CHAPTER 10

Dynamics in 1D

The scalar equation $F = ma$ introduces the concepts of motion and time derivatives to mechanics. In particular the equations of dynamics are seen to reduce to ordinary differential equations, the simplest of which have memorable analytic solutions. The harder differential equations need be solved on a computer. We explore various concepts and applications involving momentum, power, work, kinetic and potential energies, oscillations, collisions and multi-particle systems.

We now progress from statics to dynamics. As the names imply, statics generally concerns things that don't move, or at least don't accelerate much, whereas dynamics concerns things whose motion is of central interest. In statics, we neglected inertial (terms involving acceleration of mass). So, in statics the linear and angular momentum balance equations were reduced to force and moment balance. In dynamics the inertial terms in the momentum balance equations are important. In statics all the forces and moments cancel each other. In dynamics the forces and moments add up to cause the acceleration of mass.

Once you have mastered free-body diagrams and statics, the hard part of dynamics is learning how to keep track of motion. Keeping track of motion, without worrying about forces, is called *kinematics*. Kinematics is geometry in motion. The study of kinematics together with forces is dynamics, in the mechanics sense of the word①. Dynamics is called *kinetics*. We will develop our understanding of dynamics (kinetics) by considering progressively more complex kinematics.

This first dynamics chapter is limited to the unconstrained dynamics of one or more particles moving in one spatial dimension (1D). Each particle moves along a straight line (and not on a planar or spatial curve) and all forces are along that line. What is a particle?

> A *particle* is a system idealized as being totally characterized by its position (as a function of time) and its (fixed) mass (read more on page 242).

Unconstrained motion. Finally, in this chapter we only consider cases where the applied forces are either given as a function of time or can be determined from the positions and velocities of the particles. The time-varying thrust from an engine might be thought of as a force given as a function of time. Gravity and springs cause forces which are functions of position. And the drag on a particle as it moves through air or water can be modeled as a force depending on velocity. We postpone until the next chapter forces caused by geometric constraints, for example the forces between particles connected by strings or rods. Such *constraint* forces need to be solved-for using dynamics. In contrast, the forces in this chapter can be found *a priori* from position, velocity or time.

Kinematics, acceleration and calculus. The main new concept here, which stays with us until the end of the book, is that things change with time. We keep track of that change using calculus. In particular, the equation $F = ma$ is a differential equation because

$$a \equiv \frac{d^2}{dt^2}x = \ddot{x}\ .$$

Any equation that has terms which are derivatives of functions is a differential equation. Because acceleration a is defined in terms of derivatives any equation involving a is a differential equation.

The organization of this chapter

The first three sections are a review and deepening of material from freshman physics: $F = ma$, energy methods, and the harmonic oscillator. The last three sections concern multi-particle systems, collisions and collections of masses connected with springs and dashpots.

①Outside of mechanics the word *dynamics* means the study of anything which changes in time. For example, "relationship dynamics" concerns how people's interpersonal feelings change over time, and the "dynamics of the market place" concerns fluctuations of prices due to supply, demand and so on. Electro-dynamics is the study of how voltage varies in time and dynamics of the hormonal system concerns how hormone levels go up and down.

Systems with this more general dynamics are sometimes called "dynamical systems". The classic dynamical systems equation is

$$\frac{d}{dt}z = f(z)$$

where z is a list of numbers describing the state of the system and $f(z)$ is the set of rules that dictate how the system changes due to its present state. One sub-class of dynamical systems are the mechanical systems in this book where a set of equations, each of the form

$$\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}},$$

are re-written in the form

$$\dot{z} = f(z),$$

as you will see.

Before going on please get a better lay of the land by reading the introduction to mechanics in Chapter 1 and looking over the summary of mechanics on the inside cover.