10.5 1D Collisions

Preparatory Problems

10.5.1 Before a collision two particles, $m_A = 1\,\text{kg}$ and $m_B = 2\,\text{kg}$, have velocities of $v_A^- = 10\,\text{m/s}$ and $v_B^- = 5\,\text{m/s}$. After the collision the velocity of A is $v_A^+ = 6\,\text{m/s}$.

a) What is the momentum of A before the collision?

b) What is the momentum of B before the collision?

c) What is the system momentum before the collision?

d) What is the momentum of A after the collision?

e) What is the system momentum after the collision?

f) What is the momentum of B after the collision?

g) What is the impulse that A applies to B during the collision?

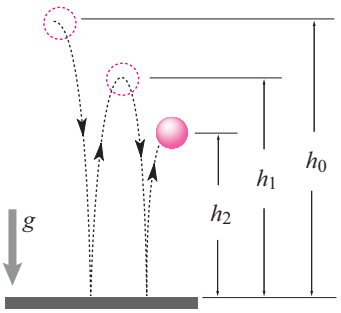
h) What is the impulse that B applies to A during the collision?

i) What is the kinetic energy of the system before the collision?

j) What is the kinetic energy of the system after the collision?

k) What is the coefficient of restitution?

10.5.2 A ball is dropped from a height of $h_0 = 10\,\text{m}$ onto a hard stationary surface. After the first bounce, it reaches a height of $h_1 = 6.4\,\text{m}$. What is the coefficient of restitution between the ball and ground? What is the height of the second bounce, h_2?



Problem 10.5.2

10.5.3 A 20 gram, 500 m/s bullet embeds in an initially-stationary 50 kg rigid block. What is the coefficient of restitution? What is the velocity of the block after this collision? *

10.5.4 A ball of mass m is dropped vertically from a height h. The only force acting on the ball in its flight is gravity. The ball strikes the ground with speed v^- and after collision it rebounds vertically with reduced speed v^+ directly proportional to the incoming speed, $v^+ = ev^-$, where $0 < e < 1$. What is the maximum height the ball reaches after one bounce, in terms of h, e, and g. *

10.5.5 Set up the following equations in matrix form and solve for v_A and v_B, if $v_0 = 2.6\,\text{m/s}$, $e = 0.8$, $m_A = 2\,\text{kg}$, and $m_B = 500\text{g}$:

$$m_A v_0 = m_A v_A + m_B v_B$$
$$-ev_0 = v_A - v_B.$$

More-Involved Problems

10.5.6 Before a collision two particles, $m_A = 7\,\text{kg}$ and $m_B = 9\,\text{kg}$, have velocities of $v_A^- = 6\,\text{m/s}$ and $v_B^- = 2\,\text{m/s}$. The coefficient of restitution is $e = .5$. Find the impulse of mass A on mass B and the velocities of the two masses after the collision.

10.5.7 Two frictionless masses $m_A = 2\,\text{kg}$ and $m_B = 5\,\text{kg}$ travel on straight collinear paths with speeds $V_A = 5\,\text{m/s}$ and $V_B = 1\,\text{m/s}$, respectively. The masses collide since $V_A > V_B$. Find the amount of energy lost in the collision. The coefficient of restitution is $e = 0.5$.

Problem 10.5.7

10.5.8 A ball of mass m is dropped from height h onto the solid hard ground where its coefficient of restitution is $e < 1$. The gravitational constant is g.

a) How many times does the ball bounce before it comes to a stop?

b) How long does it take from first release until it comes to a stop?

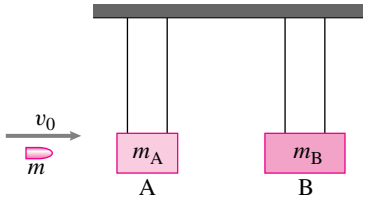
c) What is the total distance the ball travels before coming to a stop (add up and down distances)?

10.5.9 A bullet of mass m with initial speed v_0 is fired in the horizontal direction through block A of mass m_A and becomes embedded in block B of mass m_B. Each block is suspended by thin wires. The bullet causes A and B to start moving with speed of v_A and v_B respectively. Determine

a) the initial speed v_0 of the bullet in terms of v_A and v_B, *

b) the velocity of the bullet as it travels from block A to block B,and *

c) the energy loss due to friction as the bullet (1) moves through block A and (2) penetrates block B. *



Problem 10.5.9

10.5.10 A basketball with mass m_b is dropped from height h onto the hard solid ground on which it has coefficient of restitution e_b. Just on top of the basketball, falling with it and then bouncing against it after the basketball hits the ground, is a small rubber ball with mass m_r that has a coefficient of restitution e_r with the basketball.

a) In terms of some or all of m_b, m_r, h, g, e_b and e_r how high does the rubber ball bounce (measure height relative to the collision point)?

b) Assuming the coefficients of restitution are less than or equal to one, for given h, what mass and restitution parameters maximize the height of the bounce of the rubber ball and what is that height?

10.5.11 Show that it is necessary that $|e| \leq 1$ for the net kinetic energy (sum of the two kinetic energies of the colliding particles) to not increase.

10.5.12 According to the problem above, unless energy is created in the collision (as in an explosion), $-1 \leq e \leq 1$. Show that, for given masses and given initial velocities, that the loss of system kinetic energy is maximized by $e = 0$.