10.2.3 A force $F = F_0 \sin(ct)$ acts on a particle with mass $m = 3\,\text{kg}$ which has position $x = 3\,\text{m}$, velocity $v = 5\,\text{m/s}$ at $t = 2\,\text{s}$. $F_0 = 4\,\text{N}$ and $c = 2/\text{s}$. At $t = 2\,\text{s}$ evaluate (give numbers and units):

a) a,

b) E_K,

c) P,

d) $\dot{E}_K$,

e) the rate at which the force is doing work.

Answer:
(a) $-1.008\,\text{m/s}$, (b) 37.5 Joules, (c), (d) and (e) -15.12 Watts

9.2.3

$F = F_0 \sin(ct)$

$m = 3\text{kg}$

$\dot{x}\big|_{t=2} = 5\,m/s$, $x\big|_{t=2} = 3m$, $c = 2/s$

a) $m\ddot{x} = F = F_0 \sin ct$

$x = -\frac{F_0}{mc^2} \sin ct + At + B$

$a = \ddot{x} = \frac{F_0 \sin ct}{m} = \frac{4\sin(4)}{3} = \frac{4(-0.756)}{3}$

$a = -1.008\, m/s$

a) $E_K = \frac{1}{2}mv^2 = \frac{1}{2}\cdot m\cdot(5)^2 = \frac{75}{2}$ Joules

$= 37.5$ Joules

b) $P = F\cdot V = 4(-0.756)5 = -15.12$ Watts

c) $\dot{E}_K = \frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2\right) = \frac{1}{2}m\dot{x}\times 2\times\ddot{x} = m\dot{x}\ddot{x} = -3\times 5\times 1.008$

$= -15.1$ Watts

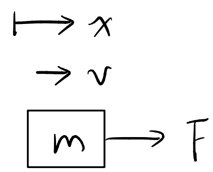
e) Rate at which Force is doing work

$W_F = F\cdot V = -15.12$ watts

MAE 2030 HW2 P6 Solution

Duan Li

Given:



1D problem

$F = F_0 \sin(ct)$, $F_0 = 4\,N$, $c = 2\,s^{-1}$

$m = 3\,kg$

@ $t = 2\,s$, $x = 3\,m$, $v = 5\,m/s$

Find: @ $t = 2\,s$

a) a acceleration

b) E_k kinetic energy

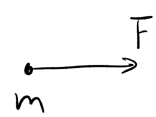
c) P power of the force

d) $\dot{E}_k$

e) the rate at which the force is doing the work

Soln:

a) FBD:



LMB: $m\ddot{x} = F$

Let $t' = 2\,s$

$$a(t') = \ddot{x}(t') = \frac{1}{m} F(t') = \frac{1}{m} F_0 \sin(ct')$$

$$= \frac{1}{3\,kg}\, 4N \sin(2\,s^{-1} \cdot 2s) = \boxed{-1.01\ m/s^2}$$

b) $$E_k(t') = \frac{1}{2} m v(t)^2 = \frac{1}{2}\, 3\,kg \cdot (5\,m/s)^2 = \boxed{37.5\ J}$$

c) $P(t') = F(t')\,v(t') = 4\text{N} \sin(2\text{s}^{-1} \cdot 2\text{s}) \cdot 5\ \text{m/s}$

$= \boxed{-15.14\ \text{W}}$

d) Power balance eqn :

$\dot{E}_k(t') = P(t') = \boxed{-15.14\ \text{W}}$

e) $\dfrac{dW}{dt} = \dfrac{F\,dx}{dt} = Fv = P$

$\left.\dfrac{dW}{dt}\right|_{t'} = P(t') = \boxed{-15.14\ \text{W}}$

10.2.4 A force only depends on position according to $F = C_0 + C_1 x$ where C_0 and C_1 are constants. What is the work done by this force when the point to which it is applied moves from x_1 to x_2? Answer in terms of some or all of C_0, C_1, x_1 and x_2.

9.31.

$$F = C_0 + C_1 x$$

workdone, $W = \int_{x_1}^{x_2} F\,dx$

$$= \int_{x_1}^{x_2} (C_0 + C_1 x)\,dx = \left[C_0 x + \frac{C_1 x^2}{2}\right]_{x_1}^{x_2}$$

$$\Rightarrow W = C_0(x_2 - x_1) + \frac{C_1}{2}\left[x_2^2 - x_1^2\right]$$

$$\Rightarrow W = (x_2 - x_1)\left[C_0 + \frac{1}{2}C_1(x_2 + x_1)\right]$$ Ans

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

10.2.5 Find the potential E_P associated with each of these force fields.

a) $F = 0$.

b) $F = F_0$ (=constant).

c) $F = kx$.

d) $F = A\sin(x/x_0)$.

e) $F = c/x^2$.

9.32.

$$F = -\frac{dE_p}{dx}.$$

a) $F=0$, $\frac{dE_p}{dx}=0 \Rightarrow \boxed{E_p = C = \text{constt.}}$

b) $F=F_0$, $\int dE_p = \int F_0 dx$

$$\Rightarrow \boxed{E_p = F_0 x + C}$$

c) $F = Kx.$

$$\Rightarrow \int dE_p = \int Kx\,dx$$

$$\Rightarrow \boxed{E_p = \frac{1}{2}Kx^2 + C.}$$

d) $F = A\sin(x/x_0)$

$$\Rightarrow \boxed{E_p = -Ax_0\cos(x/x_0) + C.}$$

10.2.7 A mass m is held in place by a spring whose restoring force is $T(x) = kx$. Derive the equation of motion of the system (that is, find the acceleration a in terms of x).

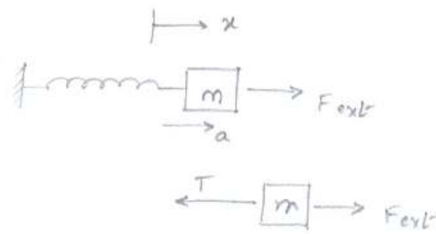
9.2.7

Equation of motion

$m\ddot{x} = F_{ext} - T$

$m\ddot{x} = F_{ext} - Kx$

$m\ddot{x} + Kx = F_{ext}$



10.2.8 The peak propulsion force on a 4-wheel-drive car is about $\mu m g$ where $\mu \approx 1$ for rubber on road (a bit more for fancy racing tires). Assume a car starts from rest at position zero. Answer the following questions with symbols and with numbers (using $\mu = 1$, $m = 1000\,\mathrm{kg}$, and $g = 10\,\mathrm{m/s^2}$).

a) What is the minimum distance required to reach $v_1 = 60\,\mathrm{mph}$?

b) What is the extra distance required to get from $v_1 = 60\,\mathrm{mph}$ up to $v_2 = 70\,\mathrm{mph}$?

c) What is the peak power used by the engine in getting up to $v_1 = 60\,\mathrm{mph}$ (assuming no dissipation and no air friction)?

9.35

$F_P = \mu m g$

$= 1 \times 1000 \times 10\ \mathrm{N}.$

$\therefore$ acceleration, $a = 10\,\mathrm{ms^{-1}}$.

Since motion is constant acceleration.

$v = u + at$ — (1)

$v^2 = u^2 + 2as$ — (2)

& $s = ut + \frac{1}{2}at^2$ — (3) can be used.

here $u = 0$.

(a) $\therefore$ from (2)

$$s = \frac{v^2}{2a}.$$

$$v = 60\,\mathrm{mph} = \frac{60 \times 1.609 \times 10^3}{60 \times 60}\,\mathrm{ms^{-1}} = 26.8166\,\mathrm{ms^{-1}}.$$

$\therefore S_1 = 35.9566\,\mathrm{m}.$ Ans

(b) from (1) $\Rightarrow$ $v_2 = 70\,\mathrm{mph} = 31.286\,\mathrm{ms^{-1}}.$

$\therefore$ $31.286 = 26.8166 + 10t$

$\Rightarrow t = 0.44695\,\mathrm{sec}.$ Ans

(c) ~~Forward~~ $F \cdot v\,dt$

peak power used by engine in getting up to $v_1 = 60\,\mathrm{mph}$

$P = F v$

10.2.9 A car (mass $m = 1000\,\text{kg}$) traveling at speed $v_0 = 30\,\text{m/s}$ crashes into a brick wall and comes to a stop as the front end of the car compresses a distance $d = 1\,\text{m}$. Answer with symbols and numbers. Assume constant deceleration during the crash. Neglect the mass of the crushing region of the car.

a) What is the total energy dissipated in the crash?

b) What is the force of the car on the wall?

c) What is the force of the wall on the car?

d) What is the deceleration of the car passengers (assuming they are strapped in and move with the bulk of the car). Answer in g's?

e) Assuming an $m_p = 50\,\text{kg}$ person, what is the force of the seat belts on the person (answer in body weight).

f) If a parent was holding a 15 kg child on his lap, what force would he need to hold on to the child through the crash (answer in N and in number of child body weights).

9.36

$m = 1000\,\text{Kg}$.

$v_0 = 30\,\text{ms}^{-1}$.

$d = 1\,\text{m}$.

a). Total energy dissipated in crash

~~[illegible]~~ $= (K.E)_{\text{before crash}} - (KE)_{\text{after crash}}$.

(neglecting change of potential energy).

$= \frac{1}{2} \times 1000 \times 30^2 - 0$

$= 450\,\text{KJ}$ Ans

(b) Since constant accelⁿ is assumed during crash.

F = Constt.

$F \times 1 = 450\,\text{KJ}$

$\Rightarrow F = 450\,\text{KN}$. Ans.

(c) Newton second law → force of the wall on car = 450 KN.

(d) $F = 450\,\text{KN}$

$\therefore\ a = \frac{450 \times 10^3}{10^3} = 450\,\text{m/s}^2$.

$\boxed{a = 45.871g.}$ $g = 9.81\,\text{ms}^{-2}$.

Ans

(e) Force of the seat belt on the person

$= 50 \times 450\,\text{N}$

$= 2293.577$ times body weight.

(f) $F = 450 \times 15\,\text{N}$. If parent is held stationary by belt.

$= 6.750\,\text{KN}. = 6750\,\text{N}$. Ans

$= 450$ times child body weight.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

10.2.10 A kid (m = 90 lbm) stands on a h = 10 ft wall and jumps down, accelerating with g = 32 ft/s^2. Upon hitting the ground with straight legs, she bends them so her body slows to a stop over a distance d = 1 ft. Neglect the mass of her legs. Assume constant deceleration as she brakes the fall.

a) What is the total distance her body falls?

b) What is the potential energy lost?

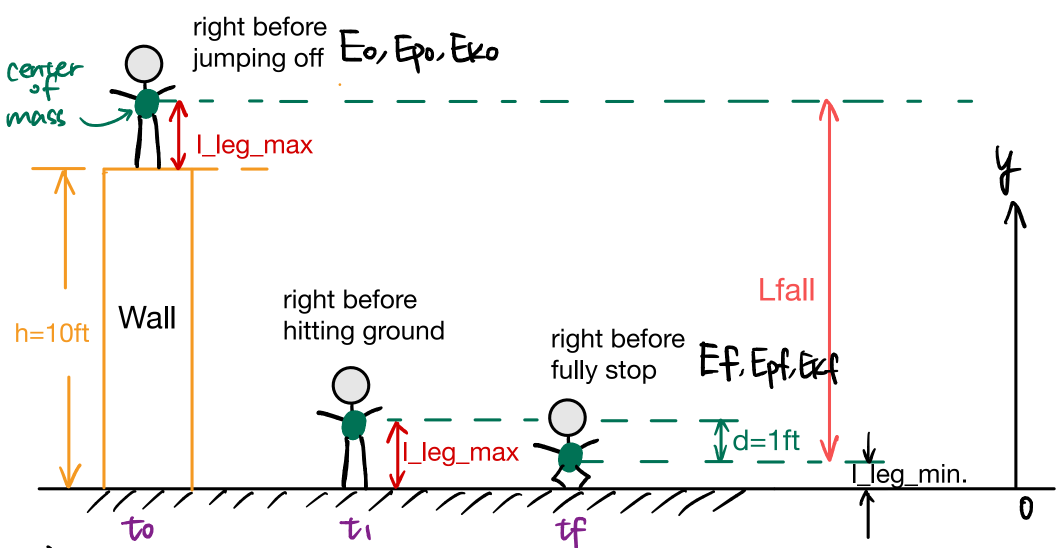
c) How much work must be absorbed by her legs?

d) What is the force of her legs on her body? Answer in symbols, numbers and numbers of body weight (*i.e.*, find F/mg).

Answer:
11 ft

Katherine Cao

Lfall



a)

$$L_{fall} = l_{legmax} + h - l_{legmin}$$
$$= l_{legmax} + h - (l_{legmax} - d)$$
$$= \boxed{h+d} = 10ft + 1ft$$
$$= \boxed{11ft}$$ (also given in HW hints)

The total distance center of mass fell equals the height of the wall h plus the distance of bending d

$$\Rightarrow L_{fall} = h+d$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

b) Change in potential energy

= final potential energy − initial potential energy

$$\Rightarrow \Delta E_p = E_{pf} - E_{p0} = mg(-L_{fall})$$

$$= -mg(h+d) = -990 \text{ lbf-ft}$$

Potential energy lost = negative of change in potential energy

See P232~233 of RuinaPratap (or read more of Chapter 4.1) for unit conversions

$$\text{Thus, } E_{plost} = -\Delta E_p = \boxed{mg(h+d)}$$

$$= (90\,\text{lbm})(32\,\text{ft/s}^2)(11\,\text{ft}) = 31680 \text{ pdl-ft}$$

$$= \boxed{990 \text{ lbf-ft}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

c) comparing the kid's state right before jumping off the wall and right before fully stop, velocity of the kid is zero for both states

$$E_0 = E_{p0} + \cancel{E_{k0}}^{\,0} = E_{p0}$$

$$E_f = E_{pf} + \cancel{E_{kf}}^{\,0} = E_{pf}$$

Change in ~~total~~ energy

= final total energy − initial ~~total~~ energy

$$\Delta E = E_f - E_0 = E_{pf} - E_{p0} = \Delta E_p$$

Total energy lost

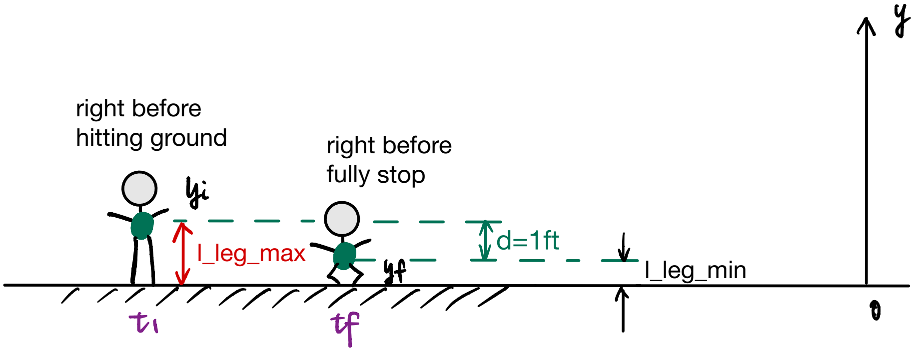
= negative of change in total energy

$$\Rightarrow \quad E_{lost} = -\Delta E = -\Delta E_p$$

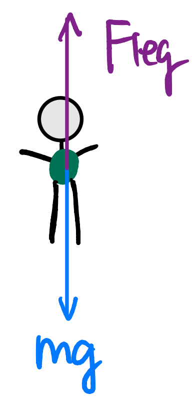
from b) $\Delta E_p = -mg(h+d) = -990 \text{ lbf-ft}$

Work absorbed by leg should equal ~~total~~ energy lost of center of mass of the kid

$$\boxed{W_{absorb} = E_{lost} = mg(h+d) = 990 \text{ lbf-ft}}$$



FBD for ti to tf



$y_i = l_{legmax}$ $\quad y_f = l_{legmin} = l_{legmax} - d$

d) Work done by leg on center of mass should equal energy change of center of mass:

$$W_{leg} = \Delta E \quad \text{from c)} \quad \Delta E = \Delta E_p = -mg(h+d)$$

With constant deceleration, using newton's 2nd law

$$\Sigma F = F_{leg} - mg = ma$$

$$F_{leg} = m(g+a) \quad m, g, a \text{ all constant value}$$

$\Rightarrow$ F_{leg} is constant

$$W_{leg} = \int_{y_i}^{y_f} F_{leg}\, dy = F_{leg}\, y \Big|_{y_i = l_{legmax}}^{y_f = l_{legmax} - d}$$

$$= F_{leg}(y_f - y_i) = F_{leg}(l_{legmax} - d - l_{legmax})$$

$$= -F_{leg}\, d = \Delta E = -mg(h+d)$$

$$\Rightarrow F_{leg} = \frac{mg(h+d)}{d} = \boxed{\frac{(h+d)}{d} mg}$$

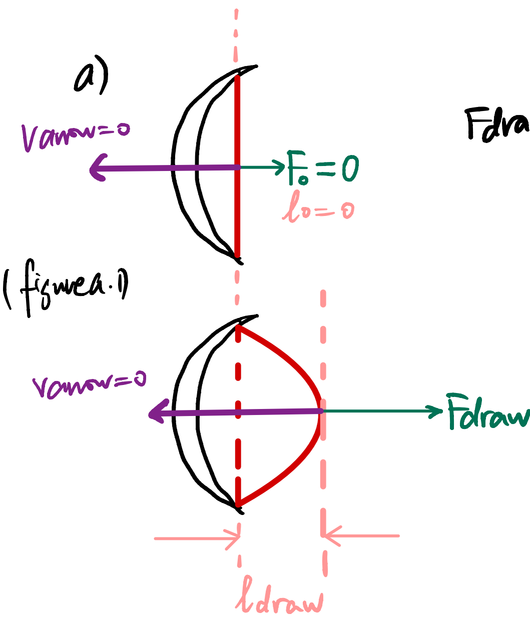
$$= \left(\frac{10\text{ft}+1\text{ft}}{1\text{ft}}\right)(90\text{lbm})(32\text{ft/s}^2) = 31680\text{ pdl} = \boxed{990\text{ lbf}}$$

10.2.11 In traditional archery, when pulling an arrow back the force starts from 0 and increases approximately linearly up to the peak 'draw force' F_{draw}. The draw force varies from about $F_{draw} =$ 25 lbf for a bow made for a small person to about $F_{draw} =$ 75 lbf for a bow made for a big strong person. The distance the arrow is pulled back, the draw length ℓ_{draw}, varies from about $\ell_{draw} =$ 2 ft for a small adult to about 30 inch for a big adult. An arrow has mass of about 300 grain (1 grain $\approx$ 64.8milli gm, so an arrow has mass of about 19.44 $\approx$ 20 gm $\approx$ 3/4 ounce). Give all answers in symbols and numbers.

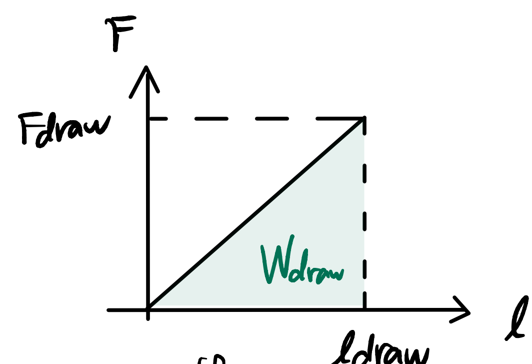
a) What is the range of speeds you can expect an arrow to fly?

b) What is the range of heights an arrow might go if shot straight up (it's a bad approximation, but for this problem neglect air friction)?

$m_{arrow} = 3/4$ ounce ≈ 0.047 lbm



(figure a.1)



(figure a.2)

The total energy of the bow-arrow system at any time consists of the kinetic energy of the arrow and the potential energy stored in the bow's string. The change in total energy of the system is due to work done by exteral force, in this case, Wdraw done by draw force.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

As stated in the prompt, draw force approximately linearly increase from 0 to peak Fdraw over a length of ldraw

$\Rightarrow F = F_{draw}\, l / l_{draw}$

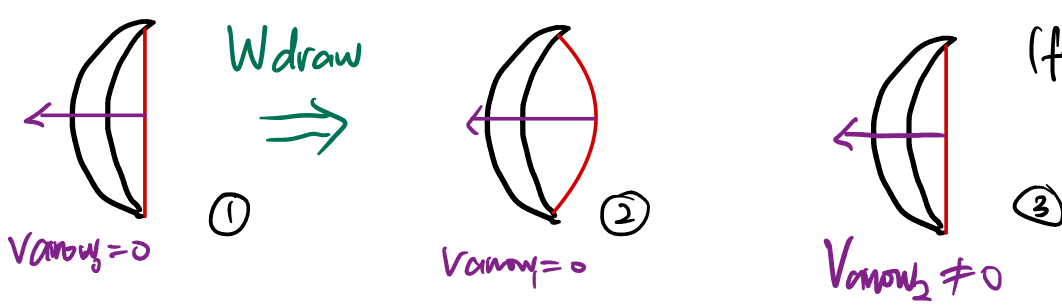
Thus,

$$W_{draw} = \int_{l_0=0}^{l_{draw}} F\, dl = \int_{l_0=0}^{l_{draw}} \frac{F_{draw}\, l}{l_{draw}}\, dl$$

(or look at the area of figure a.2)

$$= \frac{F_{draw}\, l^2}{2\, l_{draw}} \Bigg|_{l_0}^{l_{draw}} = \frac{1}{2} F_{draw}\, l_{draw}$$

The above Wdraw is the work done by the draw force to the bow-arrow system during the process of pulling the arrow back. Looking at the process of "releasing" the arrow, there is no eternal force doing work to the system, thus there is no change when considering the total energy of the bow-arrow system.



(figure a.3)

$E_0 = E_{arrow_0} + E_{string_0}$
$= 0 + 0 = 0$

$E_1 = E_{arrow_1} + E_{string_1}$
$= E_0 + W_{draw}$

$E_2 = E_{arrow_2} + E_{string_2}$
$= E_1$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

As shown in figure a.3

comparing state 1 and 3, the difference in total energy of the bow-arrow system equals work done by external force W_{draw}. Since bow string restored to resting length, W_{draw} is fully converted to kinetic energy of the arrow E_{arrow_2} if neglect energy loss throughout the process.

Thus,

$$W_{draw} = \frac{1}{2} F_{draw} \ell_{draw} = E_{arrow_2} = \frac{1}{2} m_{arrow} v_{arrow_2}^2$$

$$\Rightarrow v_{arrow_2} = \sqrt{\frac{F_{draw} \ell_{draw}}{m_{arrow}}}$$

The upper limit of v_{arrow_2} set by bow for a big, strong person and the lower limit set by bow for a small person.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$V_{arrow_2\,max} = \sqrt{\frac{F_{draw\,big}\; l_{big}}{M_{arrow}}} = \sqrt{\frac{75\text{ lbf }\; 30\text{ in}\left(\frac{1\text{ ft}}{12\text{ in}}\right)}{0.047\text{ lbm}}}$$

$$= \sqrt{\frac{(75\text{ lbf})\left(32.2\,\frac{\text{lbm}}{\text{lbf}}\,\text{ft/s}^2\right)\; 2.5\text{ ft}}{0.047\text{ lbm}}} \simeq 358\text{ ft/s}$$

$$V_{arrow_2\,min} = \sqrt{\frac{F_{draw\,small}\; l_{small}}{M_{arrow}}} = \sqrt{\frac{25\text{ lbf }\; 2\text{ ft}}{0.047\text{ lbm}}}$$

$$= \sqrt{\frac{(25\text{ lbf})\left(32.2\,\frac{\text{lbm}}{\text{lbf}}\cdot\text{ft/s}^2\right)\; 2\text{ ft}}{0.047\text{ lbm}}} \simeq 185\text{ ft/s}$$

$$\boxed{\text{Thus}\quad 185\text{ ft/s} \le V_{arrow_2} \le 358\text{ ft/s}}$$

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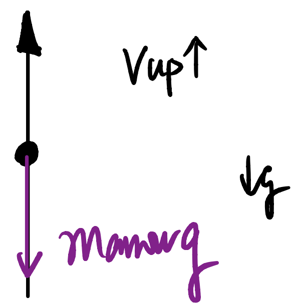
b) arrow



If shot straight up, assuming no energy loss due to drag on arrow due to air or other reasons the ~~total~~ energy of the arrow is conserved, meaning the sum of the arrow's kinetic energy and gravitational potential energy does not change.

$$E_{arrow} = E_{k\,arrow} + E_{p\,arrow}$$

$$= E_{k\,arrow\_release} + E_{p\,arrow\,0} = E_{k\,arrow\text{-}top} + E_{p\,arrow\text{-}top}$$

$$= \frac{1}{2} M_{arrow} V_{arrow}^2 + M_{arrow}\, g(h - h_0)$$

$$= \frac{1}{2} M_{arrow} V_{arrow_2}^2 + mg(h_0 - h_0)$$

$$= \frac{1}{2} M_{arrow} V_{arrow\,top}^2 + mg\,(h_{top} - h_0)$$

$$\Rightarrow \tfrac{1}{2} m_{arrow} v_{arrow_2}^2 + 0 = 0 + m_{arrow}\, g \Delta h$$

$$\Rightarrow \quad \Delta h = \frac{\tfrac{1}{2} m_{arrow} v_{arrow_2}^2}{m_{arrow}\, g}$$

using range of v_{arrow_2} in part a

Δh_{max} set by bow for big strong person.

Δh_{min} set by bow for small person

$$\Delta h_{max} = \frac{\tfrac{1}{2} m_{arrow} v_{arrow_2\,max}^2}{m_{arrow}\, g}$$

$$= \frac{\tfrac{1}{2}(0.047\,\text{lbm})(358\,\text{ft/s})^2}{(0.047\,\text{lbm})(32.2\,\text{ft/s}^2)}$$

$$\approx 1990\,\text{ft}$$

$$\Delta h_{min} = \frac{\tfrac{1}{2} m_{arrow} v_{arrow_2\,min}^2}{m_{arrow}\, g}$$

$$= \frac{\tfrac{1}{2}(0.047\,\text{lbm})(185\,\text{ft/s})^2}{(0.047\,\text{lbm})(32.2\,\text{ft/s}^2)}$$

$$\approx 531\,\text{ft}$$

$$\Rightarrow \quad \boxed{531\,\text{ft} \le \Delta h \le 1990\,\text{ft}}$$

10.2.12 A big person ($m = 100\,\text{kg}$) jumps on a trampoline which we model as a linear spring with stiffness k. You know that the trampoline deflects $d_0 = 20\,\text{cm}$ under the stationary weight mg of the person (use $g = 10\,\text{m/s}^2$). Assume there is no dissipation and the person is jumping repeatedly a height $h = 1\,\text{m}$ above the unloaded surface of the trampoline. Give all answers with symbols and numbers.

a) What is the stiffness k of the spring (answer in terms of some or all of m, g and d_0).

b) What is the maximum deflection of the trampoline during these jumps?

c) What is the peak force of the trampoline on the jumper? (answer in symbols, Newtons, and numbers of body weights).

9.39.

$m = 100\,kg$

$d_o = 20\,cm = 0.2\,m$ (under stationary weight mg).

(a) Stiffness of the spring,

$$K = \frac{mg}{d_o} = \frac{100\times 9.81}{0.2} = 4905\,N/m$$

(b) Initial (K.E + PE) of system, when person at height 1m.

$= 0 + mg\times 1$

$= 981\,J$

Final (KE + PE) of system, when spring is having maximum compression

$= 0 + \frac{1}{2}K d_{max}^2 - mg d_{max}$

Since Assuming there is no dissipation.

$$-mg d_{max} + \frac{1}{2}K d_{max}^2 = 981\,J$$

$$\Rightarrow \frac{1}{2}\times 4905\, d_{max}^2 - 981\, d_{max} - 981 = 0$$

$$\Rightarrow 2452.5\, d_{max}^2 - 981\, d_{max} - 981 = 0.$$

$$\Rightarrow d_{max} = 0.8633\,m,\ -0.46332\,m.$$

$$\boxed{d_{max} = +0.8633\,m}$$ Ans

(c)

$F_{peak} = K d_{max}$

$= 4905 \times 0.8633$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

10.2.13 For the car of problem 10.2.8 what is the average power required to reach speed v_1? There are two plausible ways to calculate this power:

$$\bar{P}_1 = \int_0^x P(x')\,dx'/x \qquad \text{and}$$

$$\bar{P}_2 = \int_0^t P(t')\,dt'/t\,.$$

Use both. Do the two methods give the same answer? If so, why, and will the answers be the same for all problems? If not, why not, in what cases will the answers agree, and, when they differ, which one is right?

9.2.13

$F_p = \mu mg \simeq mg$

$m = 1000\ kg$

$\mu = 1$

$g = 10\ m/s^2$

$v_1 = 60\ mph$

$\Rightarrow \quad a = \frac{mg}{m} = g$

for $v_1 = 60\ mph = \frac{60 \times 1.6093 \times 10^3}{60 \times 60}\ \frac{m}{sec}$

$= 26.8\ m/sec$

at any point $v = gt$; $x = \frac{1}{2}gt^2$; $v = \sqrt{2gx}$

$F = mg$;

Instantaneous power

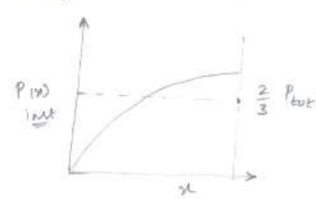
$P = F \cdot v = mgv$

$P(x) = mg\sqrt{2gx} = mg^2t$

$\bar{P}_1 = \frac{1}{x}\int_0^x mg\sqrt{2gx'}\,dx' = \frac{mg\sqrt{2g}\ x^{3/2}}{x\,(3/2)} = \frac{2}{3}mg\sqrt{2gx} = \frac{2}{3}mg^2t \longrightarrow (i)$

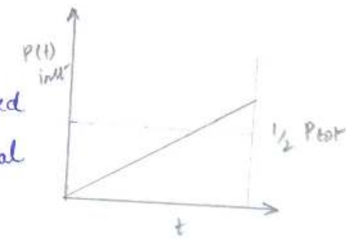
$\bar{P}_2 = \frac{1}{t}\int_0^t mg^2t'\,dt' = \frac{mg^2t}{2} \longrightarrow (ii)$

The two methods give diffrent powers. Power averged over distance travelled and since $P \propto \sqrt{x}$, this avg power is more than the time averaged power $\bar{P}(t)\,\Delta t$



Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

If acceleration is Zero and input force is balanced by some dissipative force, say friction, then the input power averaged over distance or time will be same and equal to instantaneous power during the ride.



when the differ the time averged power is right because $\bar{P}(t)\cdot t$ then represents total input energy;

ie $\bar{P}(t) = \dfrac{\Delta E}{\Delta t}$

10.2.14 For problem 10.2.9 which answers would change, and in which way, if the deceleration was not exactly constant during the crash? That is, for which quantities would the answer be bigger, which smaller, which the same, for which would the answer depend on the nature of the non-constant acceleration?

9.2.14

If deacceleration is not constant, all answer will change except for part (a) ie the total energy dissipated.

answer to (b), (c), (d), (e) and (f) will depend on instantaneous deacceleration.

10.2.15 The earth's gravitational pull on a mass m is $F = -\frac{mgR^2}{r^2}$, where mg is the pull at the surface of the earth and R is the radius of the earth. Assume a ballistic rocket is shot straight up with a launch velocity of v_0 (measured in a 'fixed' not-rotating-with-the-earth frame). Assume the rocket goes in a straight radial line as the earth turns underneath it (relative to the surface of the earth this rocket would be launched somewhat to the West to cancel the earth's rotation). Assume the period of active thrust is negligibly short (hence the word *ballistic*: "relating to or characteristic of the motion of objects moving under their own momentum and the force of gravity").

a) Solve for v as a function of r (and some or all of m, g, R and v_0).

b) Find the maximum height the rocket reaches.

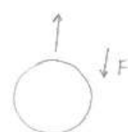
c) Find the 'escape velocity' v_{escape}, the minimum launch speed needed for the rocket to never return.

d) On one graph plot height (r or $r - R$) vs t for a v_0 just below v_{escape} and for v_0 just greater than v_{escape}. If you use numerical methods to make this plot use $g = 10\,\mathrm{m/s^2}$, $R = 6400\,\mathrm{km}$, and $m = 1\,\mathrm{kg}$. Make sure your axes are such that you can see a clear qualitative difference between the two cases.

9.2.15

a)

$$F = -\frac{mgR^2}{r^2}$$



$$m\ddot{y} = -\frac{mgR^2}{(y+R)^2}$$

where y is the altitude of rocket

$$m\left(\frac{d}{dt}v\right) = -\frac{mgR^2}{(y+R)^2}$$

$$mv\frac{dv}{dy} = -\frac{mgR^2}{(y+R)^2}$$

$$\Rightarrow \int_{v_0}^{v} mv\,dv = -\int_0^h \frac{mgR^2}{(y+R)^2}\,dy$$

$$\frac{1}{2}m(v^2 - v_0) = \left.\frac{mgR^2}{(y+R)}\right|_0^h = \left(\frac{mgR^2}{h+R}\right) - mgR$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 - \frac{mgR^2h}{R(R+h)}$$

$$v^2 = v_0^2 - \frac{2gRh}{R+h}$$

$$v = \sqrt{v_0^2 - \frac{2gRh}{R+h}}$$

b) Maximum height

$$0 = v_0^2 - \frac{2gRh_{max}}{R+h_{max}} \Rightarrow \frac{2gR}{v_0^2} = 1 + \frac{R}{h_{max}}$$

$$h_{max} = \frac{R}{\frac{2gR}{v_0^2} - 1} = \frac{Rv_0^2}{2gR - v_0^2}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

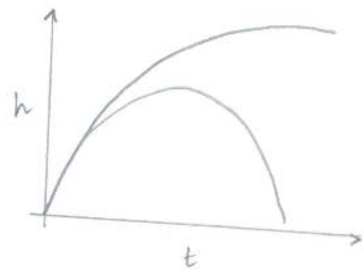
c) For V_0 = Velocity of Escape

$h_{max} \longrightarrow \infty$

$\Rightarrow 2gR - v_0^2 \longrightarrow 0$

$V_{escape} = \sqrt{2gR}$

d) $\dfrac{dh}{dt} = \sqrt{v_0^2 - \dfrac{2gRh}{R+h}}$



Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

10.2.16 The power available to a very strong accelerating cyclist over short periods of time (up to, say, about 1 minute) is about 1 horsepower. Assume a rider starts from rest and uses this constant power. Assume a mass (bike + rider) of 150 lbm, a realistic drag force of .006 lbf/(ft/s)$^2 v^2$. Neglect other drag forces.

a) What is the peak (steady state) speed of the cyclist?

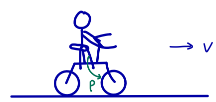
b) Using analytic or numerical methods make an accurate plot of speed vs. time.

c) What is the acceleration as $t \to \infty$ in this solution?

d) What is the acceleration as $t \to 0$ in your solution?

e) How would you improve the model to fix the problem with the answer above?

HW3 P/0 Teaya Yang



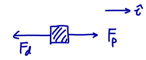
Given: $P = 1\ HP = 550\ lbf\ ft/s$ $g = 1\ lbf/lbm = 32.2\ ft/s^2$

$V_0 = 0\ m/s$

$m = 150\ lbm$

$C = 0.006\ lbf{\cdot}s^2/ft^2$

FBD:



* P = power supplied from the person's body chemistry to the mechanical energy of the system

a) Find V at steady state.

At steady state: $\Sigma \dot{E}_{in} = \Sigma \dot{E}_{out}$

Power Balance: $\Sigma\, Power = \dot{E}_K$ LMB: $F_p \hat{\imath} - F_d \hat{\imath} = ma\hat{\imath}$

$P - P_d = \frac{d}{dt}(\frac{1}{2}mv^2)$ $F_p - F_d = ma$ ②

$P - F_d V = mva$

$\frac{P}{V} - F_d = ma$ ①

① and ②: $F_d = \frac{P}{V}$

Plugging this back into ② gives ODE: $\frac{P}{V} - cV^2 = m\ddot{x}$

At steady state, $V = const.$ so $a = 0$

$0 = \frac{P}{V} - cV^2$

$V = \left(\frac{P}{c}\right)^{\frac{1}{3}} = \left(\frac{550\ \cancel{lbf}\ ft/s}{0.006\ \cancel{lbf}\ s^2/ft^2}\right)^{1/3} = \boxed{45.1\ ft/s} = \boxed{30.8\ mph}$

b) Plot $v(t)$

ODE: $m\ddot{x} = \frac{P}{V} - cv^2$

Integrated numerically in Matlab with ODE2 (Mid point method)

* V_0 had to be set to a small number to make sure $\dot{v}$ can be evaluated at $t=0$.

Plot is on next page.

Disclaimer: We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.

approximate steady state value

Terminal velocity reaches 45.1 ft/s as predicted by analytical results.

acceleration approaches ∞ at $t \to 0$

acceleration approaches 0 as $t \to \infty$

Since the difference in $a(t)$ is too significant between smaller and larger t, the axes are set to only show a section of the plot for visibility.

c) $a(t \to \infty) = ?$

Acceleration reaches 0 as $t \to \infty$. This is okay since we reach a terminal velocity at some point with the same power input.

d) $a(t \to 0) = ?$

Acceleration approaches ∞ as $t \to 0$. The largest acceleration due to friction given $\mu_{max} = 1$ is $a = \frac{\mu m g}{m} = \mu g = 32.2\ \text{ft/s}$, so an acceleration greater than this value does not make sense.

This disagreement betwee our solution and real life system indicates that our model fails at small t, see part e) for possible improvement on model.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

* See Matlab code generating the above plots below, part e) follows after.

Contents


```
% MAE 2030 HW3 P10 Sprig 2021
% Numerically integrates for the velocity of cyclist over time
% ODE2 function taken from Michelle DeSouza
```

Solving

Parameters

```
p.m = 150; % [lbm]
p.P = 550; % [lbf*ft/s]
p.c = 0.006; % [lbf*s^2/ft^2]

% IC
x0 = 0;
v0 = .0001; % Small number so that zdot can be evaluated at t=0
z0 = [x0 v0]';

% Time
tend = 1500; % Final time
tspan = [0 tend];

% Equation
eq = @(t,z) rhs(t,z,p); % Make rhs an anonymous function

% Set up options
options.RelTol = 1e-5; % Given accuracy
options.maxtime = 600; % Give up if accuracy not achieved in this many seconds.

% Solve
[tarray,zarray] = ODE2(eq,tspan,z0,options);
```

Plotting

```
% Plot v(t)
xsol = zarray(:,1); vsol = zarray(:,2);
figure(1)
hold on
box on
plot(tarray,vsol,'b-')
xlabel('t(s)')
ylabel('v(ft/s)')
title('Velocity of Cyclist')

% Plot a(t)
asol = (p.P./vsol - p.c*vsol.^2)/p.m;
figure(2)
hold on
box on
plot(tarray,asol,'b-') % Eliminating first points
```

```
axis([0 2 0 35])
xlabel('t(s)')
ylabel('a(ft/s)')
title('Acceleration of Cyclist')
```

RHS Equation

```
function zdot = rhs(t,z,p)
% Unpack variables
x = z(1);
v = z(2);

% Unpack parameters
m = p.m;
P = p.P;
c = p.c;

% Equation
xdot = v;
vdot = (P/v - c*v^2)/m;

zdot = [xdot vdot]';

end
```

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```
function [tarray zarray] = ODE2(eqn, tspan, z0, options)
%Solve any ODE with midpoint method
%From Michelle's solution to P8

tstart = tspan(1);
tend = tspan(2);

%unpack options
RelTol = options.RelTol; % max allowable relative error
maxtime = options.maxtime; % max computer time used

tic; % start keeping track of time

% The number of points in progressively more accurate solutions
nsols = 30; %number of solutions
npoints_array = 5*2.^[1:nsols]; % 10, 20, 30, 80, 160 ... 5*2^30.
%Initialize array of solutions at the final time
zendarray = zeros(nsols,length(z0)); % array of the solutions at tend

% Outer loop, progressively more accurate solutions,
% Each time through the loop is a new solution.
    for j = 1:nsols % iteration loop
    npoints = npoints_array(j); %number of points in this solution
    %disp(['Number of points this iteration is: 'num2str(npoints)])

    % Initialize various arrays
    tarray = linspace(tstart, tend, npoints); %Soln at these times
    zarray = zeros(npoints, length(z0)); % Claim memory for zarray
    z = z0; % z is present value of z, used in loop below
    zarray(1,:) = z'; % The first row is the initial condition z0.
    %%%%%%%%%%%%%%%%%%
        for i = 1:npoints-1 % Euler's method is in this 6-line loop
        ti = tarray(i);
        h = tarray(i+1) - tarray(i);
        zdoti = eqn(ti,z); %evaluate midpoint zdot!

        %evaluate midpoint zdot
        tmid = ti + h/2; %calculate time at half step
        zmid = z + zdoti*(h/2); %calculate z at half step
        zdotmid = eqn(tmid,zmid); %calculate zdot at half step

        %the next lines are the same as Euler's method
        znew = z + zdotmid*h; % use midpoint zdot to take a step
        zarray(i+1,:) = z'; % add a new row to the solution array
        z = znew; % the new value of z becomes the present value

            if toc>maxtime % Timeout if it is taking too long
            disp([' *** Max time has passed: '
            num2str(maxtime) ' seconds. ***'])
            disp([' (Solution up to timeout has been saved.' ...
            ' Increase maxtime or RelTol or both.)'])
            tarray = tarray(1:i); % the time so far
            zarray = zarray(1:i,:); % save the solution so far
            break % breaks out of Euler method loop
            end % of max time check if

        end % of Euler's method

    %%%%%%%%%%%%%%%%%%
```

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

```
    if toc>maxtime % Timeout if it is taking too long
        break % breaks out of iterative solution loop
    end % check max time if

    zendarray(j,:) = zarray(end,:); % add to list of solutions the last line of most recent solution

        if j>1 % compare solutions once you have more than one solution
        % More or less arbitrary measure of error
        error = (zendarray(j,:) - zendarray(j-1,:)); % differences between this solution and the previous
        error = sum(abs(error)); % sum all the differences as a measure of error
        error = error/sum(zendarray(j,:)); %Compare error to value of z
        %disp([' Error this iteration is: ' num2str(error)])
        %disp([' Time spent solving so far is: ' num2str(toc)' seconds.'])

            if error < RelTol
                disp(['Success! RelTol of ' num2str(RelTol) ' achieved using Midpoint method.'])
                break; % exits the outer loop
            end % of error if

        end % of error check if

    end % of iteration loop

end % of function
```

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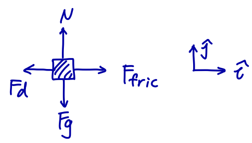
e) What can we do to change the model?

(From Prof. Ruina)

Using the maximum friction force between the bike and the ground as the initial bounded force that can be exerted on the bike, our model can be modified as follows:



$\vec{F}_{fric} = \mu m g \,\hat{\imath}$

Maximum friction coefficient between bicycle tire and asphalt road: $\mu = 1$

We can use this force until the power becomes $P = 1hp$.

$P_{fric} = \mu m g v$

ODE for low velocity: $m\ddot{x} = \mu m g - c v^2$ (I)

ODE for high velocity: $m\ddot{x} = \frac{P}{v} - c v^2$ (II) (same as before)

So use (I) until $P_{fric} = P$ and then use (II) till t_{end}.

This is done in Matlab.

Plot showing the power that we use:

So, highlighted part would be what make up our new model. The power would be constant except the initial part.

e) Results :

* The same v(t) vs. t graph is shown with 2 different time axes below to show the comparison between different times.

more rapid
increase in
old model

Same steady
state velocity

The model with constant power causes velocity to increase more rapidly initially.

The velocities converge and reach the same steady state level for $t \to \infty$.

* The same a(t) plot is shown in two different time axes below.

infinite
acceleration

accelerations
converge

decreasing
acceleration

constant
acceleration

Initially, the acceleration of the constant power model starts at infinity and decreases rapidly, whereas the constant force model ensures a constant acceleration.

Acceleration of the new model starts at a finite value due to the constant friction force, then it decreases and approaches 0 at $t \to \infty$.

* More plots on next page.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

constant power

same constant power

increasing power

This plot shows the comparison between the power variations over time of the two models. While power is constant in the old model, the new model we are using has an initially rapidly increasing power and it becomes constant once P=1hp is reached.

infinite force requred

finite maximum force initially

output force approaches constant

This is only the force due to outputted power (not including drag), so unlike acceleration this force approaches a constant. In the original model, an infinite force is required at $t=0$ to produce the target output power, while in the new model, the cyclist starts at a maximum force and the amount of force required to output the same power decreases after a few seconds due to the increasing velocity.

See next page for Matlab script for new model.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Contents


```
% MAE 2030 HW3 P10 Sprig 2021
% Cyclist with maximum friction force, drag force, and constant
% power(eventually)
% Numerically integrates for the velocity of cyclist over time
% Solved using midpoint method wrtitten by Michelle DeSouza
```

Solve the problem first using constant friction

```
% Parameters
p.m = 150; % [lbm]
p.P = 550; % [lbf*ft/s}
p.c = 0.006; % [lbf*s^2/ft^2]
p.g = 1; % [lbf/lbm]
p.mu = 1;

% IC
x0 = 0;
v0 = 0; % It's okay to have 0 initial velocity now!
z0 = [x0 v0]';

% Time
tend = 10; % time in which we search for when power becomes > 1hp
tspan = [0 tend];

% Equation
eq1 = @(t,z) rhs1(t,z,p); % Make rhs1 an anonymous function

% Set up options
options.RelTol = 1e-5; % Given accuracy
options.maxtime = 600; % Give up if accuracy not achieved in this many seconds.

% Solve
[tarray1,zarray1] = ODE2(eq1,tspan,z0,options);
```

Extract points with power below 1hp from first solution

```
% Calculate all Pfric using solved velocities
x1 = zarray1(:,1); v1 = zarray1(:,2);
Pfric = v1*p.mu*p.m*p.g;
Pgiven = p.P*ones(length(tarray1),1);

% Find element of Pfric closest to 1hp
[~,idx] = min(abs(Pfric-p.P));

% Plot What we did just to be clear
```

```
figure(1)
hold on
box on
plot(tarray1,Pfric,'b-') % plot Pfric
plot(tarray1,Pgiven,'r--'); % Plot constant P
legend('Power from friction','Constant power given')
xlabel('t(s)')
ylabel('Power(lbf*ft/s)')
title('Acceleration of Cyclist')

% Then trim everything to this length
x1 = x1(1:idx); v1 = v1(1:idx);
tarray1 = tarray1(1:idx);
Pfric = Pfric(1:idx);
a1 = (p.mu*p.m*p.g - p.c*v1.^2)/p.m; % Calculate acceleration
```

Integrating with constant power

```
% Now the last points from before become ICs
x0 = x1(end); v0 = v1(end);
z0 = [x0 v0]';

% Time
tend = 1500;
tspan = [0 tend];

% Equation
eq2 = @(t,z) rhs2(t,z,p); % Make rhs2 an anonymous function

% Set up options
options.RelTol = 1e-5; % Given accuracy
options.maxtime = 600; % Give up if accuracy not achieved in this many seconds.

% Solve
[tarray2,zarray2] = ODE2(eq2,tspan,z0,options);
```

Put two parts together

```
tarray2 = tarray2 + tarray1(end); % make tarray2 start at end of tarray1
t = [tarray1,tarray2]; % put tarrays together
x2 = zarray2(:,1); v2 = zarray2(:,2);
x = [x1;x2]; v = [v1;v2]; % put x and v together
narrays = length(x); % length of all the arrays in final solution
P = [Pfric;p.P*ones(narrays-idx,1)]; % put power together
F = P./v; % Driving force
a2 = (p.P./v2 - p.c*v2.^2)/p.m; % acceleration from second part
a = [a1;a2]; % put acceleration together
```

Solve Original Model like in HW3P10 to Compare

IC

```
x0 = 0;
v0 = .0001; % Small number so that zdot can be evaluated at t=0
z0 = [x0 v0]';
```

```
% Time
tend = 1500; % Final time
tspan = [0 tend];

% Equation
eq = @(t,z) rhs2(t,z,p); % Make rhs an anonymous function

% Set up options
options.RelTol = 1e-5; % Given accuracy
options.maxtime = 600; % Give up if accuracy not achieved in this many seconds.

% Solve
[t2,z2] = ODE2(eq,tspan,z0,options);
v2 = z2(:,2);
```

Make various plots to show final solution

Plot v(t)

```
figure(2)
hold on
box on
plot(t,v,'b-')
plot(t2,v2,'r--')
% axis([0 1500 0 50])
axis([0 100 0 30]) %zoomed in view
xlabel('t(s)')
ylabel('v(ft/s)')
title('Velocity of Cyclist')
legend('New Model','Original Model')
```

Plot a(t)

```
a2 = (p.P./v2 - p.c*v2.^2)/p.m;
figure(3)
hold on
box on
plot(t,a,'b-')
plot(t2,a2,'r--')
legend('New Model','Original Model')
axis([0 20 0 5])
xlabel('t(s)')
ylabel('a(ft/s^2)')
title('Acceleration of Cyclist')
```

Plot power

```
P2 = p.P*ones(length(t2),1);
figure(4)
hold on
box on
plot(t,P,'b-')
plot(t2,P2,'r--')
legend('New Model','Original Model')
axis([0 25 0 600])
```

```
xlabel('t(s)')
ylabel('P(lbf*ft/s)')
title('Power on Bike')
```

Plot Force

```
F2 = P2./v2;
figure(5)
hold on
box on
plot(t,F,'b-')
plot(t2,F2,'r--')
legend('New Model','Original Model')
axis([0 35 0 200])
xlabel('t(s)')
ylabel('F(lbf)')
title('Drving Force on Bike')
```

RHS Equations

```
% RHS with constant friction force
function zdot = rhs1(t,z,p)

% Unpack variables
x = z(1);
v = z(2);

% Unpack parameters
m = p.m;
c = p.c;
mu = p.mu;
g = p.g;

% Equations
xdot = v;
vdot = (mu*m*g - c*v^2)/m;

zdot = [xdot vdot]';

end

% RHS with constant power
function zdot = rhs2(t,z,p)

% Unpack variables
x = z(1);
v = z(2);

% Unpack parameters
m = p.m;
c = p.c;
P = p.P;

% Equations
xdot = v;
vdot = (P/v - c*v^2)/m;

zdot = [xdot vdot]';
```

```
end
```

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