10.2 Energy methods in 1D

Preparatory Problems

10.2.1 A mass m is at position x moving at velocity v and being acted upon by force F. For each of the quantities below:

i give the symbol used for the quantity

ii describe the quantity in words

iii give a formula to evaluate the quantity in terms of some or all of m, x, v and F and any other variables you may need.

iv Give the standard units for the quantity in the SI system.

v Give the standard units for the quantity in the English system.

a) Power

b) Kinetic energy

c) Work

d) Potential energy

10.2.2 Write an equation relating the two words in each of these pairs. If any conditions or descriptions of the situation are needed, give them. If you know more than one equation (or form for a given equation), give all that you know. All should be given in the context of this section: 1D motion.

a) work and power

b) work and kinetic energy

c) power and kinetic energy

d) work and potential energy

e) potential energy and kinetic energy

10.2.3 A force $F = F_0 \sin(ct)$ acts on a particle with mass $m = 3\,\mathrm{kg}$ which has position $x = 3\,\mathrm{m}$, velocity $v = 5\,\mathrm{m/s}$ at $t = 2\,\mathrm{s}$. $F_0 = 4\,\mathrm{N}$ and $c = 2/\mathrm{s}$. At $t = 2\,\mathrm{s}$ evaluate (give numbers and units):

a) a,

b) E_K,

c) P,

d) $\dot{E}_\mathrm{K}$,

e) the rate at which the force is doing work. *

10.2.4 A force only depends on position according to $F = C_0 + C_1 x$ where C_0 and C_1 are constants. What is the work done by this force when the point to which it is applied moves from x_1 to x_2? Answer in terms of some or all of C_0, C_1, x_1 and x_2.

10.2.5 Find the potential E_P associated with each of these force fields.

a) $F = 0$.

b) $F = F_0$ (=constant).

c) $F = kx$.

d) $F = A\sin(x/x_0)$.

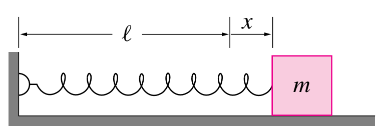
e) $F = c/x^2$.

10.2.6 Consider a spring-mass system with $m = 2\,\mathrm{kg}$ and $k = 5\,\mathrm{N/m}$. The mass is pulled to the right a distance $x = x_0 = 0.5\,\mathrm{m}$ from the unstretched position and released from rest. No external forces act on the mass.

a) What are the initial potential and kinetic energy of the system?

b) What is the potential and kinetic energy of the system as the mass passes through the static equilibrium (unstretched spring) position?

c) What is the speed of the mass when it passes through the static equilibrium position?



Problem 10.2.6

10.2.7 A mass m is held in place by a spring whose restoring force is $T(x) = kx$. Derive the equation of motion of the system (that is, find the acceleration a in terms of x).

10.2.8 The peak propulsion force on a 4-wheel-drive car is about μmg where $\mu \approx 1$ for rubber on road (a bit more for fancy racing tires). Assume a car starts from rest at position zero. Answer the following questions with symbols and with numbers (using $\mu = 1$, $m = 1000\,\mathrm{kg}$, and $g = 10\,\mathrm{m/s^2}$).

a) What is the minimum distance required to reach $v_1 = 60\,\mathrm{mph}$?

b) What is the extra distance required to get from $v_1 = 60\,\mathrm{mph}$ up to $v_2 = 70\,\mathrm{mph}$?

c) What is the peak power used by the engine in getting up to $v_1 = 60\,\mathrm{mph}$ (assuming no dissipation and no air friction)?

10.2.9 A car (mass $m = 1000\,\mathrm{kg}$) traveling at speed $v_0 = 30\,\mathrm{m/s}$ crashes into a brick wall and comes to a stop as the front end of the car compresses a distance $d = 1\,\mathrm{m}$. Answer with symbols and numbers. Assume constant deceleration during the crash. Neglect the mass of the crushing region of the car.

a) What is the total energy dissipated in the crash?

b) What is the force of the car on the wall?

c) What is the force of the wall on the car?

d) What is the deceleration of the car passengers (assuming they are strapped in and move with the bulk of the car). Answer in g's?

e) Assuming an $m_p = 50\,\mathrm{kg}$ person, what is the force of the seat belts on the person (answer in body weight).

f) If a parent was holding a 15 kg child on his lap, what force would he need to hold on to the child through the crash (answer in N and in number of child body weights).

More-Involved Problems

10.2.10 A kid ($m = 90\,\mathrm{lbm}$) stands on a $h = 10\,\mathrm{ft}$ wall and jumps down, accelerating with $g = 32\,\mathrm{ft/s^2}$. Upon hitting the ground with straight legs, she bends them so her body slows to a stop over a distance $d = 1\,\mathrm{ft}$. Neglect the mass of her legs. Assume constant deceleration as she brakes the fall.

a) What is the total distance her body falls? *

b) What is the potential energy lost?

c) How much work must be absorbed by her legs?

d) What is the force of her legs on her body? Answer in symbols, numbers and numbers of body weight (*i.e.,* find F/mg).

10.2.11 In traditional archery, when pulling an arrow back the force starts from 0 and increases approximately linearly up to the peak 'draw force' F_{draw}. The draw force varies from about $F_{draw} = 25\,\text{lbf}$ for a bow made for a small person to about $F_{draw} = 75\,\text{lbf}$ for a bow made for a big strong person. The distance the arrow is pulled back, the draw length ℓ_{draw}, varies from about $\ell_{draw} = 2\,\text{ft}$ for a small adult to about 30 inch for a big adult. An arrow has mass of about 300 grain (1 grain $\approx$ 64.8milli gm, so an arrow has mass of about $19.44 \approx 20\,\text{gm} \approx 3/4$ ounce). Give all answers in symbols and numbers.

a) What is the range of speeds you can expect an arrow to fly?

b) What is the range of heights an arrow might go if shot straight up (it's a bad approximation, but for this problem neglect air friction)?

10.2.12 A big person ($m = 100\,\text{kg}$) jumps on a trampoline which we model as a linear spring with stiffness k. You know that the trampoline deflects $d_0 = 20\,\text{cm}$ under the stationary weight mg of the person (use $g = 10\,\text{m/s}^2$). Assume there is no dissipation and the person is jumping repeatedly a height $h = 1\,\text{m}$ above the unloaded surface of the trampoline. Give all answers with symbols and numbers.

a) What is the stiffness k of the spring (answer in terms of some or all of m, g and d_0).

b) What is the maximum deflection of the trampoline during these jumps?

c) What is the peak force of the trampoline on the jumper? (answer in symbols, Newtons, and numbers of body weights).

10.2.13 For the car of problem 10.2.8 what is the average power required to reach speed v_1? There are two plausible ways to calculate this power:

$$\bar{P}_1 = \int_0^x P(x')\,dx'/x \quad \text{and}$$
$$\bar{P}_2 = \int_0^t P(t')\,dt'/t\,.$$

Use both. Do the two methods give the same answer? If so, why, and will the answers be the same for all problems? If not, why not, in what cases will the answers agree, and, when they differ, which one is right?

10.2.14 For problem 10.2.9 which answers would change, and in which way, if the deceleration was not exactly constant during the crash? That is, for which quantities would the answer be bigger, which smaller, which the same, for which would the answer depend on the nature of the non-constant acceleration?

10.2.15 The earth's gravitational pull on a mass m is $F = -\frac{mgR^2}{r^2}$, where mg is the pull at the surface of the earth and R is the radius of the earth. Assume a ballistic rocket is shot straight up with a launch velocity of v_0 (measured in a 'fixed' not-rotating-with-the-earth frame). Assume the rocket goes in a straight radial line as the earth turns underneath it (relative to the surface of the earth this rocket would be launched somewhat to the West to cancel the earth's rotation). Assume the period of active thrust is negligibly short (hence the word *ballistic*: "relating to or characteristic of the motion of objects moving under their own momentum and the force of gravity").

a) Solve for v as a function of r (and some or all of m, g, R and v_0).

b) Find the maximum height the rocket reaches.

c) Find the 'escape velocity' v_{escape}, the minimum launch speed needed for the rocket to never return.

d) On one graph plot height (r or $r - R$) vs t for a v_0 just below v_{escape} and for v_0 just greater than v_{escape}. If you use numerical methods to make this plot use $g = 10\,\text{m/s}^2$, $R = 6400\,\text{km}$, and $m = 1\,\text{kg}$. Make sure your axes are such that you can see a clear qualitative difference between the two cases.

10.2.16 The power available to a very strong accelerating cyclist over short periods of time (up to, say, about 1 minute) is about 1 horsepower. Assume a rider starts from rest and uses this constant power. Assume a mass (bike + rider) of 150 lbm, a realistic drag force of $.006\,\text{lbf}/(\text{ft/s})^2 v^2$. Neglect other drag forces.

a) What is the peak (steady state) speed of the cyclist?

b) Using analytic or numerical methods make an accurate plot of speed vs. time.

c) What is the acceleration as $t \to \infty$ in this solution?

d) What is the acceleration as $t \to 0$ in your solution?

e) How would you improve the model to fix the problem with the answer above?