10.1.1 Give three examples of real life objects where you might use the idealization, for dynamic calculations, that the object is a particle in unconstrained 1D motion.

Answer:
Some possibilities: a) a car with given thrust and drag, b) a person falling vertically during bungy jumping, c) a speaker cone oscillating due to magnetic forces on its coils and resistance from air pressure.

Answer: 9.1

- Body in pure translatory Motion (e.g. sliding object)
- Motion of car can be also described by assuming as a particle if our interest is to find displacement, velocity & Acceln.
- Motion of lift (Kinetics).

10.1.2 A car is going downhill on a constant slope straight road. For finding out the car's speed at the end of the road you model it as a particle. For specifying initial velocity, which point on the car would you consider?

Answer:
All points have equal velocity so all have the same velocity as the center of mass. Any point on the car can be used to measure the car's position.

9.2 Centre of mass.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

10.1.3 The acceleration of a particle is given as a function of time, $a(t)$. Is this information sufficient to find the speed of the particle at the end of, say, T seconds?

Answer:
No. You need also to know $v(0)$. Then $v(T) = v(0) + \int_0^T a(t)\,dt$. Knowing $a(t)$ over a given time interval determines the change of v over that interval, but not the value $v(T)$.

9.3 $\frac{dv}{dt} = a(t)$

$\int dv = \int a\,dt \Rightarrow v = \int a\,dt + C$

No, we need initial velocity to find the const C.

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10.1.4 If a particle has constant acceleration, its linear momentum (a) remains constant, (b) changes linearly with time, or (c) changes quadratically with time. Which one is true?

Answer:
(b) changes linearly in time

9.4 $\frac{m\,dv}{dt} = F = ma$

rate of change of linear momentum is constant.

Hence, linear momentum changes linearly with time (b)

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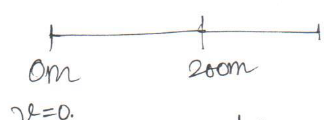
10.1.5 In a motorcycle race on a straight track, the speed v^* of a motorcyclist at the d=200 m mark is recorded. Given that the rider started from rest, find the initial acceleration of the motorcycle. Assume the acceleration (a) is constant or (b) (challenge) decreases linearly with time to zero at the end.

Answer:
a) $a = {v^*}^2/(2d)$; b) Assume $a = a_0 - ct$. If you knew a_0 and c then you could find $T = a_0/c$, $d = a_0^3/(3c^2)$, and $v^* = a_0^2/(2c)$. But you are given v^* and d and can solve to get $a_0 = 4{v^*}^2/3d$.

9.5

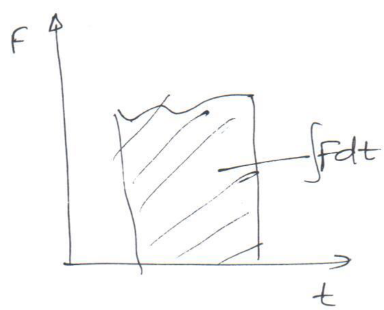


$$a=\frac{dv}{dt} \Rightarrow v_f = v_i + \int a\,dt$$

10.1.6 The force acting on a particle with mass m is a given function of time. If you plot the force vs time and find the area under the graph, from that area can you determine (a) the net displacement of the particle?, (b) the average velocity of the particle?, or (c) the change in linear momentum of the particle?

Answer:
Only (c), the change in linear momentum. You could find the displacement only if the initial velocity is also given.

9.6



$$\int F dt = \int d(mv)$$

c

10.1.7 If the linear momentum of a body remains constant in time, it must have (pick one): (a) a constant force acting on it, (b) no net force acting on it, or (c) a sinusoidal force acting on it.

Answer:
(b)

9.7 (b) No net force acting on it.

10.1.8 The distance between two points in a bicycle race is 10 km. How many minutes does a bicyclist take to cover this distance if he/she maintains a constant speed of 15 mph.

Answer:
$t = 24.8\,\text{min}$

9.1.8

$d = 10\,Km$; $v = 15\,mph$

$$t = \frac{d}{v} = \frac{10 \times 10^3}{15 \times 1609.34} \times 60 = 24.85 \text{ mins}$$

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10.1.9 A 5 kN constant force acts on a 1 kg object that was initially at rest for 5 seconds and then stops. Find the speed of the object at the end of (a) 5 seconds, and (b) 10 seconds.

From the given information:

$$F = \begin{cases} 5\,\text{kN} & 0 \le t \le 5\,\text{s} \\ 0 & t > 5\,\text{s} \end{cases}$$

$$a = F/m \equiv \frac{dv}{dt}$$

$$\Rightarrow \quad v = \int_0^t a\,dt + v_0 \quad = v_0 + \frac{F}{m} t$$

Here $v_0 = 0$, $F = 5\,\text{kN}$ for $t \le 5\,\text{s}$, $m = 1\,\text{kg}$

$$\Rightarrow \quad v(5\,\text{s}) = 0 + \frac{5\,\text{kN}}{1\,\text{kg}} \cdot 5\,\text{s} = 25000\,\text{m/s}$$

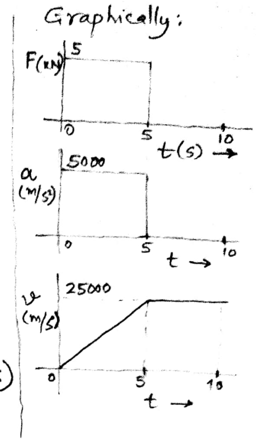
$$v(10\,\text{s}) = v(5\,\text{s}) + \int_5^{10} \frac{F}{m} dt$$

$$= 25000\,\text{m/s} + \int_5^{10} 0\,dt \quad (\because F = 0 \text{ for } t > 5\,\text{s})$$

$$= 25000\,\text{m/s}.$$

$$\boxed{\text{Ans: } v(5\,\text{s}) = 25000\,\text{m/s} = v(10\,\text{s})}$$

Graphically:



Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

10.1.10 Given that $\dot{x} = k_1 + k_2 t$, $k_1 = 1\,\text{ft/s}$, $k_2 = 1\,\text{ft/s}^2$, and $x(0) = 1\,\text{ft}$, what is the displacement at the end of 10 seconds?

Answer:
$x(10\,\text{s}) = 61\,\text{ft}$

(2)

9.10

Given

$\dot{x} = K_1 + K_2 t$

$K_1 = 1\,\text{ft}\,\text{s}^{-1}, K_2 = 1\,\text{ft}\,\text{s}^{-2}$

$x(0) = 1\,\text{ft}$

$$\frac{dx}{dt} = K_1 + K_2 t \Rightarrow \int_1^x dx = \int_0^t (K_1 + K_2 t)\,dt$$

$$\Rightarrow (x - 1) = K_1 t + K_2 t^2/2$$

$$\therefore x\Big|_{t=10\text{sec}} = 1 + 1\times10 + 1\times\frac{10^2}{2}$$

$$= 1 + 10 + 50$$

$$= 61\,\text{ft}. \quad \underline{\underline{\text{Ans}}}$$

10.1.11 Find $x(3\,\mathrm{s})$ given that

$$\dot{x} = x/(1\,\mathrm{s}) \quad \text{and} \quad x(0\,\mathrm{s}) = 1\,\mathrm{m}$$

or, expressed slightly differently,

$$\dot{x} = cx \quad \text{and} \quad x(0\,\mathrm{s}) = x_0,$$

where $c = 1\,\mathrm{s}^{-1}$ and $x_0 = 1\,\mathrm{m}$. Make a sketch of x versus t.

Answer:
$x(3\,\mathrm{s}) = 20.08\,\mathrm{m}$

9.11

given,

$\dot{x} = \frac{dx}{dt} = cx$

$\Rightarrow \frac{dx}{x} = c\,dt$

Integrating both sides, we get

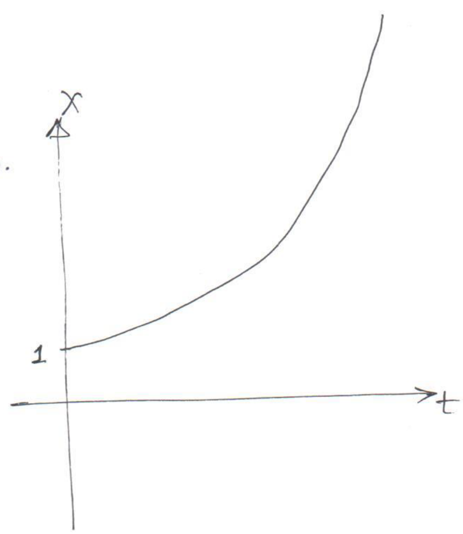
$\ln x = ct + c_1$

at $t = 0$, $x(0) = 1$, $c = 1$

$\Rightarrow 0 = 0 + c_1 \Rightarrow c_1 = 0.$

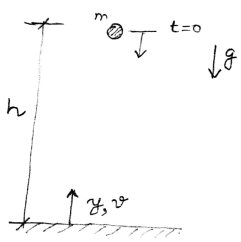
$\therefore \ln x = t$

$\Rightarrow \boxed{x = e^t}$



10.1.12 A ball of mass m is dropped from rest at a height h above the ground. Find the position and velocity as a function of time (as well as m and g, if needed). Neglect air friction. When does the ball hit the ground? What is the velocity of the ball just before it hits?

Answer:
$v = -gt$, $y = h - \frac{1}{2}gt^2$, $y = 0$ when $t = \sqrt{2g/h}$, $v = -\sqrt{2gh}$



$$a = \frac{dv}{dt} = -g$$

$$\Rightarrow v = \cancel{v_0}^{\,0} - gt$$

$$= -gt$$

Now $v = \dfrac{dy}{dt}$

$$\Rightarrow \frac{dy}{dt} = -gt \Rightarrow \int dy = \int -gt\,dt$$

$$\Rightarrow y = y_0 - \frac{1}{2}gt^2$$

at $t=0$, $y = h \Rightarrow y_0 = h$

$$\Rightarrow \boxed{y = h - \frac{1}{2}gt^2}$$

when the ball hits the ground, $y = 0$

thus $0 = h - \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

Substituting in $v = -gt$, we get the velocity at impact:

$$v = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh}$$

Thus $y=0$ (i.e., ball hits the ground) at $\boxed{t = \sqrt{\frac{2h}{g}} \text{ and } v = -\sqrt{2gh}}$

10.1.13 The speed of a particle varies sinusoidally as $v = A\sin[ct]$, where $A = 0.5\,\mathrm{m/s}$ and $c = 3\,\mathrm{rad/s}$. Let the initial position of the particle be $x(0) = 0$. Find the position of the particle at $t = \pi/2$ s.

Answer:
$x(\pi/2) = 0.17\,\mathrm{m}$

1.129 Given that $\dot{x} = A\sin(\omega t)$
with $\omega = 3\,\mathrm{rad/s}$, $A = .5\,\mathrm{m/s}$
and initial condition $x(0) = 0\,\mathrm{m}$.
Find $x(\pi/2\ \mathrm{s.})$.

$$\frac{dx}{dt} = A\sin\omega t \Rightarrow \int dx = \int A\sin\omega t\,dt$$

$$\Rightarrow x = -\frac{A}{\omega}\cos\omega t + C$$

use initial condition $x(t=0\,\mathrm{s.}) = 0$ to evaluate C:

$$0 = -\frac{A}{\omega}\cos(0) + C \Rightarrow C = A/\omega$$

Thus: $x(t) = \frac{A}{\omega}(1 - \cos\omega t)$

at $t = \pi/2$ sec., $x = \frac{.5}{3}(1 - \underbrace{\cos 3\pi/2}_{0})$ m. $\Rightarrow$

$$\boxed{x(t = \pi/2\ \mathrm{s.}) = \frac{1}{6}\ \mathrm{m.}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

9.13 (3)

$v = A\sin[ct]$

$A = 0.5\,ms^{-1}.$

$c = 3\,rad\,s^{-1}.$

$\frac{dx}{dt} = A\sin(ct)$

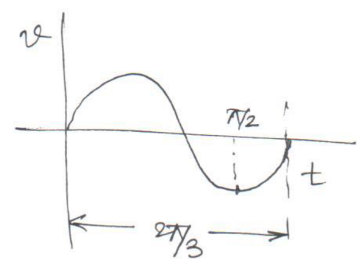
$\Rightarrow \int_0^x dx = \int_0^{\pi/2} A\sin(ct)\,dt$

$\Rightarrow x = \left[\frac{-A\cos[ct]}{c}\right]_0^{\pi/2}$

$= A/c = \frac{0.5}{3}$

$\Rightarrow x = 0.1667\,m.$ Ans.

$x = -A/c\,[\cos(ct) - 1]$



$\boxed{x = A/c\,[1 - \cos(ct)]}$

10.1.14 The speed of a particle is directly proportional to its position and is given as $\dot{x} = x/\text{s}$. If the initial position, $x(0) = 1\text{ m}$, how far would the particle be from the origin in 5 seconds?

Answer:
$x(5\text{ s}) = 148.41\text{ m}$

9.14

Given,

$\dot{x} = x/s$

$x(0s) = 1m.$

$x(t=5sec) = ?$

$\frac{dx}{dt} = x \Rightarrow \int dx/x = \int dt$

$\cancel{\Rightarrow x = e^t} \Rightarrow \ln x = t + C$

$\Rightarrow \boxed{C=0} \quad \because t=0, x=1.$

$\therefore x = e^t$

$\therefore x|_{t=5sec} = e^5 m$

$\Rightarrow x(5s) = 148.413m.$ Ans

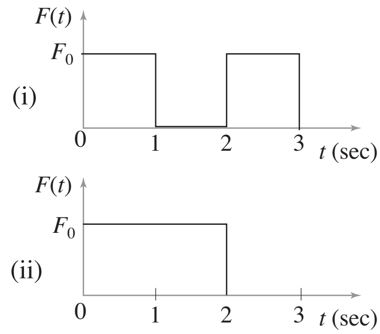
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

10.1.15 Consider a force $F(t)$ acting on a cart over a 3 second span. In case (a), the force acts in two impulses of one second duration each as shown in fig. 10.1.15. In case (b), the force acts continuously for two seconds and then is zero for the last second. Given that the mass of the cart is 10 kg, $v(0) = 0$, and $F_0 = 10\,\text{N}$, for each force profile,

a) Find the speed of the cart at the end of 3 seconds, and

b) Find the distance travelled by the cart in 3 seconds.

Comment on your answers for the two cases.

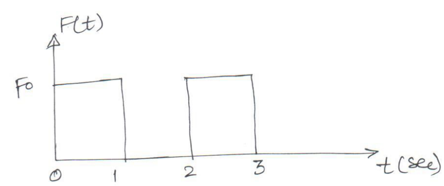


Problem 10.15

Answer:
(a) $v(3\,\text{s}) = 2\,\text{m/s}$ in each case. (b) $x(3\,\text{s}) = 3\,\text{m}$ for case (a), $x(3\,\text{s}) = 4\,\text{m}$ for case (b). During the one second with no force, in case (b) it has already got up to a higher speed.

9.15

(a)



$m\,dv = F_0\,dt$ — (1)

$\Rightarrow m(v_1 - v_0) = F_0(t_1 - t_0)$ [∵ Fo is const].

$\Rightarrow v_1 = \frac{10 \times (1-0)}{10} = 1\,ms^{-1}$.

Since external force acting b/w point (1) & (2) is 0,

$v_1 = v_2$.

Again,

$v_3 = v_2 + \frac{F_0}{m}[t_3 - t_2]$ (from (1))

$\Rightarrow v_3 = 1 + \frac{10}{10}(1)$
$= 2\,ms^{-1}$.

(b) $v_2 = v(0) + \frac{F_0}{m}[t_2 - t_0]$
$= 2\,ms^{-1}$.

$\therefore v_3 = v_2 = 2\,ms^{-1}$. [Since no force acting b/w point (2) & (3)].

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9.15

$m = 10\ kg$ $\quad F = ma \quad \therefore \quad a_o = F_o/m_o = 10N/10kg$

$v(0) = 0$ $\quad\quad = 1\ m/s^2$

$F_o = 10\ N$

Force profile (a):

@ $t = 1s$: $a = 1\ m/s^2$, $v = v_o + at = 0 + 1 m/s^2 (1s) = 1\ m/s$

$x = x_o + v_o t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(1 m/s^2)(1s)^2 = 0.5m$

@ $t = 2s$: $a = 0$, $v = v_o + at = 1\ m/s + 0 = 1\ m/s$

$x = x_o + v_o t + \frac{1}{2}at^2 = 0.5 + 1(1) + 0 = 1.5\ m$

@ $t = 3s$: $a = 1 m/s^2$, $v = v_o + at = 1 + 1(1) = \underline{2.0\ m/s}$

$x = x_o + v_o t + \frac{1}{2}at^2 = 1.5 + 1(1) + \frac{1}{2}(1)(1)^2 = \underline{3.0\ m}$

Force profile (b):

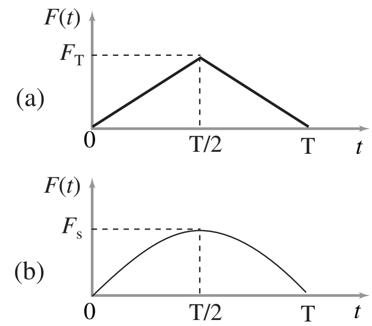
@ $t = 2s$: $a = 1\ m/s^2$, $v = v_o + at = 0 + 1(2) = 2\ m/s$

$x = x_o + v_o t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(1)(2)^2 = 2\ m$

@ $t = 3s$: $a = 0$, $v = v_o + at = 2 + 0 = \underline{2.0\ m/s}$

$x = x_o + v_o t + \frac{1}{2}at^2 = 2 + 2(1) + 0 = \underline{4.0\ m}$

10.1.16 A car of mass m is accelerated by applying a triangular force profile shown in fig. 10.1.16(a). Find the speed of the car at $t = T$ seconds. If the same speed is to be achieved at $t = T$ seconds with a sinusoidal force profile, $F(t) = F_s \sin\frac{\pi t}{T}$, find the required force magnitude F_s. Is the peak higher or lower? Why?



Problem 10.16

Answer:
b) $F_s = \frac{\pi}{4} F_T$. This is lower than for (a) because for a given peak force the sinusoidal force is bigger at every instant in time. So, to have the same effect (same impulse) the peak must be lower.
$F_s = \frac{\pi}{4} F_T$

9.16

4

$F = m\frac{dv}{dt}$

$\Rightarrow m\,dv = F\,dt$

$m(v_f - v_i) = \int F\,dt$

To get same final velocity, v_f. $\int_0^T F\,dt$ must be same for both cases.

$\therefore \frac{1}{2}\times F_T \times T = \int_0^T F_s \sin\left(\frac{\pi t}{T}\right) dt$

$\Rightarrow \frac{F_T \cdot T}{2} = \left[F_s \cdot -\cos\left(\frac{\pi t}{T}\right) \cdot \frac{T}{\pi}\right]_0^T$

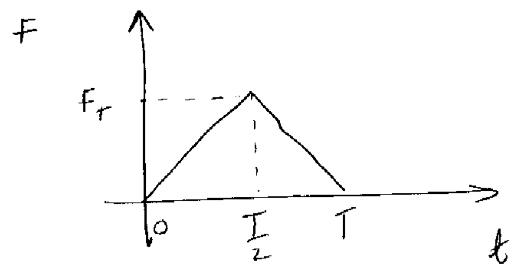
$= \frac{T F_s}{\pi}\left[1 - \cos\frac{\pi \cancel{T}}{\cancel{T}}\right]$

$\Rightarrow \frac{F_T \cdot \cancel{T}}{2} = \frac{2\cancel{T} F_s}{\pi}$

$\Rightarrow \boxed{F_s = \frac{\pi}{4} F_T.}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

9.16



a) $V = \int a\,dt$

$= \frac{1}{m} \underbrace{\int F\,dt}_{\text{area under F-t curve}}$

$= \frac{1}{m}\left[\frac{1}{2} \times T \times F_T\right]$ $\left\{\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}\right\}$

$\therefore \boxed{V = \frac{T F_T}{2m}}$

b) $F(t) = F_s \sin\left(\frac{\pi t}{T}\right)$

$V = \frac{1}{m}\int_0^T F\,dt = \frac{F_s}{m}\int_0^T \sin\left(\frac{\pi t}{T}\right) dt$

$= \frac{F_s}{m} \frac{\left[-\cos\frac{\pi t}{T}\right]_0^T}{\frac{\pi}{T}}$

$= \frac{T F_s}{m\pi}\left[-\cos\pi + \cos 0\right] = \frac{2 T F_s}{m\pi}$

But $\frac{T F_T}{2m} = \frac{2 T F_s}{\pi m} \Longrightarrow \boxed{F_s = \frac{\pi}{4} F_T}$

10.1.17 A particle of mass $m = 1\,\text{kg}$ is acted upon by a short duration force given by

$$F(t) = \begin{cases} F_0 t/\text{s} & 0 \le t \le 1\,\text{s} \\ F_0(2 - t/\text{s}) & 1\,\text{s} < t \le 2\,\text{s} \end{cases}$$

where $F_0 = 5\,\text{N}$. If the particle starts from rest, find the speed of the particle as a function of time. Sketch the given force profile as a function of time and draw the corresponding speed $v(t)$ as a function of time. What is the speed of the particle at $t = 2\,\text{s}$?

Answer:
$v(2\,\text{s}) = 5\,\text{m/s}$

9.17

Given,

$m = 1\text{ kg}.$

$$F(t) = \begin{cases} F_0 t & 0 \le t \le 1\text{s} \\ F_0(2-t) & 1\text{s} \le t \le 2\text{s} \end{cases}$$

$F_0 = 5\text{ N}.$

$v(t=0) = 0.$

For $0 \le t \le 1\text{s}$

$$m\frac{dv}{dt} = F_0 t$$

$$\Rightarrow dv = \frac{F_0}{m}\, t\,dt$$

Integrating both sides, we get

$$\int_0^v dv = \frac{F_0}{m}\int_0^t t\,dt$$

$$\Rightarrow \boxed{v = \frac{F_0}{2m}t^2} \Rightarrow \boxed{v = 2.5t^2}$$

$$\therefore v\Big|_{t=1\text{sec}} = 2.5\text{ m/s}.$$

$1\text{s} \le t \le 2\text{s}.$

$F(t) = F_0(2-t)$

$$\Rightarrow m\int_{v_1}^{v_2} dv = F_0\int_1^2 (2-t)\,dt$$

$$\Rightarrow v_2 = v_1 + \frac{F_0}{m}\cdot\left[2t - \frac{t^2}{2}\right]_1^2$$

$$\Rightarrow v_2 = 2.5 + \frac{5}{1}\left[(4-2) - (2 - 1/2)\right]$$

$$\Rightarrow v_2 = 5\text{ m s}^{-1}. \quad \text{Ans}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

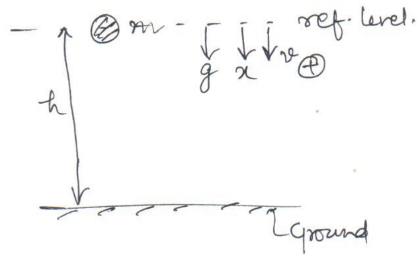
10.1.18 A ball of mass m is dropped vertically from rest at a height h above the ground. Air resistance causes a drag force on the ball directly proportional to the speed v of the ball, $F_d = bv$. Find the velocity and position of the ball as a function of time. Find the velocity as a function of position. Gravity is non-negligible, of course.

5

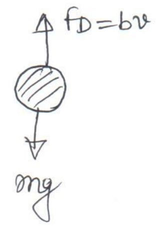
9.18

$v(t) = ?$

$x(t) = ?$



Free body diagram:—



Net force on body of mass 'm' in x-dirn.

$$mg - bv = m\ddot{x}$$

$$\Rightarrow \quad m\frac{dv}{dt} = mg - b\dot{x} \qquad [v = \dot{x}]$$

$$\Rightarrow \quad \frac{\cancel{m}\,dv}{\left(\cancel{m}g - \frac{bv}{\cancel{m}}\right)} = dt$$

$$\Rightarrow \quad \ln\left(\cancel{m}g - \frac{bv}{m}\right) = t + C$$

at $t = 0$, $v = 0$

$$\Rightarrow \quad C = \ln(\cancel{m}g).$$

$$\therefore \quad \ln\left(1 - \frac{bv}{mg}\right) = t$$

$$\Rightarrow \quad 1 - \frac{bv}{mg} = e^{t}$$

$$\Rightarrow \quad \frac{bv}{mg} = 1 - e^{t}$$

$$\Rightarrow \quad \boxed{v = \frac{mg}{b}\left[1 - e^{t}\right]}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$v = \frac{mg}{b}\left[1 - e^{t}\right]$$

$$\Rightarrow \frac{dx}{dt} = \frac{mg}{b}\left[1 - e^{t}\right]$$

$$\Rightarrow \int_0^x dx = \frac{mg}{b}\int_0^t (1 - e^{t})\,dt$$

$$\Rightarrow x = \frac{mg}{b}\left[t - e^{t}\right]_0^t$$

$$= \frac{mg}{b}\left[t - e^{t} + 1\right]$$

$$\Rightarrow \boxed{x = \frac{mg}{b}\left[1 + t - e^{t}\right]} \quad \underline{\text{Ans}}$$

10.1.19 A sinusoidal force acts on a 1 kg mass as shown in the figure and graph below. The mass is initially still; i. e.,

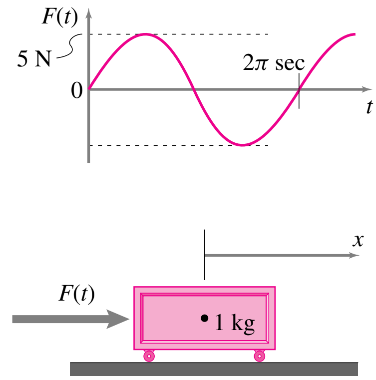
$$x(0) = v(0) = 0$$

.

a) What is the velocity of the mass after 2π seconds?

b) What is the position of the mass after 2π seconds?

c) Plot position x versus time t for the motion.



Problem 10.19

Answer:
(a) $v = 0$, (b) $x = 10\pi$ m

6

9.19

$x(0) = v(0) = 0$, $m = 1\,kg$.

$F(t) = 5\sin t$

$m\frac{dv}{dt} = 5\sin t$

$\Rightarrow \int_0^v dv = \frac{5}{m}\int_0^t \sin t\, dt$

$\Rightarrow v = 5\left[-\cos t\right]_0^t$

$\Rightarrow \boxed{v = 5[1-\cos t]}$ Ans

(a) $v\big|_{t=2\pi} = 0.$

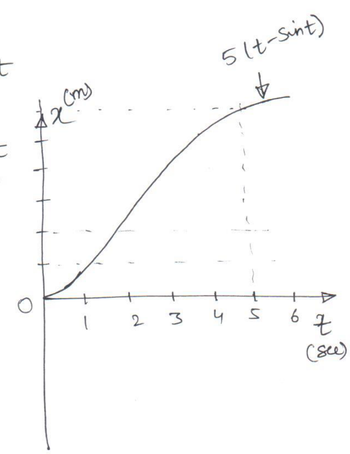
(b) $v = 5(1-\cos t)$

$\frac{dx}{dt} = 5(1-\cos t)$

$\Rightarrow \int_0^x dx = \int_0^t 5(1-\cos t)\,dt$

$\Rightarrow x = 5\left[t - \sin t\right]_0^t$

$\Rightarrow \boxed{x = 5\left[t - \sin t\right]}.$



Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

10.1.20 A motorcycle accelerates from 0 mph to 60 mph in 5 seconds. Find the average acceleration in m/s^2. How does this acceleration compare with g, the acceleration of an object falling near the earth's surface?

9.20

$v_i = 0\,\text{mph}$ 1 mile $\simeq$ 1.6 Km.

$v_f = v(t = 5\,\text{sec}) = 60\,\text{mph} \simeq \dfrac{\cancel{60} \times 1.6 \times 10^3}{\cancel{60} \times 60}$

$= 26.67\,\text{ms}^{-1}$.

Average acceleration: $a_{avg} = \dfrac{v_f - v_i}{t_f - t_i}$

$= \dfrac{26.67}{5}\,\text{ms}^{-2}$

$\Rightarrow \boxed{a_{avg} = 5.334\,\text{ms}^{-2}}$ Ans

$\Rightarrow \boxed{a_{avg} = 0.5437\,g.}$

10.1.21 A car moves on a straight road with an initial velocity $v_0 = 30\,\mathrm{m/s}$. Let its position at $t = 0$ be $x = 0$. For the first 5 s it has no acceleration, and thereafter it brakes with a retarding force that gives it a constant acceleration $a_x = -10m/s^2$. Calculate the velocity and the x-coordinate of the car when $t = 8$ s and when $t = 12$ s, and find the distance travelled by the car from start until it comes to a final stop.

9.21

$0 \le t \le 5\,\text{sec}$

$v_0 = 30\,ms^{-1}$.

$\therefore\ x(5\,\text{sec}) = v_0 t$

$= 150\,m$.

~~$5 \le t \le$~~ for $t \geq 5\,\text{sec}$, $a_x = -10\,ms^{-2}$.

$\Rightarrow \dfrac{dv}{dt} = -10$

$\Rightarrow \int_{30}^{v} dv = \int_{5}^{t} -10\,dt$

$\Rightarrow v - 30 = -10(t-5)$

$\Rightarrow \boxed{v = 80 - 10t}$

9.21 Continue ... 7

$$v = 80 - 10t \quad \text{— (1)}$$

$$\frac{dx}{dt} = 80 - 10t$$

$$\Rightarrow \int_{150}^{x} dx = \int_{5}^{t} (80 - 10t)\,dt$$

$$\Rightarrow x = 150 + \left(80t - 10t^2/2\right)_5^t$$

$$= 150 + 80t - 5t^2 - 400 + 125$$

$$\Rightarrow \boxed{x = -5t^2 + 80t - 125}$$

$$\therefore v\Big|_{t=8sec} = 0 \quad \underline{Ans}$$

$$v\Big|_{t=12sec} = 80 - 10 \times 12$$
$$= -40ms^{-1}. \quad \underline{Ans}$$

body started moving in negative x-dirn. with $40ms^{-1}$ at $t = 12sec$.

$$x\Big|_{t=8sec} = -5 \times 8^2 + 80 \times 8 - 125$$
$$= 195m. \quad \underline{Ans}$$

$$x\Big|_{t=12sec} = 115m. \quad \underline{Ans}$$

10.1.22 A grain of sugar falling through honey has a negative acceleration proportional to the difference between its velocity and its 'terminal' velocity, which is a known constant v_t. Write this sentence as a differential equation, defining any constants you need. Solve the equation assuming some given initial velocity v_0.

9.22

$$a \propto -(v - v_t)$$

$$a = -b(v - v_t) \quad ; \; b = \text{proportionality Constant.}$$

$$\Rightarrow m\frac{dv}{dt} = -b(v - v_t)$$

$$\Rightarrow \frac{dv}{dt} = -\frac{b}{m}(v - v_t)$$

$$\Rightarrow \frac{dv}{(v - v_t)} = -\frac{b}{m}\,dt$$

Integrating, we get

$$\int_{v_0}^{v} \frac{dv}{(v - v_t)} = -\frac{b}{m}\int_{t=0}^{t} dt$$

$$\Rightarrow \left[\ln(v - v_t)\right]_{v_0}^{v} = -\frac{b}{m}t$$

$$\Rightarrow \left(\frac{v - v_t}{v_0 - v_t}\right) = e^{-b/m\,t}$$

$$\Rightarrow v = (v_0 - v_t)e^{-b/m\,t} + v_t$$

$$\Rightarrow \boxed{v = v_t + (v_0 - v_t)e^{-b/m\,t}}$$

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8

9.22 Continue —

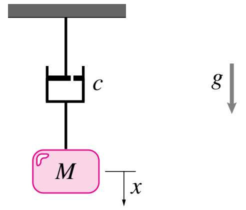
$$\frac{dx}{dt} = v_t + (v_0 - v_t)\, e^{-b/m\, t}$$

$$\int_{x_0}^{x} dx = \int_0^t \left[v_t + (v_0 - v_t)\, e^{-b/m\, t}\right] dt$$

$$\Rightarrow \quad x = x_0 + v_t \cdot t + (v_0 - v_t)\frac{m}{b}\left[1 - e^{-b/m\, t}\right]$$

Ans

10.1.23 The mass-dashpot system shown below is released from rest at $x = 0$. Determine an equation of motion for the particle of mass m that involves only $\dot{x}$ and x (a first-order ordinary differential equation). The dashpot opposes the motion of mass m with a force $F = c\dot{x}$ where c is the damping coefficient of the dashpot. (You can think of the dashpot as a model for linear friction.)

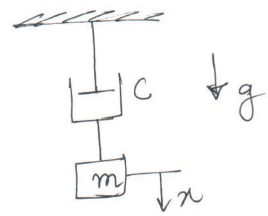


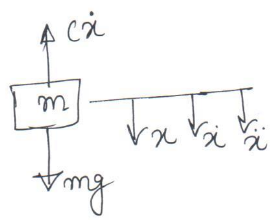
Problem 10.23

9.23

9

Free body diagram of mass "m"





~~$m\ddot{x} + mg = c\dot{x}$~~ $\quad F_{net} = ma$

$$\Rightarrow \quad mg - c\dot{x} = m\ddot{x}$$

$$\Rightarrow \quad \boxed{m\ddot{x} + c\dot{x} - mg = 0.}$$

$$\dot{x} = y$$

$$\boxed{m\dot{y} + cy = mg}$$

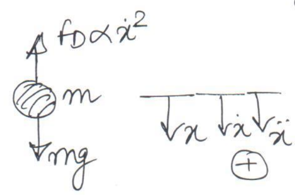
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

10.1.24 Due to gravity, a particle falls in air with a drag force proportional to the speed squared.

1. Write $\sum \vec{F} = m\vec{a}$ in terms of variables you clearly define,
2. find a constant speed motion that satisfies your differential equation,
3. pick numerical values for your constants and for the initial height. Assume the initial speed is zero
   a) set up the equation for numerical solution,
   b) solve the equation on the computer,
   c) make a plot with your computer solution and show how that plot supports your answer to (2).

9.24 1. $F_D \propto \dot{x}^2$ 10

$\Rightarrow F_D = +b\dot{x}^2$



$m\ddot{x} = mg - b\dot{x}^2$

$$\Rightarrow \boxed{m\ddot{x} + b\dot{x}^2 - mg = 0}$$

2 Constant speed motion, $\ddot{x} = 0$

$\Rightarrow b\dot{x}^2 = mg.$

$\dot{x}^2 = \frac{mg}{b}.$

$\Rightarrow \dot{x} = \sqrt{\frac{mg}{b}}.$

3. $\ddot{x} = -\frac{b}{m}\dot{x}^2 + mg.$

Matlab: For Numerical solution

```
~~function (xdot~~
function [xdot] = myeqn(t,x)

xdot = [x(2) ; (-b*x(2)^2)/m + m*g.]

(t,x) = ode45(@myeqn, [tspan], [zo]);
```

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

10.1.26 A bullet penetrating flesh slows approximately as it would if penetrating water. The drag on the bullet is about $F_D = c\rho_w v^2 A/2$ where ρ_w is the density of water, v is the instantaneous speed of the bullet, A is the cross sectional area of the bullet, and c is a drag coefficient which is about $c \approx 1$. Assume that the bullet has mass $m = \rho_l AL$ where ρ_l is the density of lead, A is the cross sectional area of the bullet and L is the length of the bullet (approximated as cylindrical). Assume $m = 2$ grams, entering velocity $v_0 = 400\,\mathrm{m/s}$, $\rho_l/\rho_w = 11.3$, and bullet diameter $d = 5.7\,\mathrm{mm}$.

a) Plot the bullet position vs time.

b) Assume the bullet has effectively stopped when its speed has dropped to $5\,\mathrm{m/s}$, what is its total penetration distance?

c) According to the equations implied above, what is the penetration distance in the limit $t \to \infty$?

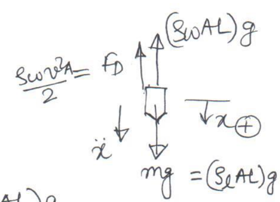
d) How would you change the model to make it more reasonable in its predictions for long time?

9.26

Free body diagram of bullet –

$\rho_L = 11.3\rho_w$

$\Rightarrow \rho_w = 0.0885\rho_l$.



$$\Rightarrow \ddot{x}(\rho_l AL) = (\rho_l AL)g - (\rho_w AL)g - \frac{\rho_w v^2 A}{2}$$

$$\Rightarrow \cancel{\rho_l A}L \frac{dv}{dt} = 0.9115(\cancel{\rho_l A}L)g - \frac{0.0885(\cancel{\rho_l A})v^2}{2}$$

$$\Rightarrow \frac{L\,dv}{dt} = 0.9115Lg - \frac{0.0885v^2}{2}$$

$$\Rightarrow \frac{l\,dv}{(8.942l - 0.04425v^2)} = dt$$

$$\Rightarrow \frac{dv}{\left(8.942 - \frac{0.04425v^2}{L}\right)} = dt$$

Integrate to find velocity & position as a function of time.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

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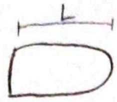
2/16/21

10.1.26

BULLET WITH DRAG

GIVEN:

(force sense only correct with positive velocity)



$$\leftarrow F_D = c\rho_w V^2 \frac{A}{2}$$

where ρ_w = density of water = 1000 kg/m³

V = instantaneous speed

A = cross-sectional area $= \pi r^2 = \pi\left(\frac{0.0057m}{2}\right)^2$

c = drag coefficient = 1

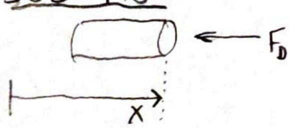
also ρ_l/ρ_w = density of lead/density of H_2O = 11.3

$V_0 = 400$ m/s

$m = 0.002$ kg $= \rho_l A L$

FIND: a) position vs. time, b) position at $V = 5$ m/s c) position at $t \to \infty$, d) model change for large-t predictions

SOLUTION:



FORCE BALANCE: $\sum \vec{F} = m\vec{a}$

1D problem, so I dot with $\hat{\imath}$, no more vector notation from here on out

$$-c\rho_w V^2 \frac{A}{2} = ma$$

$$\underbrace{-c\rho_w \frac{A}{2}\left(\frac{1}{m}\right)}_{\text{let's call this constant "B"}} V^2 = \frac{dv}{dt}$$

$(B > 0)$

NOTE: because of the square, this equation only works for $V > 0$. (imagine if V were negative – the square would get rid of the sign & cause the force to have the wrong sense.)

$$\therefore \quad \boxed{-Bv^2 = \frac{dV}{dt}}$$

and

$$\boxed{V = \frac{dx}{dt}}$$

two 1st-order ODES

pg 1

okay, we have our 2 ODES:

$$-Bv^2 = \frac{dv}{dt} \qquad V = \frac{dx}{dt}$$

we can start addressing the specific questions now :)

a) position vs. time

2 strategies:

NUMERICAL

plan: use MATLAB to estimate the position based on last position & rate of change of position @ last position

"Euler method"

code structure:

1. parameters
2. define time array
3. initial conditions
4. use numerical integrator (takes in EoM, params, time array, IC's) (returns integrated state vector)
5. plot

FUNCTIONS:

① numerical integrator

↓

ANALYTICAL

plan 1: solve by hand :(

let's use the 1st order $-Bv^2 = \frac{dv}{dt}$ ODE and try separation of variables:

$$-B\,dt = \frac{1}{V^2}\,dV$$ integrate both sides

$$-Bt + c_1 = -\frac{1}{V}$$

$$V = -\frac{1}{-Bt + c_1} = \frac{1}{Bt - c_1}$$

use initial condition V_0 to solve for integration constant:

$$V_0 = \frac{1}{B(0) - c_1} = -\frac{1}{c_1}$$

$$c_1 = -\frac{1}{V_0}$$

now we have:

$$\boxed{V = \frac{1}{Bt + \frac{1}{V_0}}}$$

which we can rewrite as:

↓

PG 2

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

↓

(2) EoM function

see $\boxed{\text{p10-1-26.m}}$ (pg.___) for full code.

see $\boxed{\text{figure (1)}}$ (pg.___) for position vs. time plot.

↓

$$\frac{dx}{dt} = \frac{1}{Bt+\frac{1}{V_0}}$$ another 1st order ODE!

separate variables again:

$$dx = \frac{1}{Bt+\frac{1}{V_0}}\,dt$$

integrate both sides

$$x = \frac{\ln(BV_0t+1)}{B} + C_2$$

use initial condition X_0 to solve for integration constant:

$$X_0 = \frac{\ln(BV_0(0)+1)}{B} + C_2$$

$$X_0 = \cancel{\frac{\ln(1)}{B}}^{\,0} + C_2$$

$$C_2 = X_0$$

so finally:

$$\boxed{x = \frac{\ln(BV_0t+1)}{B} + X_0}$$

plan 2: solve in MATLAB symbolic toolbox... cool, but not necessary for this class.

see $\boxed{\text{p10\_1\_26.m}}$ (pg.___) if interested.

phew! let's go to part b

↓

pg (3)

b) position @ $V = 5\ m/s$

strategies:

(1) NUMERICAL: open zarray in workspace.

zarray =

x_0	x_1	x_2	x_3	....
V_0	V_1	V_2	V_3	...

find where $V = 5\ m/s$, select corresponding x.

(2) NUMERICAL: plot V vs. t. click around on plot until you find $V = 5\ m/s$. note the time.
now plot x vs. t. click around on plot until you find the time you just noted. now note the position.

(3) ANALYTICAL: use the analytical solution $V = \frac{1}{Bt + \frac{1}{V_0}}$ to solve for t when $V = 5\ m/s$.
then use the solution $x = \frac{\ln(BV_0 t + 1)}{B} + x_0$ to solve for x at the t you found.

(4) ANALYTICAL: use your two analytical solutions

$V = \frac{1}{Bt + \frac{1}{V_0}}$ & $x = \frac{\ln(BV_0 t + 1)}{B} + x_0$

to produce ONE equation that eliminates t. (i.e. x as a function of V. solve.)

there are many methods. Result should be around $\boxed{x = 0.68m.}$

PG (4)

c) What happens to position as $t \to \infty$?? $x \to \infty$... but why?

ANALYTICAL: look at solution

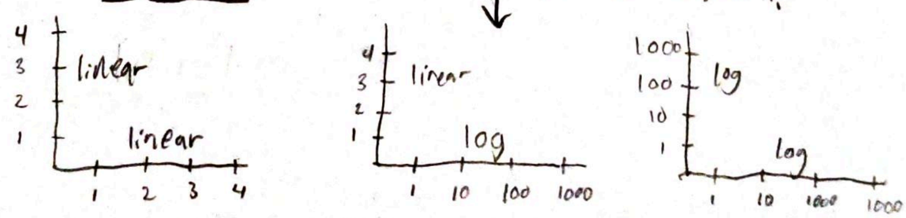
$$x = \frac{\ln(BV_0 t + 1)}{B} + x_0$$

take the limit! (or recognize that the behavior of $\ln(t)$ does not reach a horizontal asymtote. $\lim_{t \to \infty}(\ln(t)) = \infty$)

NUMERICAL: fancy MATLAB! the linear scaling on your plot makes it hard to see if $x(t)$ is approaching a horizontal asymtote or just increasing very slowly. But what if we change the t-axis to log scale? see $\boxed{\text{figure (1)}}$ & $\boxed{\text{figure (2)}}$ (pg. ___)

review:

this one works well for our system.



command: plot( ) command: semilogx( ) command: loglog( )

not super important for this class – if you are interested, spend some time thinking about why log scales better display long-term behavior. :)

RESULT: as $t \to \infty$, $\boxed{x \to \infty}$

what?? this doesn't happen in real life... see part (d)!

Pg 5

```
% p10_1_26.m

% DYNAMICS HW 2
% PROBLEM 5
% RUINA-PRATAP 10.1.26

% MARIGOT FACKENTHAL
% MKF47@CORNELL.EDU


clear; clf; close all; clc;

% SETUP
 *******************************************************************

% parameters [CHANGE THESE]
m = 0.002;
c = 1;
rhoW = 1000;
rhoL = 11.3*rhoW;
d = 0.0057;
A = pi*(d/2)^2;
L = m/(rhoL*A);

% for this problem, I'll group some of the parameters into a variable
% called "B" (it's a constant that will get carried around)
B = c*rhoW*A / (2*m);
p.B = B;  % pack B into a parameter struct for ode45

% timing
t0 = 0;  tf = 1; n = 5000;
tarray = linspace(t0,tf,n);

% initial conditions
rx0 = 0;
vx0 = 400;
z0 = [rx0; vx0];  % use semicolons to make it a 4x1
% also, make sure the entries are in the standard order (ex. x1, x2,
 v1, v2)



% NUMERICAL SOLUTION
 ******************************************************

[tarray, zarray] = myEulerSolver(@eqn, tarray, z0, p);

% outputs
rx = zarray(:,1);
vx = zarray(:,2);

% lets also find F(t)
```

```
Farray = -B.*vx.^2 .* m;


% ANALYTICAL SOLUTION
 ****************************************************

syms x(t) v(t) B_

disp('Analytical solutions w/o initial conditions or parameters:')
ode1 = diff(x) == v;
ode2 = diff(v) == -B_*v^2;
odes = [ode1; ode2];
S = dsolve(odes);
xSol(t) = S.x
vSol(t) = S.v

disp('Analytical solutions with initial conditions and parameters:')
ode1 = diff(x) == v;
ode2 = diff(v) == -B*v^2;
odes = [ode1; ode2];
cond1 = x(0) == 0;
cond2 = v(0) == 400;
conds = [cond1; cond2];
S = dsolve(odes, conds);
xSol(t) = real(S.x)
vSol(t) = S.v

xAnalytical = xSol(tarray)';
vAnalytical = vSol(tarray)';


% lets also find F(t) [analytical]
FarrayAnalytical = (-B.*vAnalytical.^2 .* m)';


disp('Find x when v = 5m/s:')
xSol(0.031)



% PLOTTING
 ***************************************************************

% ---------------
% POSITION V. TIME
figure(1)
hold on
grid on

% plot numerical solution
plot(tarray, rx, '-r', 'LineWidth', 3)

% plot analytical solution
```

```
plot(tarray, xAnalytical, '--b', 'LineWidth', 3)

% figure settings
xlabel('t [s]')
ylabel('x [m]')
title('(1) bullet with drag (position)')
legend('numerical', 'analytical')
hold off


% ---------------
% POSITION V. LOG(TIME)
figure(2)

% plot numerical solution
semilogx(tarray, rx, '-r', 'LineWidth', 3)
hold on
grid on

% plot analytical solution
semilogx(tarray, xAnalytical, '--b', 'LineWidth', 3)

% figure settings
xlabel('t [s]')
ylabel('x [m]')
title('(2) bullet with drag (logx scale, position)')
legend('numerical', 'analytical')
hold off


% ---------------
% NUMERICAL TO ANALYTICAL COMPARISON FOR POSITION V. TIME
figure(3)
hold on
grid on

% plot difference between analytical and numerical solutions
plot(tarray, xAnalytical-rx)

% figure settings
xlabel('t [s]')
ylabel('analytical - numerical [m]')
title('(3) numerical to analytical comparison (position)')
hold off


% ---------------
% FORCE V. VELOCITY
figure(4)

% plot numerical solution
loglog(vx, Farray, '-r', 'LineWidth', 3)
hold on
grid on
```

```
% plot analytical solution
loglog(vAnalytical, FarrayAnalytical, '--b', 'LineWidth', 3)

% figure settings
xlabel('v [m/s]')
ylabel('F [N]')
title('(4) bullet with drag (F vs v)')
legend('numerical', 'analytical')
hold off



% FUNCTIONS
 ****************************************************************

% equations of motion
function dz = eqn(t,z,p) % time, state, struct of matricies

    % unpack parameters ***********************************
    B = p.B;

    % unpack z vector *************************************
    % positions
    rx = z(1);
    % velocities
    vx = z(2);

    % fill dz vector **************************************
    % first entries become velocities
    dz(1) = vx;
    % next entries become accelerations
    % CHANGE THESE 2 LINES ACCORDING TO EOM
    dz(2) = -B*vx^2;

    % transpose dz from row to column vector **************
    dz = dz';

end


% Solver using Euler's method (Stolen from Teaya Yang)
function  [tarray, zarray] = myEulerSolver(eqn, tarray, z0, p)
% Generating zarray matrix
ntimes = length(tarray);
zarray = zeros(ntimes, length(z0));
zarray(1,:) = z0';
h = tarray(2) - tarray(1); % step size


    % Applying Euler's method at each step
    for i  = 1:ntimes-1
```


Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

```
        znow    = zarray(i,:)';   tnow = tarray(i);  % set present
 values
        zdot = eqn(tnow,znow,p);   % evaluate RSH (right hand side) at
 present time

        % Euler's method
        znew = znow + zdot * h;

        % Append to zarray
        zarray(i+1,:) = znew'; % Add a row to the array of solutions.
                          % Recall, z is a column, zarray is in rows.
    end % loop

end  % function
```

Analytical solutions w/o initial conditions or parameters:

xSol(t) =

```
                     C3
 C2 + log(B_*t - C1)/B_
```

vSol(t) =

```
              0
 -1/(C1 - B_*t)
```

Analytical solutions with initial conditions and parameters:

xSol(t) =

*0.1568*log(abs(6.3794*t + 0.0025)) + 0.9392*

vSol(t) =

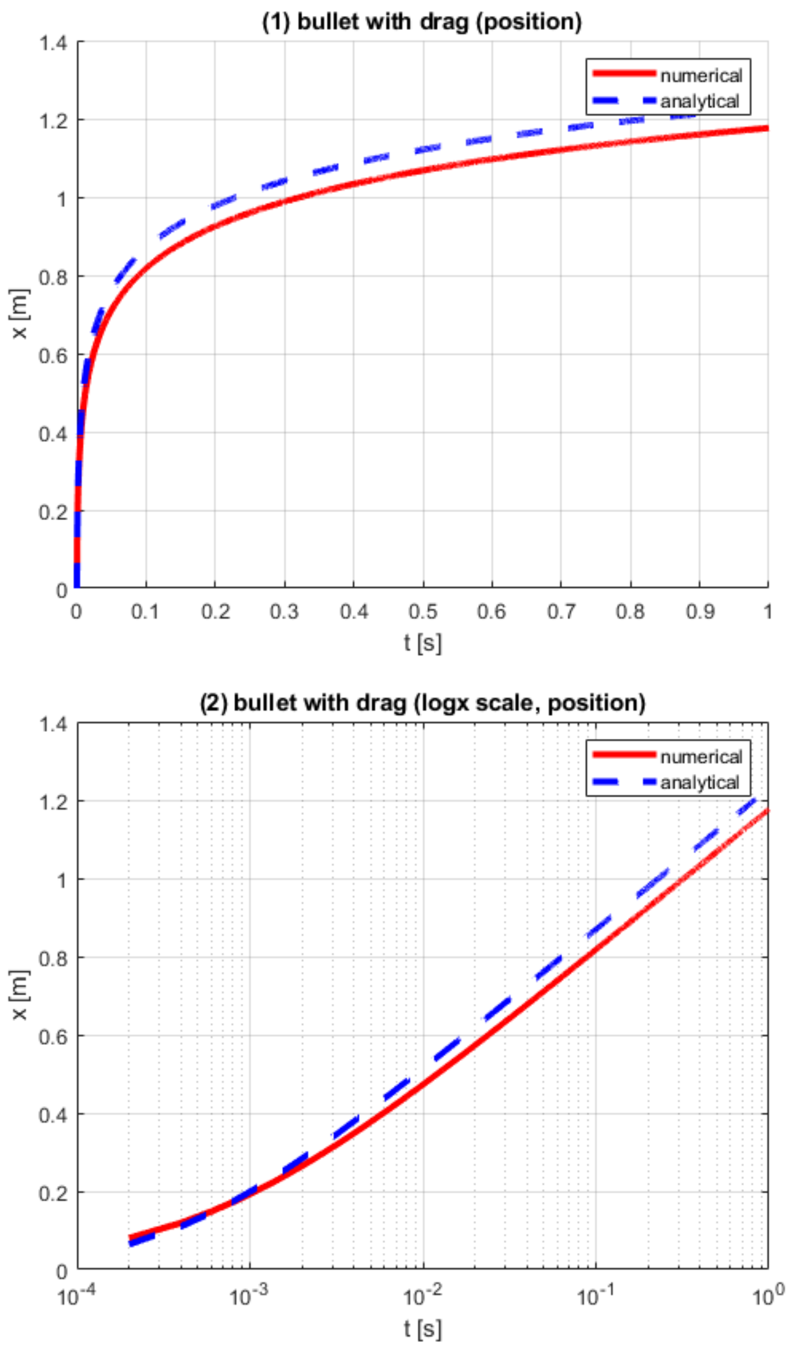
*1/(6.3794*t + 0.0025)*

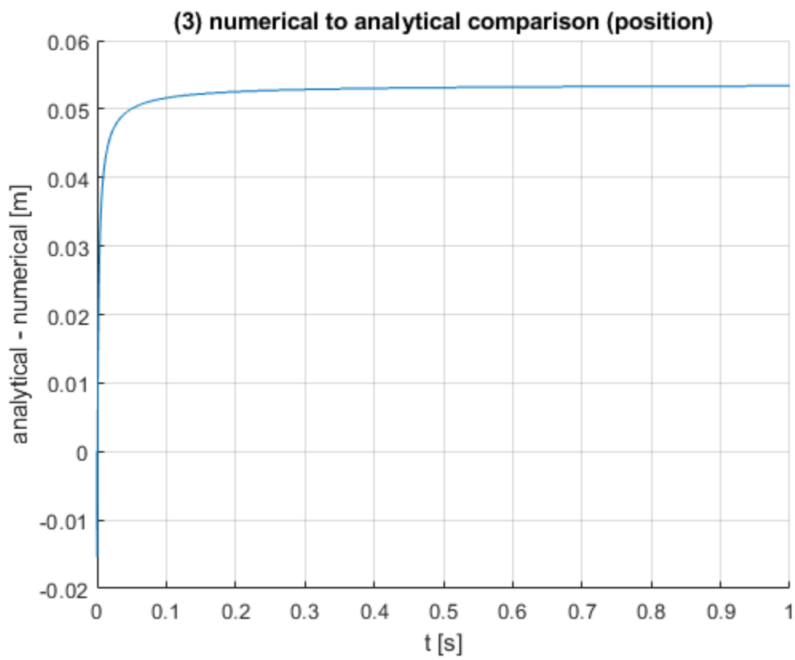
Find x when v = 5m/s:

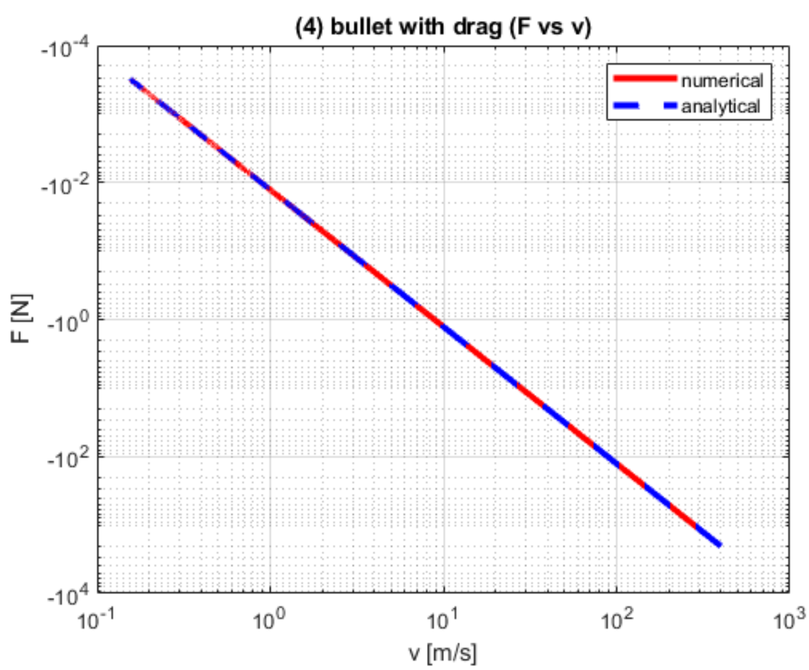
ans =

0.6871

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*


(1) bullet with drag (position)
numerical
analytical
x [m]
t [s]
(2) bullet with drag (logx scale, position)
numerical
analytical
x [m]
t [s]

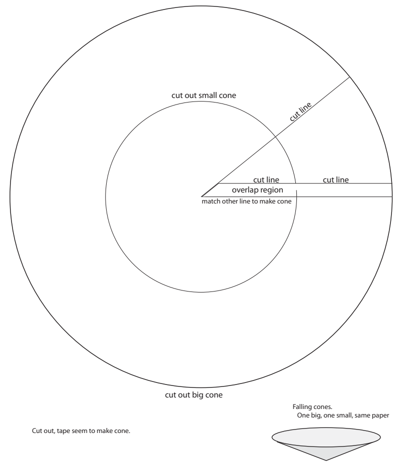




Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

10.1.28 Do the falling cone experiment: Make two cones, drop them simultaneously and see which falls faster. Explain the result in a way that would convince you if it was written by another student who you did not already trust (and you had not already seen the experiment done).

More details. Copy the drawing below onto two pieces of paper. Make one cone from each page. One has twice the radius as the other. They both have the same conical angle. Which falls faster? Use $F = ma$ to predict the result. Hint 1: the cutout lets you think about how to calculate the area of the paper used to make the cone. Hint 2: The mass of the cone is proportional to its area. The result holds for linear or quadratic drag, or a combination of both. It holds for the steady state or for the transient response.



Problem 10.28

Answer:
The cones fall the same. All of the different forces (gravity, linear drag, quadratic drag) are proportional to either the mass or the area. And, for the cones, mass is proportional to area.

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