10.3 Simple Harmonic Oscillator

Preparatory Problems

10.3.1 The basic model.

a) Draw a spring (k) mass (m) system in a configuration where the spring is stretched.

b) On the drawing indicate the variable x.

c) Draw a free-body diagram of the mass.

d) Write the equation of linear momentum balance for the mass.

e) Rearrange the momentum balance equation to get the harmonic oscillator equation in standard form.

f) Write the general solution to the harmonic oscillator equation in two different ways (one as a sum of a sine and cosine function and one as a phase shifted sine or cosine function).

g) What is the natural frequency of this system?

h) What is the period?

i) What is the frequency (or circular frequency)?

j) Find the solution for the special case that the mass is released from rest at $x(0) = x_0$.

- give the analytic expression.
- plot the position vs time for at least one whole cycle of motion.
- with the same time scale, plot velocity vs time (what is the peak velocity).
- with the same time scale, plot both the potential and kinetic energies vs time.

k) Find the solution for the special case that the mass is launched at v_0 from the rest position (just the analytic form, no need to repeat all the parts just above).

10.3.2 Does the function $x = C_1 e^{\lambda t} + C_2 e^{-\lambda t}$ satisfy the harmonic oscillator equation $\ddot{x} + \lambda^2 x = 0$ for any, possibly special, values of C_1 and C_2? Show that it does or does not.

10.3.3 Given that $\ddot{x} = -cx$, with $c = 1/\mathrm{s}^2$, $x(0) = 1\,\mathrm{m}$, and $\dot{x}(0) = 0$ find:

a) $x(\pi\,\mathrm{s}) = ?$

b) $\dot{x}(\pi\,\mathrm{s}) = ?$

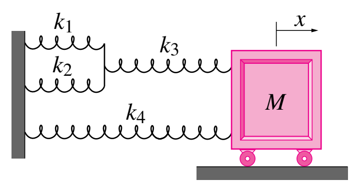
10.3.4 Given that $\ddot{x} + \lambda^2 x = C_0$, $x(0) = x_0$, and $\dot{x}(0) = 0$, find the value of x at $t = \pi/\lambda$ s.

More-Involved Problems

10.3.5 A spring and mass system is shown in the figure.

a) First, as a review, let k_1, k_2, and k_3 equal zero and k_4 be non-zero. What is the natural frequency of this system?

b) Now, let all the springs have non-zero stiffness. What is the stiffness of a single spring equivalent to the combination of k_1, k_2, k_3, k_4? What is the frequency of oscillation of mass M?



Problem 10.3.5

10.3.6 Mass m hangs from a spring with constant k and which has the length l_0 when it is relaxed (i.e., when no mass is attached). It only moves vertically.

a) Draw a Free-Body Diagram of the mass.

b) Write the equation of linear momentum balance. *

c) Reduce this equation to a standard differential equation in x, the position x of the mass. *

d) Verify that one solution is that $x(t)$ is constant at $x = l_0 + mg/k$.

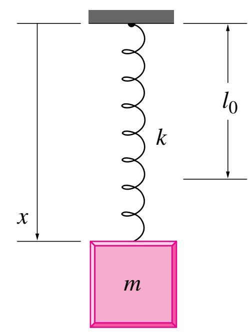
e) What is the meaning of that solution? (That is, describe in words what is going on.) *

f) Define a new variable $\hat{x} = x - (l_0 + mg/k)$. Substitute $x = \hat{x} + (l_0 + mg/k)$ into your differential equation and note that the equation is simpler in terms of the variable $\hat{x}$. *

g) Assume that the mass is released from an an initial position of $x = D$. What is the motion of the mass? *

h) What is the period of oscillation of this oscillating mass? *

i) Why might this solution not make physical sense for a long, soft spring if the initial stretch is large. In other words, what is wrong with this solution if $D > \ell_0 + 2mg/k$? *

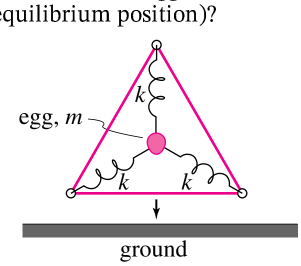


Problem 10.3.6

10.3.7 One egg-drop contestant used a structure which held the egg (mass m) at the center with rubber bands. Consider the 2D model shown. The springs are linear with spring constants k. After falling a height h the frame hits the ground on a flat edge. Assume small motions (deflection $\ll$ side-length) and that the springs do not buckle.

a) what is the vibration frequency after impact?

b) What is the maximum vertical deflection of the egg (relative to its equilibrium position)?



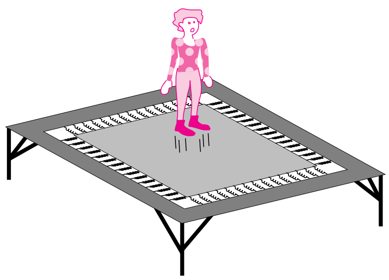
Problem 10.3.7

10.3.8 A person jumps on a trampoline. The trampoline is modeled as having an effective vertical undamped linear spring with stiffness $k = 200\,\mathrm{lbf/ft}$. The person is modeled as a rigid mass $m = 150\,\mathrm{lbm}$. $g = 32.2\,\mathrm{ft/s^2}$.

a) What is the period of motion if the person's motion is so small that her feet never leave the trampoline? *

b) What is the maximum amplitude of motion (amplitude of the sine wave) for which her feet never leave the trampoline? *

c) (harder) If she repeatedly jumps so that her feet clear the trampoline by a height $h = 5\,\mathrm{ft}$, what is the period of this motion (note, the contact time is *not* exactly half of a vibration period)? [Hint, a neat graph of height vs time will help.] *



Problem 10.3.8: A person jumps on a trampoline.