then $v_1^+ = \sqrt{2gh}(e_1 + e_2 + e_1 e_2)$ and $h_f = (e_1 + e_2 + e_1 e_2)^2 h$.

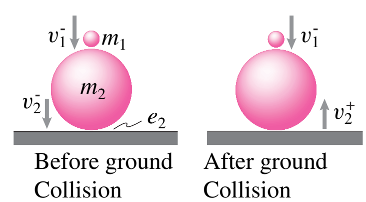

Figure 10.58: Collision of the basketball with the ground.

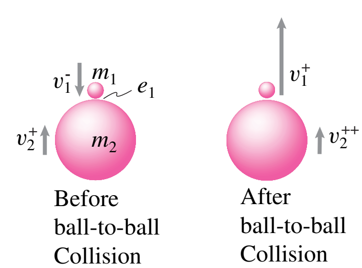

Figure 10.59: Collision of the tennis ball with the basketball.