10.1 Force and motion in 1D

Preparatory Problems

10.1.1 Give three examples of real life objects where you might use the idealization, for dynamic calculations, that the object is a particle in unconstrained 1D motion. *

10.1.2 A car is going downhill on a constant slope straight road. For finding out the car's speed at the end of the road you model it as a particle. For specifying initial velocity, which point on the car would you consider? *

10.1.3 The acceleration of a particle is given as a function of time, $a(t)$. Is this information sufficient to find the speed of the particle at the end of, say, T seconds? *

10.1.4 If a particle has constant acceleration, its linear momentum (a) remains constant, (b) changes linearly with time, or (c) changes quadratically with time. Which one is true? *

10.1.5 In a motorcycle race on a straight track, the speed v^* of a motorcyclist at the d=200 m mark is recorded. Given that the rider started from rest, find the initial acceleration of the motorcycle. Assume the acceleration (a) is constant or (b) (challenge) decreases linearly with time to zero at the end. *

10.1.6 The force acting on a particle with mass m is a given function of time. If you plot the force vs time and find the area under the graph, from that area can you determine (a) the net displacement of the particle?, (b) the average velocity of the particle?, or (c) the change in linear momentum of the particle? *

10.1.7 If the linear momentum of a body remains constant in time, it must have (pick one): (a) a constant force acting on it, (b) no net force acting on it, or (c) a sinusoidal force acting on it. *

10.1.8 The distance between two points in a bicycle race is 10 km. How many minutes does a bicyclist take to cover this distance if he/she maintains a constant speed of 15 mph. *

10.1.9 A 5 kN constant force acts on a 1 kg object that was initially at rest for 5 seconds and then stops. Find the speed of the object at the end of (a) 5 seconds, and (b) 10 seconds.

10.1.10 Given that $\dot{x} = k_1 + k_2 t$, $k_1 = 1\,\text{ft/s}$, $k_2 = 1\,\text{ft/s}^2$, and $x(0) = 1\,\text{ft}$, what is the displacement at the end of 10 seconds? *

10.1.11 Find $x(3\,\text{s})$ given that

$$\dot{x} = x/(1\,\text{s}) \quad \text{and} \quad x(0\,\text{s}) = 1\,\text{m}$$

or, expressed slightly differently,

$$\dot{x} = cx \quad \text{and} \quad x(0\,\text{s}) = x_0,$$

where $c = 1\,\text{s}^{-1}$ and $x_0 = 1\,\text{m}$. Make a sketch of x versus t. *

10.1.12 A ball of mass m is dropped from rest at a height h above the ground. Find the position and velocity as a function of time (as well as m and g, if needed). Neglect air friction. When does the ball hit the ground? What is the velocity of the ball just before it hits? *

10.1.13 The speed of a particle varies sinusoidally as $v = A\sin[ct]$, where $A = 0.5\,\text{m/s}$ and $c = 3\,\text{rad/s}$. Let the initial position of the particle be $x(0) = 0$. Find the position of the particle at $t = \pi/2$ s. *

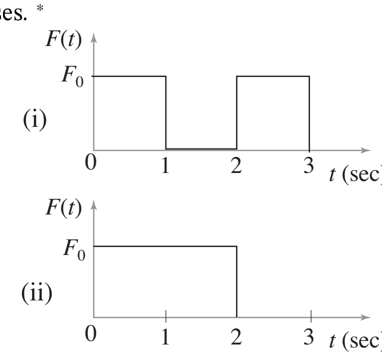
10.1.14 The speed of a particle is directly proportional to its position and is given as $\dot{x} = x/\text{s}$. If the initial position, $x(0) = 1\,\text{m}$, how far would the particle be from the origin in 5 seconds? *

More-Involved Problems

10.1.15 Consider a force $F(t)$ acting on a cart over a 3 second span. In case (a), the force acts in two impulses of one second duration each as shown in fig. 10.1.15. In case (b), the force acts continuously for two seconds and then is zero for the last second. Given that the mass of the cart is 10 kg, $v(0) = 0$, and $F_0 = 10\,\text{N}$, for each force profile,

a) Find the speed of the cart at the end of 3 seconds, and
b) Find the distance travelled by the cart in 3 seconds.

Comment on your answers for the two cases. *



Problem 10.1.15

10.1.16 A car of mass m is accelerated by applying a triangular force profile shown in fig. 10.1.16(a). Find the speed of the car at $t = T$ seconds. If the same speed is to be achieved at $t = T$ seconds with a sinusoidal force profile, $F(t) = F_s \sin\frac{\pi t}{T}$, find the required force magnitude F_s. Is the peak higher or lower? Why? *
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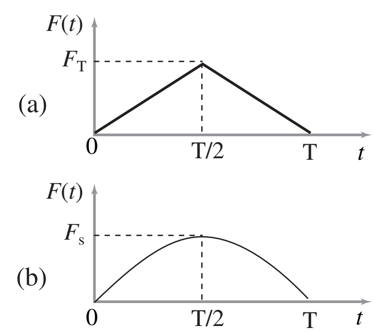

Problem 10.1.16

10.1.17 A particle of mass $m = 1\,\text{kg}$ is acted upon by a short duration force given by

$$F(t) = \begin{cases} F_0 t/\text{s} & 0 \le t \le 1\,\text{s} \\ F_0(2 - t/\text{s}) & 1\,\text{s} < t \le 2\,\text{s} \end{cases}$$

where $F_0 = 5\,\text{N}$. If the particle starts from rest, find the speed of the particle as a function of time. Sketch the given force profile as a function of time and draw the corresponding speed $v(t)$ as a function of time. What is the speed of the particle at $t = 2\,\text{s}$? *

10.1.18 A ball of mass m is dropped vertically from rest at a height h above the ground. Air resistance causes a drag force on the ball directly proportional to the speed v of the ball, $F_d = bv$. Find the velocity and position of the ball as a function of time. Find the velocity as a function of position. Gravity is non-negligible, of course.

10.1.19 A sinusoidal force acts on a 1 kg mass as shown in the figure and graph below. The mass is initially still; i. e.,

$$x(0) = v(0) = 0$$

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a) What is the velocity of the mass after 2π seconds?

b) What is the position of the mass after 2π seconds?

c) Plot position x versus time t for the motion.

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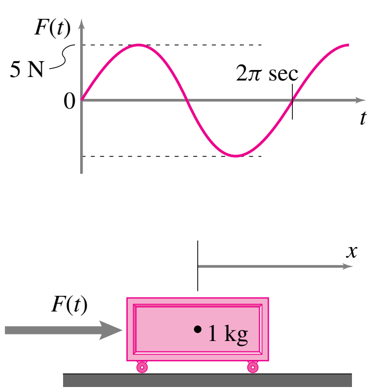


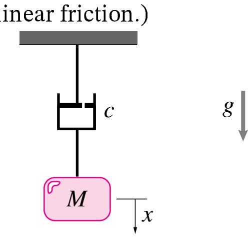
Problem 10.1.19

10.1.20 A motorcycle accelerates from 0 mph to 60 mph in 5 seconds. Find the average acceleration in m/s^2. How does this acceleration compare with g, the acceleration of an object falling near the earth's surface?

10.1.21 A car moves on a straight road with an initial velocity $v_0 = 30\,\text{m/s}$. Let its position at $t = 0$ be $x = 0$. For the first 5 s it has no acceleration, and thereafter it brakes with a retarding force that gives it a constant acceleration $a_x = -10m/s^2$. Calculate the velocity and the x-coordinate of the car when $t = 8\,\text{s}$ and when $t = 12\,\text{s}$, and find the distance travelled by the car from start until it comes to a final stop.

10.1.22 A grain of sugar falling through honey has a negative acceleration proportional to the difference between its velocity and its 'terminal' velocity, which is a known constant v_t. Write this sentence as a differential equation, defining any constants you need. Solve the equation assuming some given initial velocity v_0.

10.1.23 The mass-dashpot system shown below is released from rest at $x = 0$. Determine an equation of motion for the particle of mass m that involves only $\dot{x}$ and x (a first-order ordinary differential equation). The dashpot opposes the motion of mass m with a force $F = c\dot{x}$ where c is the damping coefficient of the dashpot. (You can think of the dashpot as a model for linear friction.)



Problem 10.1.23

10.1.24 Due to gravity, a particle falls in air with a drag force proportional to the speed squared.

1. Write $\sum \vec{F} = m\vec{a}$ in terms of variables you clearly define,
2. find a constant speed motion that satisfies your differential equation,
3. pick numerical values for your constants and for the initial height. Assume the initial speed is zero
   a) set up the equation for numerical solution,
   b) solve the equation on the computer,
   c) make a plot with your computer solution and show how that plot supports your answer to (2).

10.1.25 In quadratic drag problems, the deceleration is proportional to the square of velocity, *i.e.,* $a = \frac{dv}{dt} = -kv^2$. Assume that a particle with initial velocity $v(0) = v_0$ experiences quadratic drag.

a) How long does it take for the particle to reduce its speed to half of its initial speed (*i.e.,* find t such that $v(t) = \frac{1}{2}v_0$)?

b) Find the position of the particle as a function of velocity. How far does the particle move from its initial position when its velocity drops to half its initial value?

10.1.26 A bullet penetrating flesh slows approximately as it would if penetrating water. The drag on the bullet is about $F_D = c\rho_w v^2 A/2$ where ρ_w is the density of water, v is the instantaneous speed of the bullet, A is the cross sectional area of the bullet, and c is a drag coefficient which is about $c \approx 1$. Assume that the bullet has mass $m = \rho_l AL$ where ρ_l is the density of lead, A is the cross sectional area of the bullet and L is the length of the bullet (approximated as cylindrical). Assume $m = 2$ grams, entering velocity $v_0 = 400\,\text{m/s}$, $\rho_l/\rho_w = 11.3$, and bullet diameter $d = 5.7\,\text{mm}$.

a) Plot the bullet position vs time.

b) Assume the bullet has effectively stopped when its speed has dropped to $5\,\text{m/s}$, what is its total penetration distance?

c) According to the equations implied above, what is the penetration distance in the limit $t \to \infty$?

d) How would you change the model to make it more reasonable in its predictions for long time?

10.1.27 A force pulls a particle of mass m towards the origin according to the law (assume same equation works for $x > 0, x < 0$)

$$F = Ax + Bx^2 + C\dot{x}$$

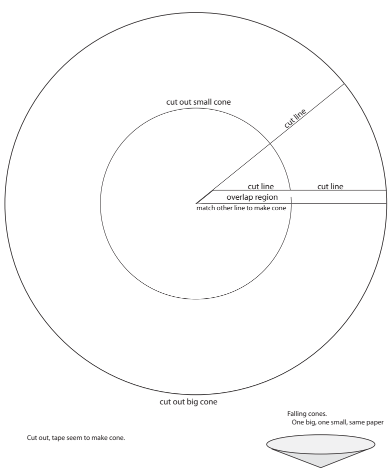
Assume $\dot{x}(0) = 0$.
Using numerical solution, find values of A, B, C, m, and x_0 so that

1. the mass never crosses the origin,
2. the mass crosses the origin once,
3. the mass crosses the origin many times.

[Hint: Vary one parameter at a time and choose a different set of parameter values for each case.]

10.1.28 Do the falling cone experiment: Make two cones, drop them simultaneously and see which falls faster. Explain the result in a way that would convince you if it was written by another student who you did not already trust (and you had not already seen the experiment done).

More details. Copy the drawing below onto two pieces of paper. Make one cone from each page. One has twice the radius as the other. They both have the same conical angle. Which falls faster? Use $F = ma$ to predict the result. Hint 1: the cutout lets you think about how to calculate the area of the paper used to make the cone. Hint 2: The mass of the cone is proportional to its area. The result holds for linear or quadratic drag, or a combination of both. It holds for the steady state or for the transient response. *



Problem 10.1.28