Partial Solutions Manual
Ruina and Pratap
Introduction to Statics and Dynamics

This draft: August 21, 2010

Have a suggestion? Want to contribute a solution?
Contact ruina@cornell.edu with Subject: Solutions Manual

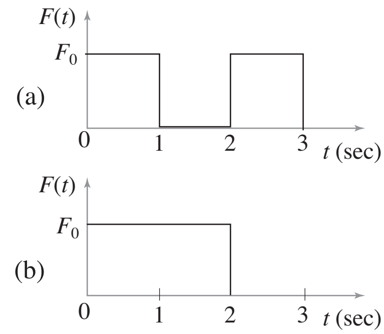
Note, the numbering of hand-written solutions is most-often wrong (corresponding to an old numbering scheme). The hand-written problem numbers should be ignored.

9.1.15 Consider a force $F(t)$ acting on a cart over a 3 second span. In case (a), the force acts in two impulses of one second duration each as shown in fig. 9.1.15. In case (b), the force acts continuously for two seconds and then goes to zero. Given that the mass of the cart is 10 kg, $v(0\,\mathrm{s}) = 0$, and $F_0 = 10\,\mathrm{N}$, for each force profile,

a) Find the speed of the cart at the end of 3 seconds, and

b) Find the distance travelled by the cart in 3 seconds.

Comment on your answers for the two cases.



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Problem 9.15

9.15

$m = 10$ kg

$v(0) = 0$

$F_0 = 10$ N

$F = ma \quad \therefore \quad a_0 = F_0/m_0 = 10\,\mathrm{N}/10\,\mathrm{kg} = 1\ \mathrm{m/s^2}$

Force profile (a):

@ $t = 1s$: $a = 1\ \mathrm{m/s^2}$, $v = v_0 + at = 0 + 1\,\mathrm{m/s^2}(1s) = 1\ \mathrm{m/s}$

$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(1\,\mathrm{m/s^2})(1s)^2 = 0.5\mathrm{m}$

@ $t = 2s$: $a = 0$, $v = v_0 + at = 1\ \mathrm{m/s} + 0 = 1\ \mathrm{m/s}$

$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0.5 + 1(1) + 0 = 1.5\,\mathrm{m}$

@ $t = 3s$: $a = 1\,\mathrm{m/s^2}$, $v = v_0 + at = 1 + 1(1) = \underline{2.0\ \mathrm{m/s}}$

$x = x_0 + v_0 t + \frac{1}{2}at^2 = 1.5 + 1(1) + \frac{1}{2}(1)(1)^2 = \underline{3.0\ \mathrm{m}}$

Force profile (b):

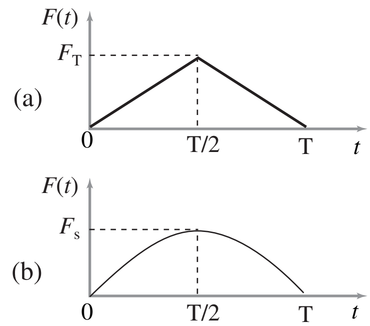
@ $t = 2s$: $a = 1\ \mathrm{m/s^2}$, $v = v_0 + at = 0 + 1(2) = 2\ \mathrm{m/s}$

$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(1)(2)^2 = 2\ \mathrm{m}$

@ $t = 3s$: $a = 0$, $v = v_0 + at = 2 + 0 = \underline{2.0\ \mathrm{m/s}}$

$x = x_0 + v_0 t + \frac{1}{2}at^2 = 2 + 2(1) + 0 = \underline{4.0\ \mathrm{m}}$

9.1.16 A car of mass m is accelerated by applying a triangular force profile shown in fig. 9.1.16(a). Find the speed of the car at $t = T$ seconds. If the same speed is to be achieved at $t = T$ seconds with a sinusoidal force profile, $F(t) = F_s \sin\frac{\pi t}{T}$, find the required force magnitude F_s. Is the peak higher or lower? Why?



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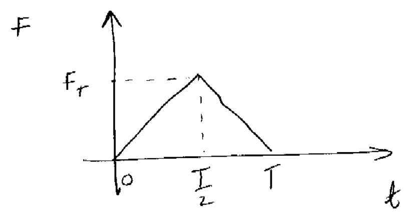
Problem 9.16

9.16



a) $V = \int a\,dt$

$= \frac{1}{m} \underbrace{\int F\,dt}_{\text{area under F-t curve}}$

$= \frac{1}{m}\left[\frac{1}{2} \times T \times F_T\right]$ {Area of triangle = $\frac{1}{2}$ x base x height}

$\therefore \boxed{V = \frac{T F_T}{2m}}$

b) $F(t) = F_s \sin\left(\frac{\pi t}{T}\right)$

$V = \frac{1}{m}\int_0^T F\,dt = \frac{F_s}{m}\int_0^T \sin\left(\frac{\pi t}{T}\right) dt$

$= \frac{F_s}{m} \frac{\left[-\cos\frac{\pi t}{T}\right]_0^T}{\frac{\pi}{T}}$

$= \frac{T F_s}{m\pi}\left[-\cos\pi + \cos 0\right] = \frac{2 T F_s}{m\pi}$

But $\frac{T F_T}{2m} = \frac{2 T F_s}{\pi m} \implies \boxed{F_s = \frac{\pi}{4} F_T}$

9.1.22 A grain of sugar falling through honey has a negative acceleration proportional to the difference between its velocity and its 'terminal' velocity, which is a known constant v_t. Write this sentence as a differential equation, defining any constants you need. Solve the equation assuming some given initial velocity v_0.

9.22. A grain of sugar falling through honey with negative acceleration $\propto (V - V_{terminal})$

Write a differential eqn, and solve using given initial velocity V_0

Kinematics only no FBD needed

let V = velocity

V_t = terminal velocity

V_0 = initial velocity

$\dot{V}$ = acceleration

constant

$$\dot{V} = -k(V - V_t)$$

$$\frac{dv}{dt} = -k(v - v_t)$$

$$\int_{v_0}^{v} \frac{dv}{v - v_t} = -\int k\, dt$$

$$\ln(v - v_t)\Big]_{v_0}^{v} = -kt$$

$$\ln(v - v_t) - \ln(v_0 - v_t) = -kt$$

$$\ln\left(\frac{V - V_t}{V_0 - V_t}\right) = -kt$$

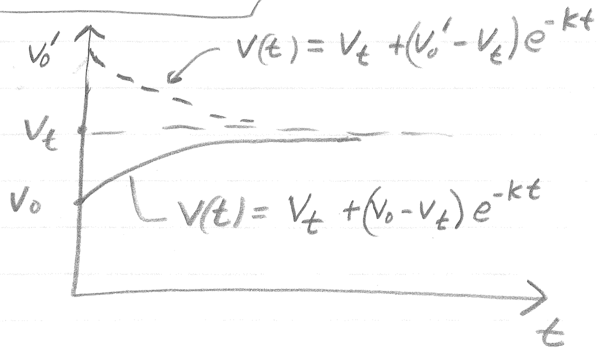
$$\frac{V - V_t}{V_0 - V_t} = e^{-kt}$$

$$\boxed{V = V_t + (V_0 - V_t)e^{-kt}}$$

Two solns:

$V_0' > V_t$

$V_0 < V_t$ $\Rightarrow$



9.1.26 A bullet penetrating flesh slows approximately as it would if penetrating water. The drag on the bullet is about $F_D = c\rho_w v^2 A/2$ where ρ_w is the density of water, v is the instantaneous speed of the bullet, A is the cross sectional area of the bullet, and c is a drag coefficient which is about $c \approx 1$. Assume that the bullet has mass $m = \rho_l AL$ where ρ_l is the density of lead, A is the cross sectional area of the bullet and L is the length of the bullet (approximated as cylindrical). Assume $m = 2$ grams, entering velocity $v_0 = 400\,\mathrm{m/s}$, $\rho_l/\rho_w = 11.3$, and bullet diameter $d = 5.7\,\mathrm{mm}$.

a) Plot the bullet position vs time.

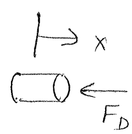
b) Assume the bullet has effectively stopped when its speed has dropped to $5\,\mathrm{m/s}$, what is its total penetration distance?

c) According to the equations implied above, what is the penetration distance in the limit $t \to \infty$?

d) How would you change the model to make it more reasonable in its predictions for long time?

9.26

FBD



$$\Sigma F_{ext} = \dot{L}$$

$$-F_D = m\frac{dv}{dt}$$

$$-\frac{c\rho_w v^2 A}{2} = m\frac{dv}{dt}$$

$$\therefore \quad m\frac{dv}{dt} + \frac{c\rho_w v^2 A}{2} = 0$$

But $m = \rho_l A l$

$$\therefore \quad \rho_l A l\frac{dv}{dt} + \frac{c\rho_w v^2 A}{2} = 0$$

$$\frac{dv}{dt} + \frac{1}{2}\cdot\left(\frac{c}{l}\right)\left(\frac{\rho_w}{\rho_l}\right)v^2 = 0$$

Let's calculate l.

$$m = \rho_l A l$$

$$\therefore l = \frac{m}{\rho_l A}$$

$m = 2\,gm$; $\rho_w = 1$ (known) $\rho_l = 11.3\,\rho_w = 11.3$

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4}(0.57)^2 = 0.26\,cm^2$$

$$\therefore l = \frac{2}{(11.3)(0.26)} = 0.68\,cm = 0.68 \times 10^{-2}\,m$$

Now set $\lambda = \left(\frac{1}{2}\right)\left(\frac{c}{l}\right)\left(\frac{\rho_w}{\rho_c}\right)$

$$\lambda = \left(\frac{1}{2}\right)\left(\frac{1}{0.68\times10^{-2}}\right)\left(\frac{1}{11.3}\right)$$

$$\lambda = 6.5$$

Thus $\frac{dv}{dt} + 6.5v^2 = 0$

⇒ Setting equation for matlab: see bullet.m

$$\left.\begin{aligned}\dot{x} &= v\\ \dot{v} &= -6.5v^2\end{aligned}\right\}\text{ rhs for ode45}$$

⇒ Analytical solution

$$\dot{v} = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dx}(v)\frac{dx}{dt} = \frac{dv}{dx}\,v$$

$$\therefore\ v\frac{dv}{dx} = \dot{v} = -6.5v^2$$

$$\therefore\ v\frac{dv}{dx} + 6.5v^2 = 0$$

$$\therefore\ \frac{dv}{dx} + 6.5v = 0$$

$$v = C_1 e^{-6.5x} \quad \text{(I)}$$

{On substituting $v = C_1 e^{sx}$ & solving for s}

Given $x = 0$; $V_0 = 400$

Solving for C_1 in I gives $C_1 = 400$

$$\therefore \quad V = 400\, e^{-6.5x}$$

$$\frac{dx}{dt} = 400\, e^{-6.5x}$$

$$\int_0^x e^{6.5x}\, dx = 400 \int_0^t dt$$

$$\left[\frac{e^{6.5x}}{6.5}\right]_0^x = 400\,[t]_0^t$$

$$\therefore \quad \frac{e^{6.5x}}{6.5} - \frac{1}{6.5} = 400t$$

$$\boxed{e^{6.5x} = 2600t + 1} \quad \text{— II}$$

a) See attached plot. Done using matlab.

b) Put $V = 5$ in I

$$5 = 400\, e^{-6.5x}$$

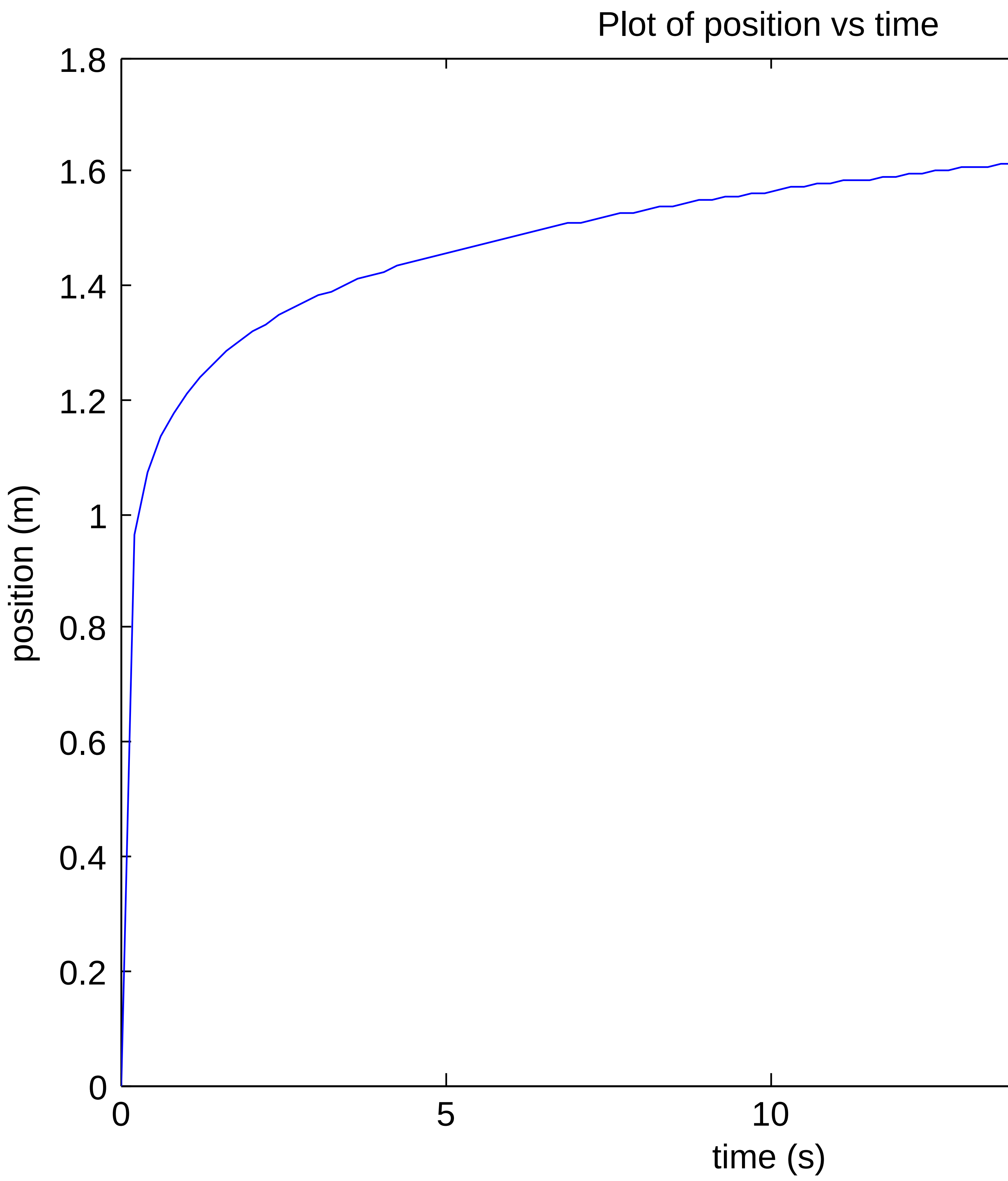
Take ln $\quad \ln\left(\frac{5}{400}\right) = -6.5x$

solving $\boxed{x = 0.67 \text{ m}}$

c) From II as $t \to \infty$ $\quad x \to \infty$

Thus the bullet would penetrate infinite distance
(clearly impossible in reality)

d) Add frictional resistance in addition to drag.


Plot of position vs time
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
position (m)
0
5
10
time (s)

9.2.3 A force $F = F_0 \sin(ct)$ acts on a particle with mass $m = 3\,\text{kg}$ which has position $x = 3\,\text{m}$, velocity $v = 5\,\text{m/s}$ at $t = 2\,\text{s}$. $F_0 = 4\,\text{N}$ and $c = 2/\text{s}$. At $t = 2\,\text{s}$ evaluate (give numbers and units):

a) a,

b) E_K,

c) P,

d) $\dot{E}_K$,

e) the rate at which the force is doing work.

9.30

A force $F = F_0 \sin(ct)$ acts on a particle with $m = 3\,kg$. At $t = 2s$; $x = 3m$, $v = 5m/s$

$F_0 = 4N$, $c = 2/s$

a) Find a at $t = 2s$.

$$F = (4N)\sin(2/s \cdot t)$$

$$a = \frac{F}{m} = \frac{4N}{3kg}\sin(2/s \cdot t)$$

$$= \left(\tfrac{4}{3}\sin(2t)\right)\ m/s^2$$

$$\boxed{a(2s) = -1.01\,m/s^2}$$

b) Find E_K at $t = 2s$

$$E_K = \tfrac{1}{2} m v^2$$

At $t = 2s$, $v = 5m/s$

$$E_K = \tfrac{1}{2}(3kg)(5m/s)^2$$

$$\boxed{E_K = 37.5\ J}$$

c) Find P at $t = 2s$

$$P = Fv$$

$$F(2) = (4\sin(2\cdot 2))\,N$$

$$= -3.03\,N$$

$$P = (-3.03N)(5m/s)$$

$$\boxed{P \approx -15.14\,W}$$

9.30 continued.

d) Find $\dot{E}_K$ at $t = 2s$

$P = \dot{E}_K$

$\boxed{\dot{E}_K = -15.14\, W}$

e) Find the rate at which force is doing work

$W = \int P\, dt$

$\dot{W} = \frac{d}{dt}\left(\int P\, dt\right)$

$= P$

$\boxed{\dot{W} = -15.14\, W}$

9.2.10 A kid ($m = 90\,\text{lbm}$) stands on a $h = 10\,\text{ft}$ wall and jumps down, accelerating with $g = 32\,\text{ft/s}^2$. Upon hitting the ground with straight legs, she bends them so her body slows to a stop over a distance $d = 1$ ft. Neglect the mass of her legs. Assume constant deceleration as she brakes the fall.

a) What is the total distance her body falls?

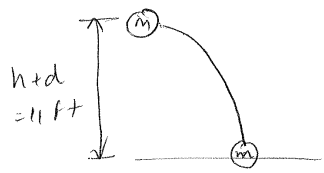
b) What is the potential energy lost?

c) How much work must be absorbed by her legs?

d) What is the force of her legs on her body? Answer in symbols, numbers and numbers of body weight (*i.e.,* find F/mg).

9.37

Given $m = 90$ lbm, $h = 10$ ft, $g = 32\ \text{ft/s}^2$, $d = 1$ ft



Treat the entire body as a particle concentrated at the center of mass

a) Total distance = 11 feet

b) $\Delta PE = mgh = 90\ \text{lbm}\ (32\ \text{ft/s}^2)\ (11\ \text{ft})$

$\boxed{\Delta PE = 31,680\ \text{lb-ft}}$

c) All work must be absorbed.

Thus $\boxed{W = 31,680\ \text{lb-ft}}$

d) $W = Fd$

so $F = \dfrac{W}{d} = \dfrac{31,680\ \text{lb-ft}}{1\ \text{ft}}$

$= \dfrac{mg(h+d)}{d}$

$= 11\,mg$

$\boxed{F = 11\,mg = 11\,wt}$

9.2.11 In traditional archery, when pulling an arrow back the force increases approximately linearly up to the peak 'draw force' F_{draw} that varies from about $F_{draw} = 25$ lbf for a bow made for a small person to about $F_{draw} = 75$ lbf for a bow made for a big strong person. The distance the arrow is pulled back, the draw length ℓ_{draw}, varies from about $\ell_{draw} = 2$ ft for a small adult to about 30 inch for a big adult. An arrow has mass of about 300 grain (1 grain $\approx$ 64.8milli gm, so an arrow has mass of about 19.44 $\approx$ 20 gm $\approx$ 3/4 ounce). Give all answers in symbols and numbers.

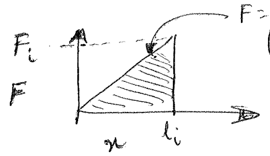
a) What is the range of speeds you can expect an arrow to fly?

b) What is the range of heights an arrow might go if shot straight up (it's a bad approximation, but for this problem neglect air friction)?

9.38

Given $F_1 = 25$ lbf $\quad l_1 = 2$ ft

$F_2 = 75$ lbf $\quad l_2 = 2.5$ ft



$F = \left(\frac{F_i}{l_i}\right)x$ where $i = 1, 2$

Energy stored in pulled arrow

$$W = \int F\,dx$$

$$= \int_0^{l_i} \left(\frac{F_i}{l_i}\right) x\,dx$$

$$W = \frac{F_i l_i}{2}$$ {which is area of triangle above}

$$W_1 = \frac{1}{2} \times 25 \times 2 = 25 \text{ lbf-ft}$$

$$= 25 \times 32.2 \text{ lbm-ft/s}^2$$

$$\therefore W_1 = 805 \text{ lbm-ft/s}^2$$

$$W_2 = \frac{1}{2} \times 75 \times 2.5 = 93.75 \text{ lbf-ft}$$

$$= 93.75 \times 32.2 \text{ lbm-ft/s}^2$$

$$\therefore W_2 = 3018.75 \text{ lbf-ft/s}^2$$

a) $W = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{\frac{2W}{m}}$

Given $m = \frac{3}{4}$ ounce $= 0.047$ lbm

$\therefore \; v_1 = \sqrt{\frac{2W_1}{m}} = \sqrt{\frac{2 \times 805}{0.047}}$

$\therefore \; v_1 = 185.1$ ft/s

$v_2 = \sqrt{\frac{2W_2}{m}} = \sqrt{\frac{2 \times 3018.75}{0.047}}$

$\therefore \; v_2 = 358.2$ ft/s

Thus $\boxed{185.1 \text{ ft/s} \leq v \leq 358.2 \text{ ft/s}}$

b) $W = mgh \Rightarrow h = \frac{W}{mg}$

$h_1 = \frac{W_1}{mg} = \frac{805}{0.047 \times 32.2} = 532$ ft

$h_2 = \frac{W_2}{mg} = \frac{3018.75}{0.047 \times 32.2} = 1994.6$ ft

$\boxed{532 \text{ ft} \leq h \leq 1994.6 \text{ ft}}$

9.2.16 The power available to a very strong accelerating cyclist over short periods of time (up to, say, about 1 minute) is about 1 horsepower. Assume a rider starts from rest and uses this constant power. Assume a mass (bike + rider) of $150\,\text{lbm}$, a realistic drag force of $.006\,\text{lbf}/(\text{ft/s})^2 v^2$. Neglect other drag forces.

a) What is the peak (steady state) speed of the cyclist?

b) Using analytic or numerical methods make an accurate plot of speed vs. time.

c) What is the acceleration as $t \to \infty$ in this solution?

d) What is the acceleration as $t \to 0$ in your solution?

e) How would you improve the model to fix the problem with the answer above?

9.43

$P = 1\text{ HP (constant)} = 550\ \frac{\text{lbf}\cdot\text{ft}}{\text{s}}$

$v_0 = 0,\quad m = 150\,\text{lbm},\quad F_d = 0.006 v^2\ [\text{lbf}]$

1) The peak speed is the speed at which all power is resisted by drag.

$$550\ \frac{\text{lbf}\cdot\text{ft}}{\text{s}} = F_d v = 0.006 v^3$$

$$\therefore\ \boxed{v_{max} = 45.1\ \text{ft/s}}$$

2) $F_d \leftarrow (m) \rightarrow F$

$\sum F = ma = F - F_d \quad \therefore\ m\dot{v} = \frac{P}{v} - 0.006 v^2$

Solve numerically using Matlab.

9.43)

```
function homework943()
% Problem 9.43 Solution
% Feb 5, 2008

% CONSTANTS
P= 550  ; % power in lbf*ft/s
m= 150;  % lbm
g= 32.2;  % ft/s^2

% INTIAL CONDITIONS
v0= 0.001; % initial velocity, zero makes the solution explode

tspan =[0 1000]; %time interval of integration

error = 1e-4;
% Set error tolerance and use 'event detection'
options = odeset('abstol', error, 'reltol', error) ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Ask Matlab to SOLVE odes in function 'rhs'
[t v] = ode45(@rhs,tspan, v0, options, P, m, g)

%UNPACK the zarray (the solution) into sensible variables
plot (t,v)
title('Problem 9.43')
xlabel('Time, t (s)'); ylabel('Speed, v (ft/s)')
axis([0 inf -inf inf])  %inf self scales plot

end  % end of main function
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%
% THE DIFFERENTIAL EQUATION 'The Right Hand Side'
function vdot = rhs(t,v,P,m,g)

vdot = P/(m*v)-0.006*v^2/m;  %  F = m a

end % end of rhs
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

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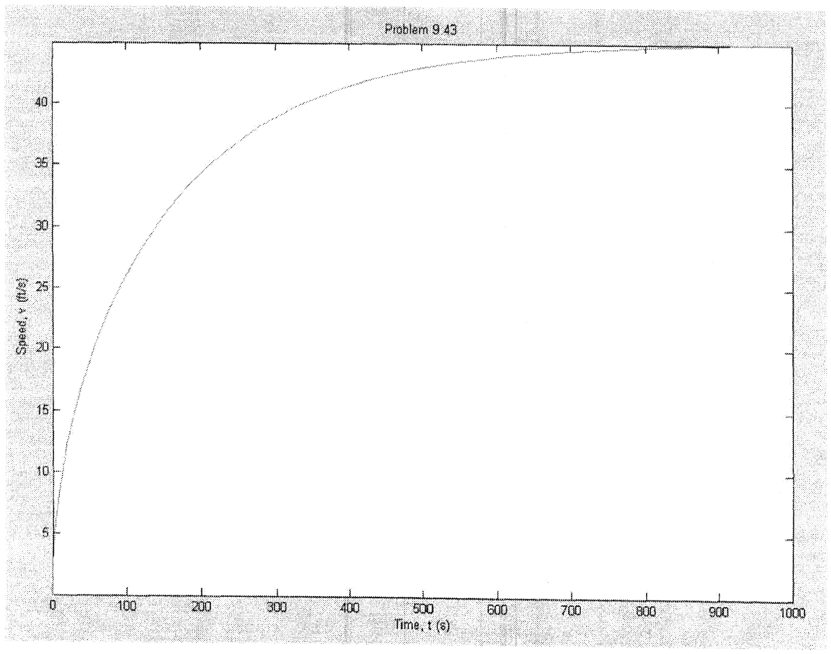
Results from Matlab Code



3) Acceleration is the slope of the velocity on the plot above. As time goes to infinity, the acceleration goes to zero.

4) As time goes to zero, the acceleration goes to infinity. This is why the initial velocity had to be inputted as a very small number (i.e. 0.001 ft/s) instead of zero.

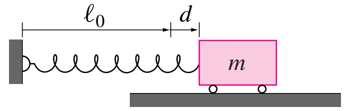
9.3.6 A spring k with rest length ℓ_0 is attached to a mass m which slides frictionlessly on a horizontal ground as shown. At time $t = 0$ the mass is released from rest with the spring stretched a distance d. Measure the mass position x relative to the wall.

a) What is the acceleration of the mass just after release?

b) Find a differential equation which describes the horizontal motion x of the mass.

c) What is the position of the mass at an arbitrary time t?

d) What is the speed of the mass when it passes through $x = \ell_0$ (the position where the spring is relaxed)?



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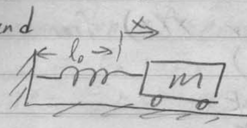
Problem 9.6

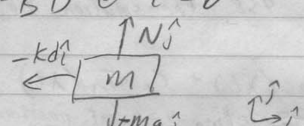
9.49

A spring with rest length l_0 attached to mass m slides frictionless on a horizontal ground



At $t=0$ mass is released with $v_0 = 0$ and spring stretch a distance d.

FBD @ $t=0$



$N\hat{j} = mg\hat{j}$

a) Find a of mass just after release

$\{ m\ddot{x}\hat{i} = -kd\hat{i} \}$

$\{\}\cdot\hat{i} : \; m\ddot{x} = -kd$

$$\boxed{\ddot{x} = \frac{-kd}{m}}$$

b) Find a differential eqn that describes horizontal motion of mass

$\{ m\ddot{x}\hat{i} = -kx\hat{i} \}$

$\{\}\cdot\hat{i} : \; \boxed{m\ddot{x} = -kx}$

9.49 continued

c) Find position of mass at arbitrary time t.

- we will solve for $x(t)$ from the eqn we find in (b)

$$m\ddot{x} = -kx$$

$$\ddot{x} = -\frac{k}{m}x$$

$$\ddot{x} + \frac{k}{m}x = 0$$

let $\lambda^2 = \frac{k}{m}$

$$\ddot{x} + \lambda^2 x = 0$$

according to Page 438, the solution to the above eqn is

$$x = C_1 \cos(\lambda t) + C_2 \sin(\lambda t)$$

apply initial conditions

At $t=0$, $x=d$ → letting $x=0$ at position spring relaxes

$$x(0) = C_1 \cos(\lambda \cdot 0) + C_2 \cancel{\sin(\lambda \cdot 0)}$$

$$d = C_1$$

At $t=0$, $v=0$

$$\dot{x} = -\lambda C_1 \sin(\lambda t) + \lambda C_2 \cos(\lambda t)$$

$$\dot{x}(0) = -\lambda C_1 \cancel{\sin(\lambda \cdot 0)} + \lambda C_2 \cos(\lambda \cdot 0)$$

$$0 = \lambda C_2$$

$$C_2 = 0$$

$$\boxed{x = d\cos\left(\sqrt{\tfrac{k}{m}}\, t\right)}$$

9.49 continued

d) Find speed of mass when it passes through the position where spring is relaxed

conservation of energy

$E_{total} = E_p + E_k$

$\frac{1}{2} k d^2 = \frac{1}{2} k x^2 + \frac{1}{2} m v^2$

for $x = 0$

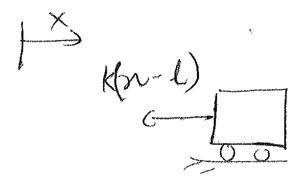
$\frac{1}{2} k d^2 = \frac{1}{2} m v^2$

$\rightarrow \boxed{v = \pm \sqrt{\frac{k}{m}} d}$

9.49 — additional note

If you assume x to be from the wall as stated in the problem, then

FBD



$$\Sigma F_{ext} = \dot{L}$$

$$-k \cdot (x - l_0) = m\ddot{x}$$

Part b:

$$\boxed{m\ddot{x} + kx = kl_0}$$

Part c:

$$\boxed{x = l_0 + d \cos\left(\sqrt{\frac{k}{m}}\, t\right)}$$

Part a, d: will have the same answer as above!

9.3.10 Mass m hangs from a spring with constant k and which has the length l_0 when it is relaxed (i.e., when no mass is attached). It only moves vertically.

a) Draw a Free Body Diagram of the mass.

b) Write the equation of linear momentum balance.

c) Reduce this equation to a standard differential equation in x, the position x of the mass.

d) Verify that one solution is that $x(t)$ is constant at $x = l_0 + mg/k$.

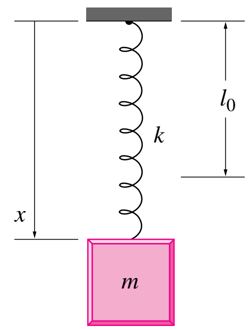
e) What is the meaning of that solution? (That is, describe in words what is going on.)

f) Define a new variable $\hat{x} = x-(l_0+mg/k)$. Substitute $x = \hat{x} + (l_0 + mg/k)$ into your differential equation and note that the equation is simpler in terms of the variable $\hat{x}$.

g) Assume that the mass is released from an an initial position of $x = D$. What is the motion of the mass?

h) What is the period of oscillation of this oscillating mass?

i) Why might this solution not make physical sense for a long, soft spring if the initial stretch is large. In other words, what is wrong with this solution if $D > \ell_0 + 2mg/k$?



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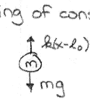
Problem 9.10

9.53

Mass m hanging from spring of constant k, ℓ_0

a) Free body diagram:



b) $\Sigma F_x = ma \Rightarrow mg - k(x-\ell_0) = ma$

c) $m\ddot{x} + k(x-\ell_0) = mg \Rightarrow \ddot{x} + \frac{k}{m}(x-\ell_0) = g$

d) Check $x(t) = \ell_0 + \frac{mg}{k}$, $\ddot{x} = 0$

$\Rightarrow 0 + \frac{k}{m}(\ell_0 + \frac{mg}{k} - \ell_0) = \frac{k}{m}\frac{mg}{k} = g$ ✓

e) This is the deformed equilibrium position of the system under gravity load.

f) Let $\hat{x} = x - (\ell_0 + \frac{mg}{k}) \Rightarrow x = \hat{x} + (\ell_0 + \frac{mg}{k})$

$\therefore$ we have $\ddot{\hat{x}} + \frac{k}{m}(\hat{x} + \ell_0 - \ell_0 + \frac{mg}{k}) = g$

OR $\ddot{\hat{x}} + \frac{k}{m}(\hat{x} + \frac{mg}{k}) = g \Rightarrow \ddot{\hat{x}} + \frac{k}{m}\hat{x} = 0$

g) We solve $\ddot{\hat{x}} + \frac{k}{m}\hat{x} = 0$ to get $\hat{x} = c_1 \sin(t\sqrt{\frac{k}{m}}) + c_2\cos(t\sqrt{\frac{k}{m}})$

$\Rightarrow$ Initial conditions: $\hat{x}(0) = D$, $\dot{\hat{x}}(0) = 0$

$\dot{x} = c_1\sqrt{\frac{k}{m}}\cos(t\sqrt{\frac{k}{m}}) - c_2\sqrt{\frac{k}{m}}\sin(t\sqrt{\frac{k}{m}})$

$\dot{x}(0) = 0 = c_1\sqrt{\frac{k}{m}} \therefore c_1 = 0$

$x(0) = c_2 = D \Rightarrow \boxed{\hat{x}(t) = D\cos(t\sqrt{k/m})}$

h) Period $= \frac{2\pi}{\omega}$, where $\omega = \sqrt{k/m}$ $\therefore \boxed{T = 2\pi\sqrt{m/k}}$

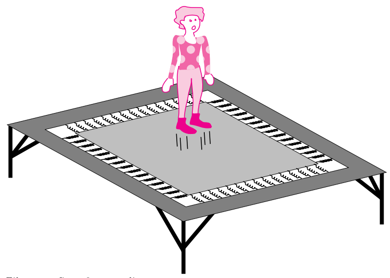
i) This would not make sense because the spring would want to oscillate upwards past the top of the spring (at its support).

9.3.12 A person jumps on a trampoline. The trampoline is modeled as having an effective vertical undamped linear spring with stiffness $k = 200\,\text{lbf}/\,\text{ft}$. The person is modeled as a rigid mass $m = 150\,\text{lbm}$. $g = 32.2\,\text{ft}/\text{s}^2$.

a) What is the period of motion if the person's motion is so small that her feet never leave the trampoline?

b) What is the maximum amplitude of motion (amplitude of the sine wave) for which her feet never leave the trampoline?

c) (harder) If she repeatedly jumps so that her feet clear the trampoline by a height $h = 5\,\text{ft}$, what is the period of this motion (note, the contact time is *not* exactly half of a vibration period)? [Hint, a neat graph of height vs time will help.]



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Problem 9.12: A person jumps on a trampoline.

9.55

Given: $k = 200 \frac{\text{lbf}}{\text{ft}}$, $m = 150$ lbm, $g = 32.2\ \text{ft}/\text{s}^2$

a) If contact with trampoline never breaks,

$m\ddot{x} + kx = -mg$ OR $\ddot{x} + \omega^2 x = -g$, where $\omega = \sqrt{k/m}$

Solution: $x = C_1 \sin(\omega t) + C_2 \cos(\omega t) - g/\omega^2$

$x(0) = 0$ and $\dot{x}(0) = 0$ (INITIAL COND.)

$\dot{x} = C_1 \omega \cos(\omega t) - C_2 \omega \sin(\omega t)$

$\Rightarrow \dot{x}(0) = C_1 \omega = 0 \Rightarrow C_1 = 0$

$x(0) = C_2 - g/\omega^2 = 0 \quad \therefore \quad C_2 = mg/k$

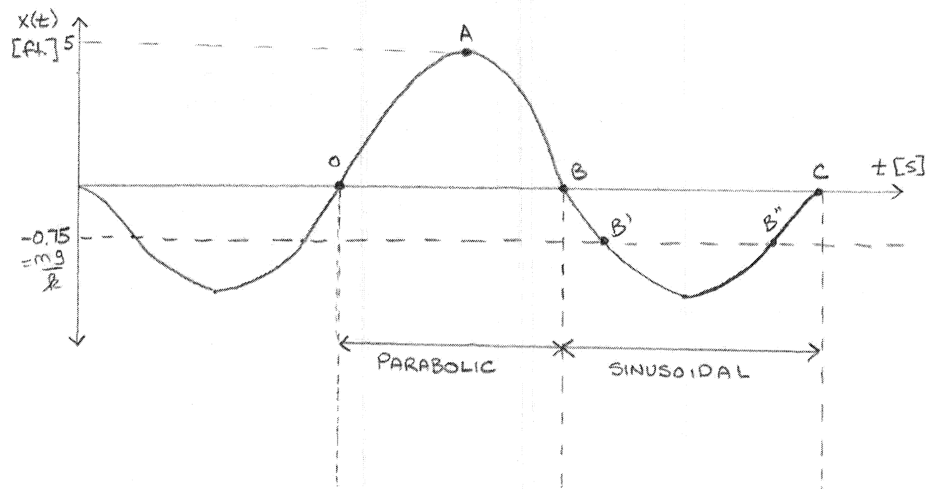
$\therefore \; x(t) = \frac{mg}{k}\left[\cos(\omega t) - 1\right]$

Period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{150\ \text{lbm}}{200\ \text{lbf}/\text{ft}} \times \frac{1\ \text{lbf}}{1\ \text{lbm}\cdot g}}$

$= 2\pi\sqrt{\frac{150}{200\ 1/\text{ft} \times 32.2\ \text{ft}/\text{s}^2}} = \boxed{0.959 \text{ seconds}}$

b) Max amplitude $= \frac{mg}{k} = \frac{150\ \text{lbm}\ g}{200\ \text{lbf}/\text{ft}} \times \frac{1\ \text{lbf}}{1\ \text{lbm}\cdot g} = \boxed{0.75 \text{ feet}}$

c) If the jumper loses contact with the trampoline, to jump to a height of 5 feet, the motion is sinusoidal while in contact and parabolic (from projectile motion theory) once the jumper is in the air. See next page for a plot of what this looks like.



To analyze, assume we begin at point A, a height of 5 feet above the trampoline. ($x=5$, $\dot{x}=0$)

From $A \to B$, $x(t) = -\frac{1}{2}gt^2 + h_o = 5 - \frac{1}{2}gt^2$

$\therefore$ $x(t) = 0$ when $5 - \frac{1}{2}gt^2 = 0$, or $t = \sqrt{\frac{2(5\text{ ft})}{32.2\text{ ft/s}^2}}$

$t = 0.5573$ seconds

$\therefore$ Total time from O to B is $2t = 1.115$ seconds

@ B, $x = 0$ and $\dot{x} = -gt\,/_{t=0.5573} = -17.945$ ft/s $= \dot{x}_0$

We use this as an initial condition to define a new sine wave, setting point B as $t=0$.

$$x(t) = c_1 \sin\left(\sqrt{\tfrac{k}{m}}\,t\right) + c_2 \cos\left(\sqrt{\tfrac{k}{m}}\,t\right) - \frac{gm}{k}$$

$$x(0) = 0 = C_1(0) + C_2 - \frac{mg}{k} \quad \therefore \quad C_2 = \frac{mg}{k}$$

$$\dot{x}(0) = \dot{x}_0 = C_1\sqrt{\frac{k}{m}}\cos(0) - C_2\sqrt{\frac{k}{m}}\sin(0)$$

$$\therefore \quad C_1\sqrt{\frac{k}{m}} = \dot{x}_0 \quad \text{OR} \quad C_1 = \dot{x}_0\sqrt{\frac{m}{k}}$$

$$\therefore \quad x(t) = \dot{x}_0\sqrt{\frac{m}{k}}\sin\left(\sqrt{\frac{k}{m}}t\right) + \frac{mg}{k}\cos\left(\sqrt{\frac{k}{m}}t\right) - \frac{mg}{k}$$

We want to find when this expression $= -\frac{mg}{k}$

$\Rightarrow$ This is point B' on previous graph

Using Matlab or a calculator to solve,

$$t = -\tan^{-1}\left(\frac{g}{\dot{x}_0}\sqrt{\frac{m}{k}}\right) \Big/ \sqrt{\frac{k}{m}}$$

$$= -\tan^{-1}\left(\frac{32.2\ \text{ft/s}^2}{-17.945\ \text{ft/s}}\sqrt{\frac{150\ \text{lbm}}{200\ \text{lbf/ft} \times 32.2\ \text{ft/s}^2}}\right) \Big/ \sqrt{\frac{200 \times 32.2}{150}} = 0.04079\ \text{s}$$

$\therefore$ Distance from B to B' $= 0.04079$ s

We know distance from B' to B'' $= \frac{T}{2} = \frac{0.959\ \text{s}}{2}$

$= 0.4795$ seconds

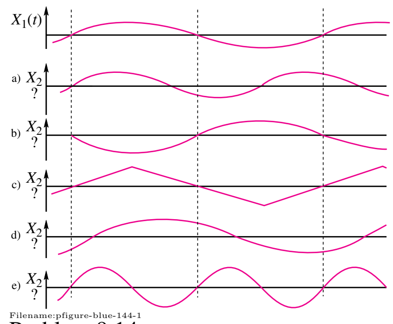
From B'' to C is the same as B to B' $= 0.04079$ s

$\therefore$ Total period $T = 1.115\ \text{s} + 2(0.0408\ \text{s}) + 0.4795\ \text{s}$

$$= \boxed{1.676 \text{ seconds}}$$

The primary emphasis of this section is setting up correct differential equations (without sign errors) and solving these equations on the computer.

9.4.14 $x_1(t)$ and $x_2(t)$ are measured positions on two points of a vibrating structure. $x_1(t)$ is shown. Some candidates for $x_2(t)$ are shown. Which of the $x_2(t)$ could possibly be associated with a normal mode vibration of the structure? Answer "could" or "could not" next to each choice and briefly explain your answer (If a curve looks like it is meant to be a sine/cosine curve, it is.)



Problem 9.14

9.73
* Look at page 466 of text
a) could not because they have different freq.

b) could

c) could not because it is not simple harmonic motion

d) could not because not exactly in (or out) of phase

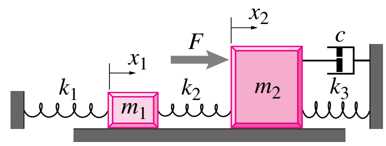
e) could not, reason same as (a)

9.4.17 Two masses are connected to fixed supports and each other with the three springs and dashpot shown. The force F acts on mass 2. The displacements x_1 and x_2 are defined so that $x_1 = x_2 = 0$ when the springs are unstretched. The ground is frictionless. The governing equations for the system shown can be written in first order form if we define $v_1 \equiv \dot{x}_1$ and $v_2 \equiv \dot{x}_2$.

a) Write the governing equations in a neat first order form. Your equations should be in terms of any or all of the constants m_1, m_2, k_1, k_2, k_3, C, the constant force F, and t. Getting the signs right is important.

b) Write computer commands to find and plot $v_1(t)$ for 10 units of time. Make up appropriate initial conditions.

c) For constants and initial conditions of your choosing, plot x_1 vs t for enough time so that decaying erratic oscillations can be observed.



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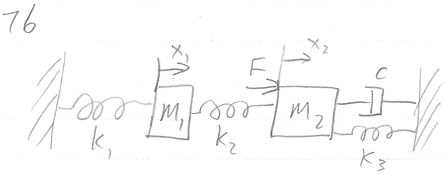
Problem 9.17

9.76

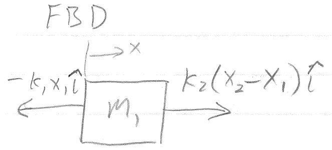


$x_1 = x_2 = 0$ when springs unstretched

ground is frictionless

a) Write governing equation in first order form.

FBD



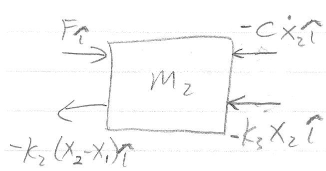
$\{m_1\ddot{x}_1\hat{\imath} = k_2(x_2-x_1)\hat{\imath} - k_1x_1\hat{\imath}\}$

$\{\}\cdot\hat{\imath} \Rightarrow$

$\ddot{x}_1 = \frac{k_2}{m}(x_2-x_1) - \frac{k_1}{m}x_1$

since $\boxed{\dot{x}_1 = v_1}$ ① $\boxed{\dot{v}_1 = \frac{k_2}{m}(x_2-x_1) - \frac{k_1}{m}x_1}$ ③

$\ddot{x}_1 = \dot{v}_1$



$\{m_2\ddot{x}_2\hat{\imath} = F\hat{\imath} - c\dot{x}_2\hat{\imath} - k_3x_2\hat{\imath} - k_2(x_2-x_1)\hat{\imath}\}$

$\{\}\cdot\hat{\imath} \Rightarrow$

$m_2\ddot{x}_2 = F - cx_2 - k_3x_2 - k_2(x_2-x_1)$

since $\boxed{\dot{x}_2 = v_2}$ ② $\boxed{\dot{v}_2 = \frac{F}{m} - \frac{c}{m}\dot{x}_2 - \frac{k_3}{m}x_2 - \frac{k_2}{m}(x_2-x_1)}$ ④

$\ddot{x}_2 = \dot{v}_2$

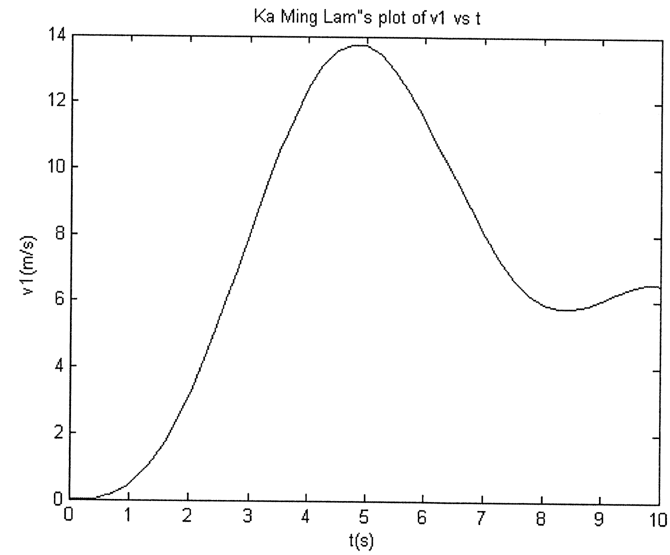
①-④ are gov. eqs in 1st order form

b). See attached files

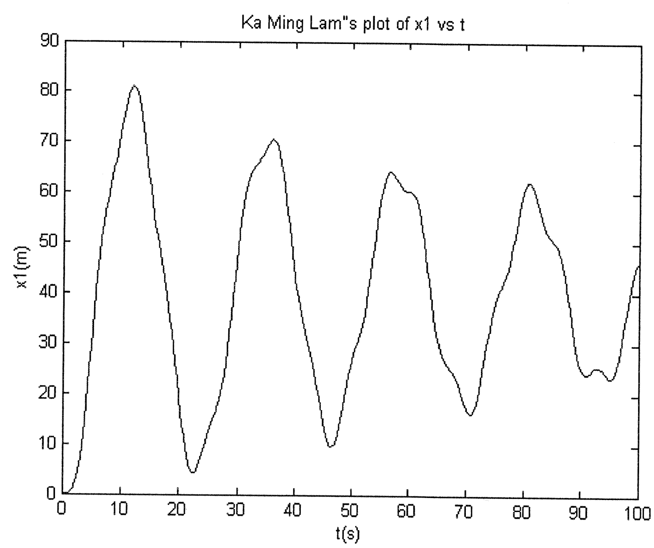
c). Modify the code in b) a little bit to plot $x_1 \sim t$., increase time to see the decaying oscillations.

```
% problem 9.76

function question976
%time span
tspan = [0,10];          %integrate for 10 sec
z0 = [0, 0, 0, 0]';       %initial position and velocity
                    %[x0, vx0, y0, vy0]
%solves the ODEs
[t z] = ode45(@rhs,tspan,z0);

%Unpack the variables
x1 = z(:,1);
v1 = z(:,2);
x2 = z(:,3);
v2 = z(:,4);


%plot the results
plot(t,v1)
title('Ka Ming Lam''s plot of v1 vs t')
xlabel('t(s)')
ylabel('v1(m/s)')
%set grid, xmin, xmax, ymin, ymax


end

%--------------------------------------------------------\
function zdot = rhs(t,z)
x1 = z(1);  v1 = z(2);  x2 = z(3);  v2 = z(4);

%put in vlaues for mass, C and g below
m1 = 2;
m2 = 20;  % masses in kg
C = 0.4;  % in kg/s
F = 120;
k1 = 1;
k2 = 1;
k3 = 1;   % k in N/m

% the linear momentum balance eqns:
x1dot = v1;
v1dot = (-k2-k1)/m1*x1+k2/m1*x2;
x2dot = v2;
v2dot = F/m2-C/m2*v2-(k3+k2)/m2*x2+k2/m2*x1;

zdot = [x1dot;v1dot ; x2dot;v2dot];   %this is what the function returns (column vector)


end
```

b). Here is the plot



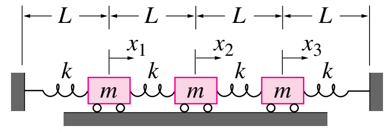
c). In order to have a decaying erratic oscillation we need to increase tspan to [0 100] for this case



9.4.23 For the three-mass system shown, assume $x_1 = x_2 = x_3 = 0$ when all the springs are fully relaxed. One of the normal modes is described with the initial condition $(x_{10}, x_2, x_3) = (1, 0, -1)$.

a) What is the angular frequency ω for this mode? Answer in terms of L, m, k, and g. (Hint: Note that in this mode of vibration the middle mass does not move.)

b) Make a neat plot of x_2 versus x_1 for one cycle of vibration with this mode.



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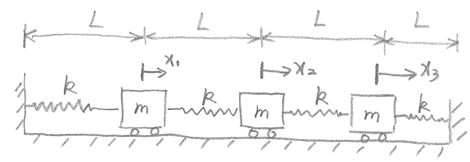
Problem 9.23

①

9.82 Normal modes



3-Mass system, connected by springs with stiffness k. All the mass is m.
Assume $x_1 = x_2 = x_3$ where all springs are at rest.
A normal mode of this system can be described as eigenvector (1, 0, -1).

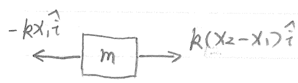
Q: 1). What's the frequency corresponding to this normal mode?
2). Plot x_2 versus x_1 for one cycle of vibration in this mode.

Solution: 1). First, we want to derive the equations of motion for this system, starting from FBD.

FBD: $\longrightarrow \hat{\imath}$

Eq. of motions

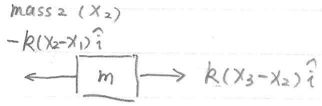
mass 1 (x_1)



$\{ m\ddot{x}_1\hat{\imath} = k(x_2-x_1)\hat{\imath} - kx_1\hat{\imath} \}$

$\{\}\cdot\hat{\imath} \Rightarrow \boxed{m\ddot{x}_1 = -2kx_1 + kx_2}$

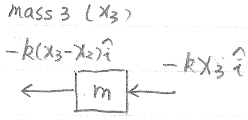
mass 2 (x_2)



$\{ m\ddot{x}_2\hat{\imath} = k(x_3-x_2)\hat{\imath} - k(x_2-x_1)\hat{\imath} \}$

$\{\}\cdot\hat{\imath} \Rightarrow \boxed{m\ddot{x}_2 = kx_1 - 2kx_2 + kx_3}$

mass 3 (x_3)



$\{ m\ddot{x}_3\hat{\imath} = -kx_3\hat{\imath} - k(x_3-x_2)\hat{\imath} \}$

$\{\}\cdot\hat{\imath} \Rightarrow \boxed{m\ddot{x}_3 = kx_2 - 2kx_3}$

(9.82. cont'd) ②

Second, by definition of normal mode, for this normal mode with (1, 0, −1), we can write

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \underbrace{A}_{\text{amplitude}} \underbrace{\cos}_{\text{simple harmonic function}} (\underbrace{\omega}_{\text{same frequency, we want to solve it}} t + \underbrace{\phi}_{\text{exactly in or out of phase.}})$$

$$\Rightarrow \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} A \frac{d^2 \cos(\omega t + \phi)}{dt^2} = -\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} A\omega^2 \cos(\omega t + \phi)$$

Third, substitute $[x]$, $[\ddot{x}]$ for this normal mode into equations of motions:

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 2k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} A\omega^2 \cos(\omega t + \phi) + \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 2k \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} A\cos(\omega t + \phi) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -\begin{bmatrix} m \\ 0 \\ -m \end{bmatrix} \omega^2 + \begin{bmatrix} 2k \\ 0 \\ -2k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -m\omega^2 + 2k = 0 \quad \Rightarrow \quad \boxed{\omega = \sqrt{\frac{2k}{m}}}$$

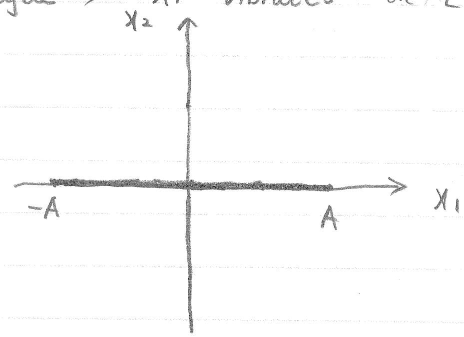
(9.82, cont'd) 3

(2) From (1), we know in this mode

$x_1 = A\cos(\sqrt{\frac{2k}{m}}t + \phi)$

$x_2 = 0$

During one cycle, x_1 vibrates in $[-A, A]$ and x_2 remains 0.



9.5.6 Before a collision two particles, $m_A = 7\,\text{kg}$ and $m_B = 9\,\text{kg}$, have velocities of $v_A^- = 6\,\text{m/s}$ and $v_B^- = 2\,\text{m/s}$. The coefficient of restitution is $e = .5$. Find the impulse of mass A on mass B and the velocities of the two masses after the collision.

9.84

Before a collision two particles,

$m_A = 1\,kg$ $\quad$ $m_B = 2\,kg$

$v_A^- = 10\,m/s$ $\quad$ $v_B^- = 5\,m/s$

After the collision

$v_A^+ = 8\,m/s$

a) momentum of A before collision?

momentum $= m_A v_A^-$

$\boxed{= 10\,kg\,m/s}$

b) momentum of B before collision?

momentum $= m_B v_B^-$

$\boxed{= 10\,kg\,m/s}$

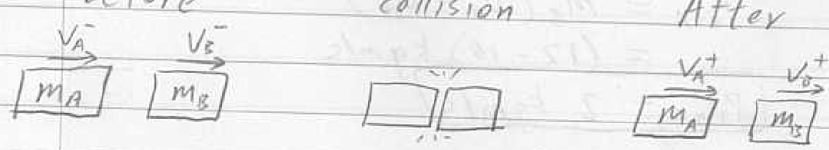
c) System momentum before collision

System momentum $= m_A v_A^- + m_B v_B^-$

$\boxed{= 20\,kg\,m/s}$

FBD

before $\quad$ collision $\quad$ After



9.84 continued

d) momentum of A after collision

momentum $= m_A V_A^+$

$\boxed{= 8\ kgm/s}$

e) System momentum after collision

System momentum after = System momentum before

$\boxed{= 20\ kgm/s}$

f) momentum of B after collision

System momentum after = momentum$_A$ + momentum$_B$

momentum$_B$ = $\boxed{12\ kgm/s}$

g) impulse A applies to B?

$P_{A \to B} = m_A (V_A^+ - V_A^-)$

$= (8 - 10)\ kgm/s$

$\boxed{P_{A \to B} = -2\ kgm/s}$

h) impulse B applies to A?

$P_{B \to A} = m_B (V_B^+ - V_B^-)$

$= (12 - 10)\ kgm/s$

$\boxed{P_{B \to A} = 2\ kgm/s}$

9.84 continued

i) E_K before collision?

$$E_K = \frac{1}{2} m_A (v_A^-)^2 + \frac{1}{2} m_B (v_B^-)^2$$

$$\boxed{E_K = 75\ \text{J}}$$

j) E_K after collision?

$$E_K = \frac{1}{2} m_A (v_A^+)^2 + \frac{1}{2} m_B (v_B^+)^2$$

$$= \frac{1}{2}(1\,\text{kg})(8\,\text{m/s})^2 + \frac{1}{2}(2\,\text{kg})(6\,\text{m/s})^2$$

$$\boxed{E_K = 68\ \text{J}}$$

k) Coefficient of restitution?

$$(v_B' - v_A') = e\,(v_A - v_B)$$

$$(6 - 8)\,\text{m/s} = e\,(10 - 5)\,\text{m/s}$$

$$-2 = 5e$$

$$\boxed{e = -2/5 = -0.4}$$

Problem 9.84

If you assumed $v_A^+ = 6\ m/s$, than the following answers will change

d) $6\ kg\ m/s$

f) $14\ kg\ m/s$

g) $-4\ kg\ m/s$. You get this by solving $v_B^+ = 7\ m/s$

h) $4\ kg\ m/s$

j) $67\ J$

k) 0.2

9.5.10 A basketball with mass m_b is dropped from height h onto the hard solid ground on which it has coefficient of restitution e_b. Just on top of the basketball, falling with it and then bouncing against it after the basketball hits the ground, is a small rubber ball with mass m_r that has a coefficient of restitution e_r with the basketball.

a) In terms of some or all of m_b, m_r, h, g, e_b and e_r how high does the rubber ball bounce (measure height relative to the collision point)?

b) Assuming the coefficients of restitution are less than or equal to one, for given h, what mass and restitution parameters maximize the height of the bounce of the rubber ball and what is that height?

9.92

Basketball with mass m_b dropped from height h, $e = e_b$

Small rubber ball with mass m_r, $e = e_r$

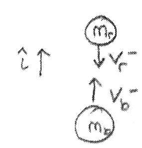
a) Treat this as two collisions:

i) basketball hits ground

$V_0 = 0$, cons. of energy $V_f = -\sqrt{2gh}$ (before hit)

After collison $V = e_b V_f = e_b\sqrt{2gh}$

ii) basketball and rubber ball collide



$V_r^- = -\sqrt{2gh}$, $V_b^- = e_b\sqrt{2gh}$

$m_b V_b^- + m_r V_r^- = m_b V_b^+ + m_r V_r^+$ (1)

$V_r^+ - V_b^+ = e_r(V_b^- - V_r^-)$ (2)

From (2), $V_b^+ = V_r^+ - e_r V_b^- + e_r V_r^-$

$= V_r^+ - e_r e_b\sqrt{2gh} - e_r\sqrt{2gh}$

$= V_r^+ - e_r\sqrt{2gh}(1+e_b)$

Plug into (1), $m_b e_b\sqrt{2gh} - m_r\sqrt{2gh} = m_r V_r^+ + m_b(V_r^+ - e_r\sqrt{2gh}(1+e_b))$

OR $\sqrt{2gh}(m_b e_b - m_r) = m_r V_r^+ + m_b V_r^+ - m_b e_r\sqrt{2gh}(1+e_b)$

$= V_r^+(m_r + m_b) - m_b e_r\sqrt{2gh}(1+e_b)$

$\therefore$ $V_r^+(m_r+m_b) = \sqrt{2gh}\left[m_b e_b - m_r + m_b e_r(1+e_b)\right]$

OR $V_r^+ = \dfrac{\sqrt{2gh}\left[m_b e_b - m_r + m_b e_r(1+e_b)\right]}{m_r + m_b}$

next

Conservation of energy:

$$\cancel{m_r} g h_r = \frac{1}{2}\cancel{m_r}(v_r^+)^2 \quad \therefore \quad h_r = \frac{1}{2g}(v_r^+)^2$$

$$h_r = \frac{1}{\cancel{2g}}(\cancel{2g}h)\left[\frac{m_b e_b - m_r + m_b e_r(1+e_b)}{m_r + m_b}\right]^2$$

$$\therefore \quad \boxed{h_r = h\left[\frac{m_b e_b - m_r + m_b e_r(1+e_b)}{m_b + m_r}\right]^2}$$

b) To maximize h_r, we can begin by recognizing that letting $e_b = e_r = 1$ maximizes the numerator of the bracketed expression.

$$\therefore \quad h_r = h\left(\frac{m_b - m_r + 2m_b}{m_b + m_r}\right)^2 = h\left(\frac{3m_b - m_r}{m_b + m_r}\right)^2$$

This is maximized by increasing m_b and decreasing m_r.

$\therefore$ We want $e_b = e_r = 1$ and the largest ratio of m_b to m_r possible.

$$h_{r,max} = h\left(\frac{3m_b - 0}{m_b + 0}\right)^2 = 9h$$

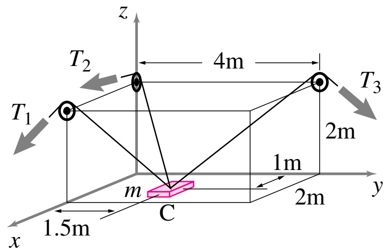
$$\boxed{\text{Theoretical max } h_r = 9h}$$

10.1.22 An object C of mass 2 kg is pulled by three strings as shown. The acceleration of the object at the position shown is $\boldsymbol{a} = \left(-0.6\hat{\imath} - 0.2\hat{\jmath} + 2.0\hat{\boldsymbol{k}}\right)\ \mathrm{m/s^2}$.

a) Draw a free body diagram of the mass.

b) Write the equation of linear momentum balance for the mass. Use $\boldsymbol{\lambda}$'s as unit vectors along the strings.

c) Find the three tensions T_1, T_2, and T_3 at the instant shown. You may find these tensions by using hand algebra with the scalar equations, using a computer with the matrix equation, or by using a cross product on the vector equation.



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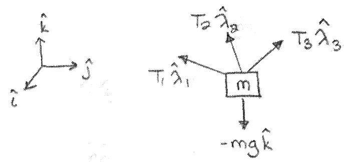
Problem 10.22

10.22

$\vec{a} = -0.6\hat{\imath} - 0.2\hat{\jmath} + 2.0\hat{k}$ [m/s²] , m = 2 kg

a) Free body diagram:



$\vec{r}_m = 1.0\hat{\imath} + 1.5\hat{\jmath}$ [m]

$\vec{r}_1 = 2.0\hat{\imath} + 2.0\hat{k}$ [m]

$\vec{r}_2 = 2.0\hat{k}$ [m]

$\vec{r}_3 = 4.0\hat{\jmath} + 2.0\hat{k}$ [m]

b) $\vec{r}_{m1} = \vec{r}_1 - \vec{r}_m = 1.0\hat{\imath} - 1.5\hat{\jmath} + 2.0\hat{k}$ $\quad |\vec{r}_{m1}| = \frac{1}{2}\sqrt{29}$

$\vec{r}_{m2} = \vec{r}_2 - \vec{r}_m = -1.0\hat{\imath} - 1.5\hat{\jmath} + 2.0\hat{k}$ $\quad |\vec{r}_{m2}| = \frac{1}{2}\sqrt{29}$

$\vec{r}_{m3} = \vec{r}_3 - \vec{r}_m = -1.0\hat{\imath} + 2.5\hat{\jmath} + 2.0\hat{k}$ $\quad |\vec{r}_{m3}| = \frac{3}{2}\sqrt{5}$

$\sum \vec{F} = m\vec{a} = T_1\hat{\lambda}_1 + T_2\hat{\lambda}_2 + T_3\hat{\lambda}_3 - mg\hat{k}$

OR $2\,\mathrm{kg}(-0.6\hat{\imath} - 0.2\hat{\jmath} + 2.0\hat{k}) = T_1\left(\frac{2\sqrt{29}}{29}\right)(\hat{\imath} - 1.5\hat{\jmath} + 2.0\hat{k})$

$+ T_2\left(\frac{2\sqrt{29}}{29}\right)(-\hat{\imath} - 1.5\hat{\jmath} + 2.0\hat{k}) + T_3\left(\frac{2\sqrt{5}}{15}\right)(-\hat{\imath} + 2.5\hat{\jmath} + 2.0\hat{k}) - 19.62\hat{k}$

In component form:

$\hat{\imath}\,(-1.2) = \hat{\imath}\,(0.3714T_1 - 0.3714T_2 - 0.2981T_3)$

$\hat{\jmath}\,(-0.4) = \hat{\jmath}\,(-0.5571T_1 - 0.5571T_2 + 0.7454T_3)$

$\hat{k}\,(4 + 19.62) = \hat{k}\,(0.7428T_1 + 0.7428T_2 + 0.5963T_3)$

CONTINUED ON PAGE 4

10.22 continued.

In matrix form, we have:

$$\begin{bmatrix} -1.2 \\ -0.4 \\ 23.62 \end{bmatrix} = \begin{bmatrix} 0.3714 & -0.3714 & -0.2981 \\ -0.5571 & -0.5571 & 0.7454 \\ 0.7428 & 0.7428 & 0.5963 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}$$

Solving in Matlab yields:

$$\boxed{\begin{aligned} T_1 &= 14.28\ \mathrm{N} \\ T_2 &= 5.86\ \mathrm{N} \\ T_3 &= 14.52\ \mathrm{N} \end{aligned}}$$

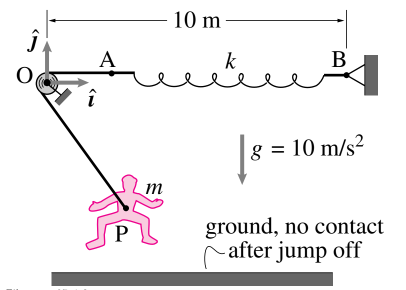
Alternately, see Matlab code (modified from problem 10.17) on previous page.

10.1.26 Bungy Jumping. In a relatively safe bungy jumping system, people jump up from the ground while being pulled up by a rope that runs over a pulley at O and is connected to a stretched spring anchored at B. The ideal pulley has negligible size, mass, and friction. For the situation shown the spring AB has rest length $\ell_0 = 2\,\text{m}$ and a stiffness of $k = 200\,\text{N/m}$. The inextensible massless rope from A to P has length $\ell_r = 8\,\text{m}$, the person has a mass of 100 kg. Take O to be the origin of an xy coordinate system aligned with the unit vectors $\hat{\imath}$ and $\hat{\jmath}$

a) Assume you are given the position of the person $\vec{r} = x\hat{\imath} + y\hat{\jmath}$ and the velocity of the person $\boldsymbol{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$. Find her acceleration in terms of some or all of her position, her velocity, and the other parameters given. Then use the numbers given, where supplied, in your final answer.

b) Given that bungy jumper's initial position and velocity are $\vec{r}_0 = 1\,\text{m}\hat{\imath} - 5\,\text{m}\hat{\jmath}$ and $\boldsymbol{v}_0 = \boldsymbol{0}$ write computer commands to find her position at $t = \pi/\sqrt{2}\,\text{s}$.

c) Find the answer to part (b) with pencil and paper (that is, find an analytic solution to the differential equations, a final numerical answer is desired).



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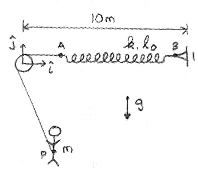
Problem 10.26: Conceptual setup for a bungy jumping system.

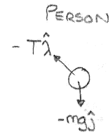
10.26



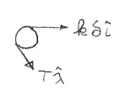
Given: $k = 200$ N/m
$\ell_0 = 2$ m
$\ell_{AP} = 8\text{m} = \ell_r$
$m = 100$ kg
Assume no air drag.

a) $\vec{r} = x\hat{\imath} + y\hat{\jmath}$, $\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$, Find $\vec{a}$

PERSON: (1)



PULLEY: (2)



$$\hat{\lambda} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{\imath} + y\hat{\jmath}}{\sqrt{x^2+y^2}}$$

δ = extension of spring = $\ell - \ell_0$, where $\ell = 10 - (\ell_r - \sqrt{x^2+y^2})$

$\therefore \ell = 10 - 8 + \sqrt{x^2+y^2} = 2 + \sqrt{x^2+y^2}$

$\therefore \delta = \ell - \ell_0 = \sqrt{x^2+y^2}$

From FBD 2: $T = k\delta$ (tension on both sides of pulley must be equal) $\therefore T = k\sqrt{x^2+y^2}$

From FBD 1: $\sum \vec{F} = m\vec{a} = -T\hat{\lambda} - mg\hat{\jmath}$

$$\therefore -k\sqrt{x^2+y^2}\,\frac{x\hat{\imath} + y\hat{\jmath}}{\sqrt{x^2+y^2}} - mg\hat{\jmath} = m\vec{a}$$

OR $-kx\hat{\imath} - (ky + mg)\hat{\jmath} = m\vec{a}$ - plug in values

$$\vec{a} = \frac{-200\,\text{N/m}\times}{100\,\text{kg}}\hat{\imath} - \frac{200\,\text{N/m}\,y + 100\,\text{kg}\,(10\,\text{m/s}^2))}{100\,\text{kg}}\hat{\jmath} = \boxed{-2x\,[\text{s}^{-2}]\,\hat{\imath} - (2y + 10\,\text{m})\,[\text{s}^{-2}]\,\hat{\jmath}}$$

Problem 10.26 (b).

```
function Prob1026()
% Problem 10.26 Solution
% March 11, 2008

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  VARIABLES (Assume consistent units)
%    r  = displacement (vector with x and y components)
%    v  = dr/dt

% INTIAL CONDITIONS
r0= [1 -5]'; % initial position
v0= [0 0]'; % initial velocity
z0= [r0;v0]; % pack variables

tspan =[0 pi/sqrt(2)]; %time interval of integration

[t zarray] = ode45(@rhs,tspan, z0);

% Unpack Variables
r= zarray(:,1:2);

disp(r(end,:));

% ANSWER:
% ans =
%
%    -1.0000    -5.0000   (meters)
%

end

% THE DIFFERENTIAL EQUATION 'The Right Hand Side'
function zdot = rhs(t,z)

%Unpack variables
r= z(1:2);
v= z(3:4);

%The equations
rdot= v;
vdot= [-2*r(1) -2*r(2)-10]';

% Pack the rate of change of r and v
zdot= [rdot; vdot];

end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

b) See Matlab code on previous page

c) From part (a),

$$\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} = -2x\hat{\imath} - (2y+10)\hat{\jmath}$$

$$\therefore \quad (\ddot{x} = -2x)\hat{\imath}\cdot\hat{\imath} \Rightarrow \ddot{x} = -2x \quad (1)$$

$$(\ddot{y} = -2y - 10)\hat{\jmath}\cdot\hat{\jmath} \Rightarrow \ddot{y} = -2y - 10 \quad (2)$$

Solve (1) and (2) with $\vec{r}(0) = \hat{\imath} - 5\hat{\jmath}$, $\vec{v}(0) = \vec{0}$

(1) $\ddot{x} = -2x$, so $x(t) = A\sin(\sqrt{2}t) + B\cos(\sqrt{2}t)$

$\dot{x}(t) = \sqrt{2}A\cos(\sqrt{2}t) - \sqrt{2}B\sin(\sqrt{2}t)$

$\dot{x}(0) = 0 = \sqrt{2}A \quad \therefore A = 0$

$x(0) = 1 = B\cos(0) \quad \therefore B = 1$

$\therefore \quad x(t) = \cos(\sqrt{2}t)$

(2) $\ddot{y} = -2y - 10$, so $y(t) = C\sin(\sqrt{2}t) + D\cos(\sqrt{2}t) \underbrace{- 5}_{y_P}$

$\dot{y}(0) = 0 = \sqrt{2}C \quad \therefore C = 0$

$y(0) = -5 = D\cos(0) - 5 \quad \therefore D = 0$

$\therefore \quad y(t) = -5$

$$\vec{r}(t) = \cos\sqrt{2}t\,\hat{\imath} - 5\hat{\jmath} = x(t)\hat{\imath} + y(t)\hat{\jmath}$$

$$\therefore \quad \vec{r}\left(\tfrac{\pi}{\sqrt{2}}\right) = \cos\left(\sqrt{2}\tfrac{\pi}{\sqrt{2}}\right)\hat{\imath} - 5\hat{\jmath} = \boxed{-\hat{\imath} - 5\hat{\jmath}\ [\text{m}]}$$

10.1.30 The equations of motion from problem **??** are nonlinear and cannot be solved in closed form for the position of the baseball. Instead, solve the equations numerically. Make a computer simulation of the flight of the baseball, as follows.

a) Convert the equation of motion into a system of first order differential equations.

b) Pick values for the gravitational constant g, the coefficient of resistance b, and initial speed v_0, solve for the x and y coordinates of the ball and make a plots its trajectory for various initial angles θ_0.

c) Use Euler's, Runge-Kutta, or other suitable method to numerically integrate the system of equations.

d) Use your simulation to find the initial angle that maximizes the distance of travel for ball, with and without air resistance.

e) If the air resistance is very high, what is a qualitative description for the curve described by the path of the ball? Show this with an accurate plot of the trajectory. (Make sure to integrate long enough for the ball to get back to the ground.)

10.30

a) $v^2 = \dot{x}^2 + \dot{y}^2$ ← speed of ball

$\vec{F}_d = -bv^2\hat{e}_t$ ← taking into account air resistance

$$m\vec{a} = \underbrace{-bv^2\hat{e}_t}_{\text{air resistance}} \underbrace{-mg\hat{j}}_{\text{gravity}}$$

$$m\ddot{x} = -b(\dot{x}^2+\dot{y}^2)\cdot(\hat{e}_t\cdot\hat{\imath})$$

$$m\ddot{y} = -b(\dot{x}^2+\dot{y}^2)\cdot(\hat{e}_t\cdot\hat{\jmath}) - mg$$

$$\boxed{\begin{aligned}\ddot{x} &= -\tfrac{b}{m}(\dot{x}^2+\dot{y}^2)\cos\theta \\ \ddot{y} &= -g - \tfrac{b}{m}(\dot{x}^2+\dot{y}^2)\sin\theta\end{aligned}}$$

$$\theta = \tan^{-1}\frac{dy}{dx} = \tan^{-1}\frac{dy/dt}{dx/dt} = \tan^{-1}\frac{\dot{x}}{\dot{y}}$$

$$\cos\theta = \frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}} \qquad \sin\theta = \frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}}$$

$$\ddot{x} = -\frac{b}{m}\dot{x}\sqrt{\dot{x}^2+\dot{y}^2}$$

$$\ddot{y} = -\frac{b}{m}\dot{y}\sqrt{\dot{x}^2+\dot{y}^2} - g$$

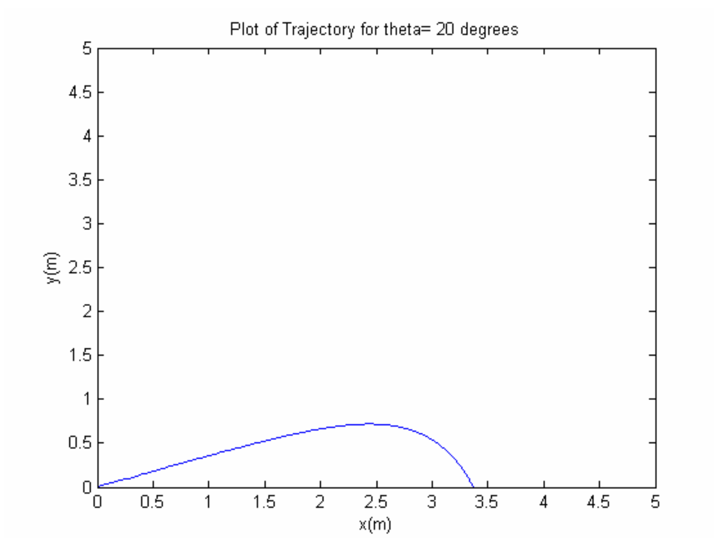
$$\boxed{\begin{aligned}&\text{for } v_x = \dot{x},\quad v_y = \dot{y} \\ &\dot{v}_x = -\tfrac{b}{m}v_x\sqrt{v_x^2+v_y^2} \\ &\dot{v}_y = -\tfrac{b}{m}v_y\sqrt{v_x^2+v_y^2} - g\end{aligned}}$$

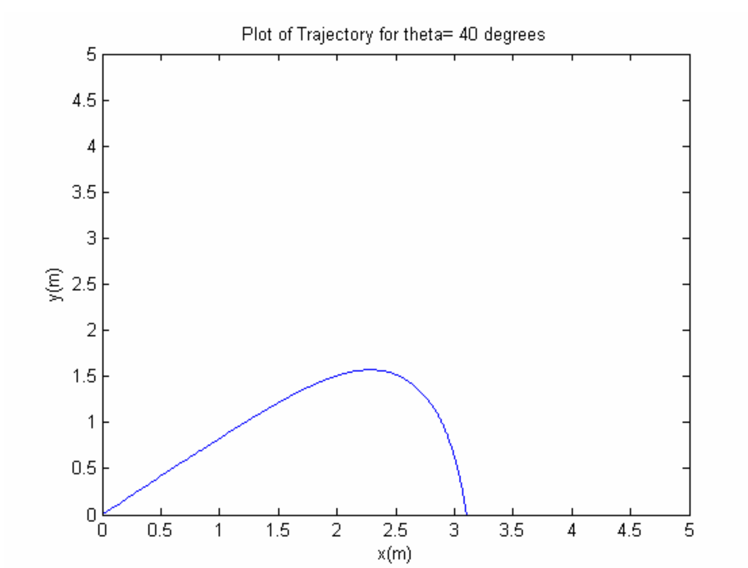
10.30 (continued)

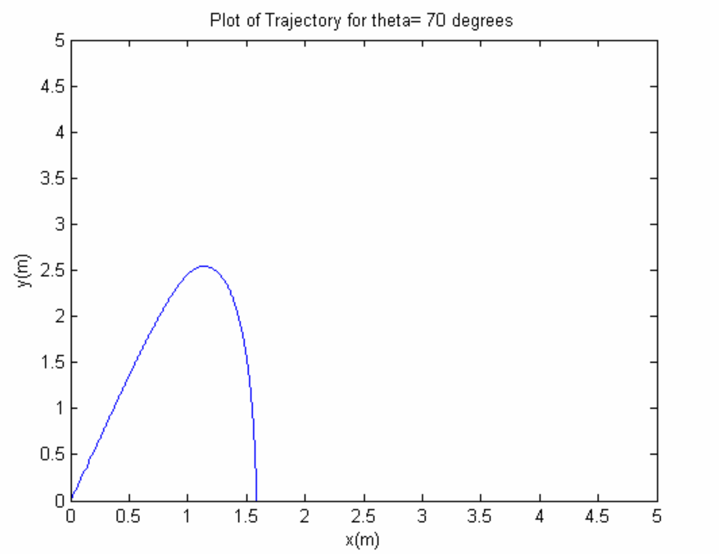
b). See attached codes and results

```
%problem 10.30(a)

function solution1030a
%solution to 10.30
%September 23,2008
b=1; m=1; g=10; % give values for b,m and g here
%Initial conditions and time span
tspan=[0:0.001:5]; %integrate for 50 seconds
x0=0;
y0=0;           %initial position
v0=50;       %magnitude of initial velocity (m/s)
theta0=20;    %angle of initial velocity (in degrees)

z0=[x0,y0,v0*cos(theta0*pi/180),v0*sin(theta0*pi/180)]';

%solves the ODEs
[t,z] = ode45(@rhs,tspan,z0,[],b,m,g);

%Unpack the variables
x= z(:,1);
y =z(:,2);
v_x = z(:,3);
v_y=z(:,4);

%plot the results
plot(x,y);
xlabel('x(m)');
ylabel('y(m)');
%set grid,xmin,xmax,ymin,ymax
axis([0,5,0,5]);
title(['Plot of Trajectory for theta= ',num2str(theta0),' degrees']);

end

%-----------------------------------------------------------------------%
function zdot = rhs(t,z,b,m,g)                    %function to define ODE
x=z(1); y=z(2); v_x=z(3); v_y=z(4);

%the linear momentum balance eqns
xdot=v_x;
v_xdot=-(b/m)*v_x*(v_x^2+v_y^2)^0.5;
ydot=v_y;
```

```
v_ydot=-g-(b/m)*v_y*(v_x^2+v_y^2)^0.5;

zdot=[xdot; ydot; v_xdot; v_ydot]; %this is what the function returns (column vector)

end
%-----------------------------------------------------------------------%
```







c). Disregard this question. This question intends to ask you develop your own ode solver similar to ode45, using Euler's method or more sophisticated method (Ruger-Kutta method).

d). To find out x distance, we use 'stopevent' to terminate the integration at y=0. Then loop over for theta from 0.1 to 89.1 degree with an increment of 1 degree.

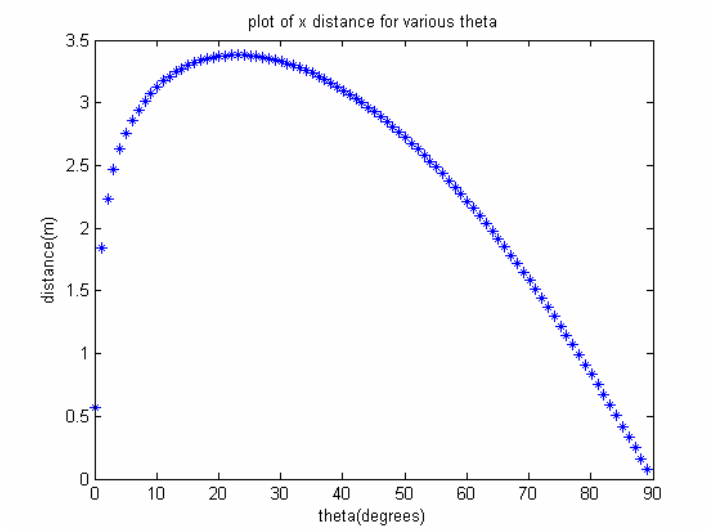
```
%problem 10.30(d)

function solution1030d
%solution to 10.30
%September 23,2008

b=1; m=1; g=10; % give values for b,m and g here

%Initial conditions and time span
tspan=[0 50]; %integrate for 50 seconds
x0=0;
y0=0;          %initial position
v0=50;         %magnitude of initial velocity (m/s)

theta0=[0.1:1:89.1]';    %angle of initial velocity (in degrees)
distance=zeros(size(theta0));    %arrays to record x distance at y=0 for each angle

for i=1:length(theta0)
```

```
z0=[x0,y0,v0*cos(theta0(i)*pi/180),v0*sin(theta0(i)*pi/180)]';

options=odeset('events', @stopevent);
%solves the ODEs
[t,z] = ode45(@rhs,tspan,z0,options,b,m,g);

%Unpack the variables
x= z(:,1);
distance(i)=x(end);% the last component of x is the distance we want
end
plot(theta0,distance,'*')
xlabel('theta(degrees)');
ylabel('distance(m)');
%set grid,xmin,xmax,ymin,ymax
title(['plot of x distance for various theta']);

[maxd,j]=max(distance);
fprintf(1,'\nThe maximum distance is %6.4f m when theta=%2.0f degrees\n', maxd,theta0(j));
%print the results
end

%----------------------------------------------------------------------%
function zdot = rhs(t,z,b,m,g)                    %function to define ODE
x=z(1); y=z(2); v_x=z(3); v_y=z(4);

%the linear momentum balance eqns
xdot=v_x;
v_xdot=-(b/m)*v_x*(v_x^2+v_y^2)^0.5;
ydot=v_y;
v_ydot=-g-(b/m)*v_y*(v_x^2+v_y^2)^0.5;

zdot=[xdot; ydot; v_xdot; v_ydot]; %this is what the function returns (column vector)

end
%----------------------------------------------------------------------%
function [value, isterminal, dir]= stopevent(t,z,b,m,g,v0,theta)
% terminate the integration at y=0
x=z(1);
y=z(2);
value= y;
isterminal=1;
dir=-1;
end
```

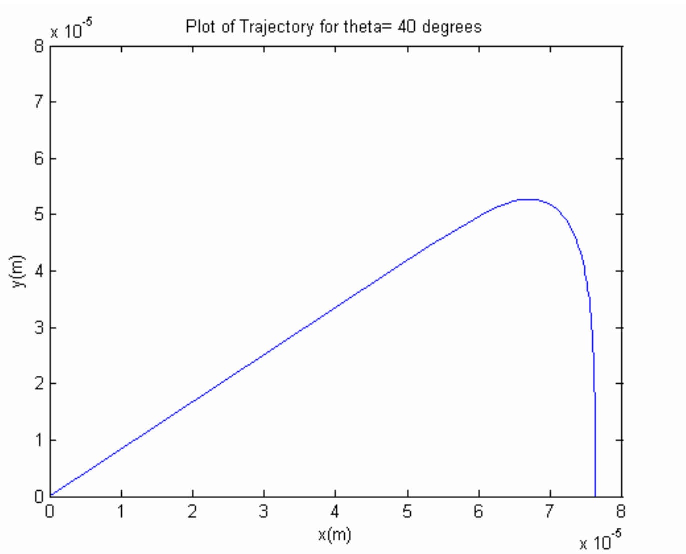
Matlab out put: The maximum distance is 3.3806 m when theta=23 degrees

10.30 (Continued)

The x distance at y=0 for various theta is plotted below



e). Use the code for (a) and change b to a very large number, 100000. The trajectory looks like



,

which is approximately a triangle.

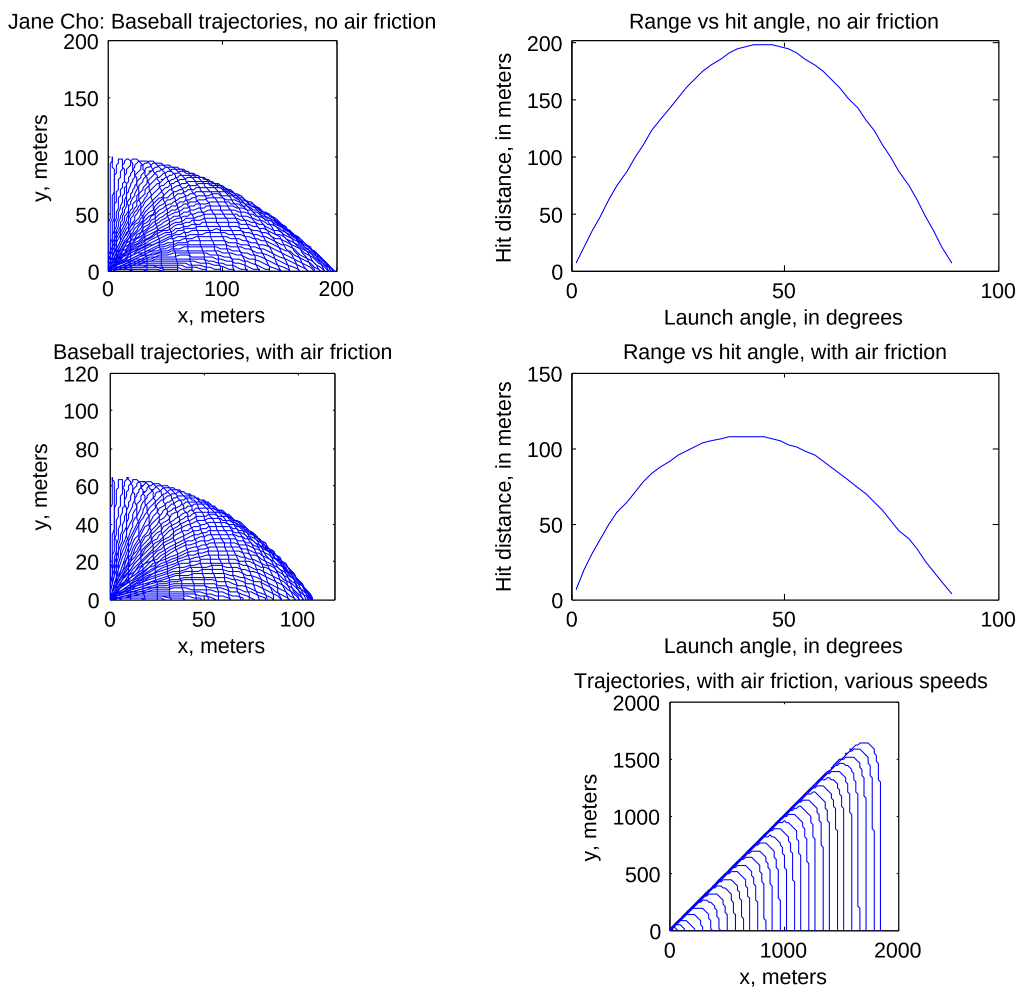
```
10.30 Another solution (more detailed)

The m file attached does the following.

a) uses events and  x(end) to calculate range.
b) has that embedded in a loop so that there is an angle(i) and
   a range(i)
c) Makes a nice plot of range vs angle
d) uses MAX to find the maximum range and corresponding angle
e) has good numerics to show that the trajectory shape converges to
   a triangle as the speed -> infinity.
```

```
function baseball_trajectory
% Calculates the trajectory of a baseball.
% Calculates maximum range for given speed,
% with and without air friction.
% Shows shape of path at high speed.
disp(['Start time:  ' datestr(now)])
cla

% (a) ODEs are in the function rhs far below.
%     The 'event' fn that stops the integration
%     when the ball hits the ground is in 'eventfn'
%     even further below.
% (b) Coefficients for a real baseball taken
% from a google search, which finds a paper
% Sawicki et al, Am. J. Phys. 71(11), Nov 2003.
% Greg Sawicki, by the way, learned some dynamics
% in TAM 203 from Ruina at Cornell.

% All parameters in MKS.
m   = 0.145;    % mass of baseball, 5.1 oz
rho = 1.23;     % density of air in kg/m^3
r   = 0.0366;   % baseball radius (1.44 in)
A   = pi*r^2;   % cross sectional area of ball
C_d = 0.35;     % varies, this is typical
g   = 9.81;     % typical g on earth
b   = C_d*rho*A/2; % net coeff of v^2 in drag force

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% (b-d)  Use typical homerun hit speed and look
% at various  angles of hit.

tspan=linspace(0,100,1001); % give plenty of time
n = 45;  % number of simulations
angle = linspace(1,89,n);  %  launch from 1 to 89 degrees
r0=[0 0]';   % Launch x and y position.

% First case:  No air friction.
b = 0;
subplot(3,2,1)
hold off

% Try lots of launch angles, one simulation for
% each launch angle.
for i = 1:n
inspeed = 44;  % typical homerun hit (m/s), 98 mph.

theta0 = angle(i)*pi/180; % initial angle this simulation
v0=inspeed*[cos(theta0) sin(theta0)]'; %launch velocity
z0=[r0; v0]; % initial position and velocity
```

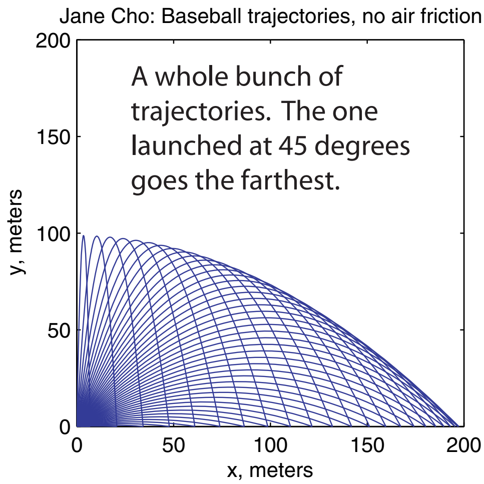
```
options=odeset('events',@eventfn);
[t zarray]=ode45(@rhs,tspan,z0,options,g,b,m); %Solve ODE

x=zarray(:,1); y=zarray(:,2); %Unpack positions
range(i)= x(end); % x value at end,  when ball hits ground

plot(x,y); title('Jane Cho: Baseball trajectories, no air friction')
xlabel('x, meters'); ylabel('y, meters'); axis('equal')
axis([0 200 0 200])
hold on  % save plot for over-writing
end % end of for loop for no-friction trajectories

%Plot range vs angle, no friction case
subplot(3,2,2); hold off;
plot(angle,range);
title('Range vs hit angle, no air friction')
xlabel('Launch angle, in degrees')
ylabel('Hit distance, in meters')

% Pick out best angle and distance
[bestx besti] = max(range);
disp(['No friction case:'])
best_theta_deg = angle(besti)
bestx


% Second case:  WITH air friction
% Identical to code above but now b is NOT zero.
b   = C_d*rho*A/2; % net coeff of v^2 in drag force

subplot(3,2,3)
hold off % clear plot overwrites

% Try lots of launch angles
for i = 1:n  %
inspeed = 44;  % typical homerun hit (m/s), 98 mph.

theta0 = angle(i)*pi/180; % initial angle this simulation
v0=inspeed*[cos(theta0) sin(theta0)]'; %launch velocity
z0=[r0; v0]; % initial position and velocity

options=odeset('events',@eventfn);
[t zarray]=ode45(@rhs,tspan,z0,options,g,b,m); %Solve ODE

x=zarray(:,1); y=zarray(:,2); %Unpack positions
range(i)= x(end); % x value at end,  when ball hits ground

plot(x,y); title('Baseball trajectories, with air friction')
xlabel('x, meters'); ylabel('y, meters'); axis('equal')
axis([0 120 0 120])
```

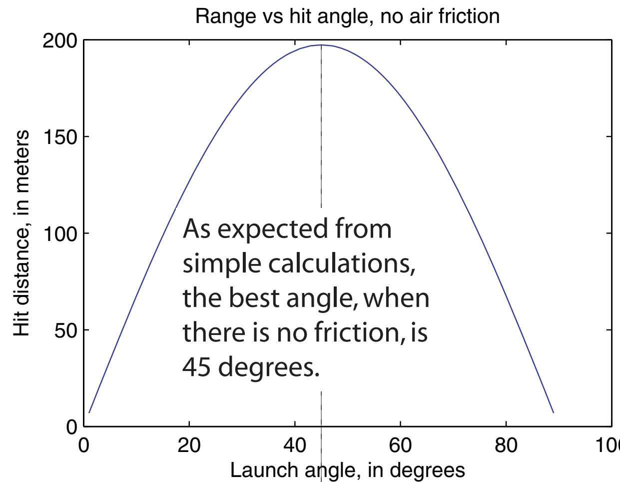
```
hold on  % save plot for over-writing
end % end of for loop for with-friction trajectories

%Plot range vs angle, no friction case
subplot(3,2,4);
plot(angle,range);
title('Range vs hit angle, with air friction')
xlabel('Launch angle, in degrees')
ylabel('Hit distance, in meters')

%Find Max range and corresponding launch angle
[bestx besti] = max(range);
disp(['With Friction:'])
best_theta_deg = angle(besti)
bestx


%%%%%%%%%%%%%%%%%%%%%%%%%%
% Now look at trajectories at a variety of speeds
% Try lots of launch angles
subplot(3,2,6)
hold off
speeds = 10.^linspace(1,8,30); % speeds from 1 to 100 million m/s
for i = 1:30  %
inspeed = speeds(i);  % typical homerun hit (m/s), 98 mph.

theta0 = pi/4; % initial angle is 45 degrees at all speeds
v0=inspeed*[cos(theta0) sin(theta0)]'; %launch velocity
z0=[r0; v0]; % initial position and velocity

options=odeset('events',@eventfn);
[t zarray]=ode45(@rhs,tspan,z0,options,g,b,m); %Solve ODE

x=zarray(:,1); y=zarray(:,2); %Unpack positions
range(i)= x(end); % x value at end,  when ball hits ground

plot(x,y); title('Trajectories, with air friction, various speeds ')
xlabel('x, meters'); ylabel('y, meters'); axis('equal')
axis([0 2000 0 2000])
hold on  % save plot for over-writing
end % end of for loop for range at various speeds

disp(['End time:  ' datestr(now)])
end % end of Baseball_trajectory.m



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Governing Ord Diff Eqs.
```

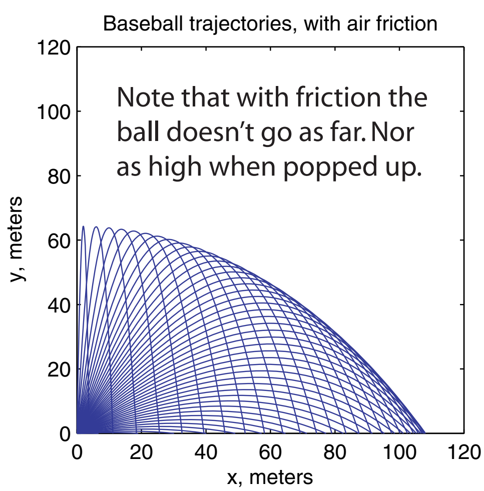
```
function zdot=rhs(t,z,g,b,m)
% Unpack the variables
x=z(1); y=z(2);
vx=z(3); vy=z(4);

%The ODEs
xdot=vx; ydot=vy; v = sqrt(vx^2+vy^2);
vxdot=-b*vx*v/m;
vydot=-b*vy*v/m - g;

zdot= [xdot;ydot;vxdot;vydot]; % Packed up again.
end



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 'Event' that ball hits the ground
function [value isterminal dir] = eventfn(t,z,g,b,m)
y=z(2);
value = y;      % When this is zero, integration stops
isterminal = 1; % 1 means stop.
dir= -1;        % -1 means ball is falling when it hits
end
```

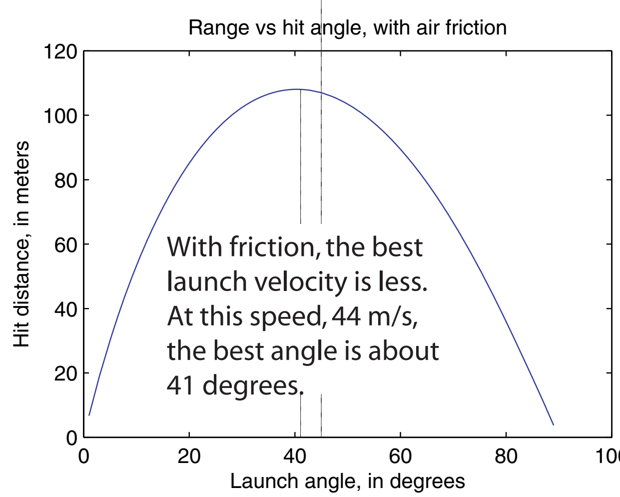

Jane Cho: Baseball trajectories, no air friction
y, meters
x, meters
Range vs hit angle, no air friction
Hit distance, in meters
Launch angle, in degrees
Baseball trajectories, with air friction
y, meters
x, meters
Range vs hit angle, with air friction
Hit distance, in meters
Launch angle, in degrees
Trajectories, with air friction, various speeds
y, meters
x, meters

Baseball. For the first 4 plots realistic ball properties are used and the launch speed is always 44 m/s (typical home run hit). Spin is ignored.

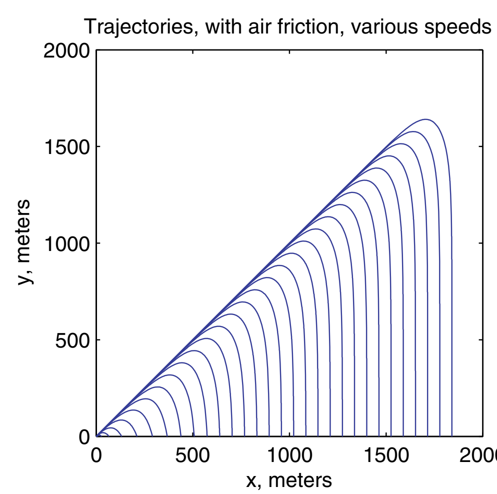








At right are a bunch of trajectories. The slowest launch is 10 m/s, the fastest is 100,000,000 m/s. Such a ball would burn up, tear apart etc... but ignore that.



Note that as the speed gets large the trajectory gets closer and closer to, its a strange and beautiful shape, to a triangle. The same would happen if the speed were fixed and the drag progressively increased.

With no friction the range increases with the square of the speed. With quadratic drag, at high speeds the range goes up with the log of the launch speed. Like the penetration distance of a bullet.

10.2.22 At a time of interest, a particle with mass $m_1 = 5\,\text{kg}$ has position, velocity, and acceleration $\vec{r}_1 = 3\,\text{m}\hat{\imath}$, $\vec{v}_1 = -4\,\text{m/s}\hat{\jmath}$, and $\vec{a}_1 = 6\,\text{m/s}^2\hat{\jmath}$, respectively. Another particle with mass $m_2 = 5\,\text{kg}$ has position, velocity, and acceleration $\vec{r}_2 = -6\,\text{m}\hat{\imath}$, $\vec{v}_2 = 5\,\text{m/s}\hat{\jmath}$, and $\vec{a}_2 = -4\,\text{m/s}^2\hat{\jmath}$, respectively. For this system of two particles, and at this time, find its

a) linear momentum $\vec{L}$,

b) rate of change of linear momentum $\dot{\vec{L}}$

c) angular momentum about the origin $\vec{H}_{/\text{O}}$,

d) rate of change of angular momentum about the origin $\dot{\vec{H}}_{/\text{O}}$,

e) kinetic energy E_K, and

f) rate of change of kinetic energy $\dot{E}_\text{K}$.

10.55

At a particular instant, two particles of interest has the mass, position, velocity and accelerative below

$m_1 = 5\,kg$ $\quad$ $m_2 = 5\,kg$

$\vec{r}_1 = 3m\hat{i}$ $\quad$ $\vec{r}_2 = -6m\hat{i}$

$\vec{v}_1 = -4m/s\,\hat{j}$ $\quad$ $\vec{v}_2 = 5\,m/s\,\hat{j}$

$\vec{a}_1 = 6\,m/s^2\,\hat{j}$ $\quad$ $\vec{a}_2 = -4\,m/s^2\,\hat{j}$

a) Find linear momentum $\vec{L}$

$\vec{L} = m\vec{v}$

$\vec{L}_1 = m_1\vec{v}_1 = \boxed{-20\,kgm/s\,\hat{j} = \vec{L}_1}$

$\vec{L}_2 = m_2\vec{v}_2 = \boxed{25\,kg\,m/s\,\hat{j} = \vec{L}_2}$

$\vec{L}_{system} = (-20+25)\,kgm/s\,\hat{j} = \boxed{5\,kgm/s\,\hat{j}}$

b) Find $\dot{\vec{L}}$

$\dot{\vec{L}} = \vec{F} = m\vec{a}$

$\dot{\vec{L}}_1 = m_1\vec{a}_1 = \boxed{30\,N\hat{j} = \dot{\vec{L}}_1}$

$\dot{\vec{L}}_2 = m_2\vec{a}_2 = \boxed{-20N\hat{j} = \dot{\vec{L}}_2}$

$\dot{\vec{L}}_{system} = (30-20)\,N\hat{j} = \boxed{10\,N\hat{j}}$

c) Find $H_{/o}$

$\vec{H}_{/o} = \vec{r}_{/o} \times (m\vec{v})$

$\vec{H}_1 = \vec{r}_1 \times (m_1\vec{v}_1) = \boxed{-60\,kgm^2/s\,\hat{k}}$

$\vec{H}_2 = \vec{r}_2 \times (m_2\vec{v}_2) = \boxed{-150\,kgm^2/s\,\hat{k}}$

$\vec{H}_{system} = (-60-150)\,kgm^2/s\,\hat{k} = \boxed{-210\,kgm^2/s\,\hat{k}}$

10.55 continued

d) Find $\dot{\vec{H}}_{/0}$

$\dot{\vec{H}} = \vec{r}_{/0} \times (m\vec{a})$

$\dot{\vec{H}}_1 = \vec{r}_1 \times (m_1 \vec{a}_1) = \boxed{90\,Nm\,\hat{k}}$

$\dot{\vec{H}}_2 = \vec{r}_2 \times (m_2 \vec{a}_2) = \boxed{120\,Nm\,\hat{k}}$

$\dot{\vec{H}}_{system} = (90 + 120)\,Nm\,\hat{k} = \boxed{210\,Nm\,\hat{k}}$

e) $E_K = \frac{1}{2} m v^2$

$E_{K1} = \frac{1}{2} m_1 |\vec{v}_1|^2$

$= \frac{1}{2}(5\,kg)(4\,m/s)^2$

$\boxed{E_{K1} = 40\,J}$

$E_{K2} = \frac{1}{2} m_2 |\vec{v}_2|^2$

$= \frac{1}{2}(5\,kg)(5\,m/s)^2$

$\boxed{E_{K2} = 62.5\,J}$

$\boxed{E_{K\,system} = 102.5\,J}$

f) Find $\dot{E}_K$

$\dot{E}_K = \vec{F} \cdot \vec{v}$

$\dot{E}_{K1} = (m_1 \vec{a}_1) \cdot \vec{v}_1$

$= (30N\hat{j}) \cdot (-4\,m/s\,\hat{j}) = \boxed{-120\,W}$

$\dot{E}_{K2} = (m_2 \vec{a}_2) \cdot \vec{v}_2$

$= (-20N\hat{j}) \cdot (5\,m/s\,\hat{j}) = \boxed{-100\,W}$

$\dot{E}_{K\,system} = -220\,W$

Experts note that these problems do not use polar coordinates or any other fancy coordinate systems. Such descriptions come later in the text. At this point we want to lay out the basic equations and the qualitative features that can be found by numerical integration of the equations using Cartesian (xyz) coordinates.

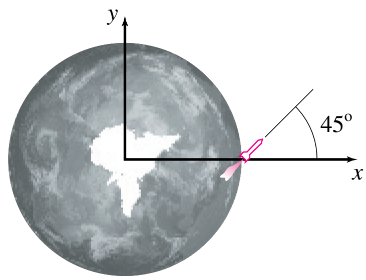
10.3.5 An intercontinental misile, modelled as a particle, is launched on a ballistic trajectory from the surface of the earth. The force on the missile from the earth's gravity is $F = mgR^2/r^2$ and is directed towards the center of the earth. When it is launched from the equator it has speed v_0 and in the direction shown, 45° from horizontal (both measured relative to a Newtonian reference frame). For the purposes of this calculation ignore the earth's rotation. You can think of this problem as two-dimensional in the plane shown. If you need numbers, use the following values:

$m = 1000\,\text{kg}$ = missile mass
$g = 10\,\text{m/s}^2$ at the earth's surface,
$R = 6,400,000\,\text{m}$ = earth's radius, and
$v_0 = 9000\,\text{m/s}$.

The distance of the missile from the center of the earth is $r(t)$.

a) Draw a free body diagram of the missile. Write the linear momentum balance equation. Break this equation into x and y components. Rewrite these equations as a system of-4 first order ODE's suitable for computer solution. Write appropriate initial conditions for the ODE's.

b) Using the computer (or any other means) plot the trajectory of the rocket after it is launched for a time of 6670 seconds. [Hint: use a much shorter time when debugging your program.] On the same plot draw a (round) circle for the earth.



Filename:pfigure-s94q12p1

Problem 10.5: In intercontinental ballistic missile launch.

TAM 203
Homework Solutions
Due 3/27/08

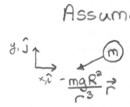
10.61

Given: $F = mgR^2/r^2$, V_0, 45° from horizontal (θ)

$m = 1000$ kg, $R = 6.4\times10^6$ m, $V_0 = 9000$ m/s

a) Free body diagram:

Assume $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$



$\sum \vec{F} = m\vec{a}$, $\quad -\frac{mgR^2}{r^3}\vec{r} = m\vec{a}$

$$-\frac{\cancel{m}gR^2}{(x^2+y^2)^{3/2}}(x\hat{\imath}+y\hat{\jmath}) = \cancel{m}\ddot{x}\hat{\imath} + \cancel{m}\ddot{y}\hat{\jmath}$$

In component form,

$$\ddot{x} + \frac{gR^2}{(x^2+y^2)^{3/2}}x = 0 \quad \text{AND} \quad \ddot{y} + \frac{gR^2}{(x^2+y^2)^{3/2}}y = 0$$

Differential equations:

(1) $\dot{x} = V_x$ (3) $\dot{y} = V_y$

(2) $\dot{V}_x = \frac{-gR^2}{(x^2+y^2)^{3/2}}x$ (4) $\dot{V}_y = \frac{-gR^2}{(x^2+y^2)^{3/2}}y$

Initial conditions:

(1) $x(0) = R$ (3) $y(0) = 0$

(2) $V_x(0) = V_0\cos\theta$ (4) $V_y(0) = V_0\sin\theta$

b) See Matlab code and plot on pages 2-3.

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10.61b – Matlab code

```
function Prob1061()
% Problem 10.61 Solution
% March 27, 2008

% VARIABLES (Assume consistent units)
% r = displacement vector [x,y]
% v = velocity vector = dr/dt [vx,vy]

m= 1000;      % Mass of satellite (kg)
R= 6400000;   % Radius of Earth (m)
g= 9.81;      % Gravity acceleration (m/s^2)
v0= 9000;     % Initial velocity (m/s)
theta= 45;    % Launch angle (degrees)

% INITIAL CONDITIONS
x0= R;
y0= 0;
vx0= v0*cosd(theta);
vy0= v0*sind(theta);
z0= [x0 y0 vx0 vy0]'; % pack variables

tspan= [0 6670]; % seconds

[t zarray]= ode45(@rhs,tspan,z0,[],m,R,g);

% Unpack Variables
x= zarray(:,1);
y= zarray(:,2);

plot(x,y,'r--');
  title('Plot of Earth and Satellite Orbit')
  xlabel('x [m]')
  ylabel('y [m]')
  axis(1000000*[-8 15 -8 15])
hold on;

% Draw the Earth
t= 0:pi/100:2*pi;
ex= R*cos(t);
ey= R*sin(t);
plot(ex,ey,'b');

end

% THE DIFFERENTIAL EQUATION 'The Right Hand Side'
function zdot = rhs(t,z,m,R,g)

% Unpack variables
x= z(1);
y= z(2);
vx= z(3);
vy= z(4);
```

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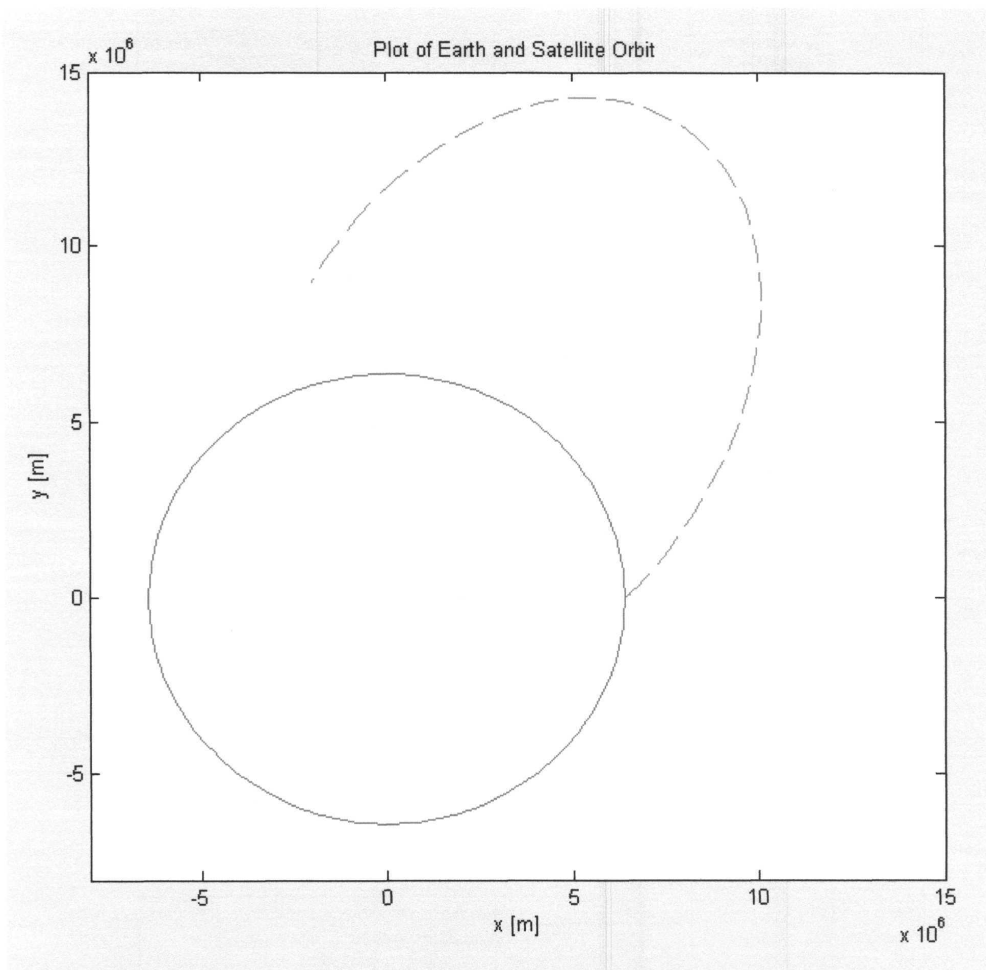
```
% The equations
xdot= vx;
vxdot= -g*R^2/(x^2+y^2)^(3/2)*x;
ydot= vy;
vydot= -g*R^2/(x^2+y^2)^(3/2)*y;

% Pack the rate of change of x,y,vx and vy
zdot= [xdot ydot vxdot vydot]';

end
```

10.61b – Satellite Orbit Plot



11.1.10 Montgomery's eight. Three equal masses, say $m = 1$, are attracted by an inverse-square gravity law with $G = 1$. That is, each mass is attracted to the other by $F = Gm_1 m_2/r^2$ where r is the distance between them. Use these unusual and special initial positions:

$$\begin{aligned}(x1, y1) &= (-0.97000436, 0.24308753)\\ (x2, y2) &= (-x1, -y1)\\ (x3, y3) &= (0, 0)\end{aligned}$$

and initial velocities

$$\begin{aligned}(vx3, vy3) &= (0.93240737, 0.86473146)\\ (vx1, vy1) &= -(vx3, vy3)/2\\ (vx2, vy2) &= -(vx3, vy3)/2.\end{aligned}$$

For each of the problems below show accurate computer plots and explain any curiosities.

a) Use computer integration to find and plot the motions of the particles. Plot each with a different color. Run the program for 2.1 time units.

b) Same as above, but run for 10 time units.

c) Same as above, but change the initial conditions slightly.

d) Same as above, but change the initial conditions more and run for a much longer time.

11.10

a) See attached Matlab code and plots for (a) - (d), recognizing that:

$$m\ddot{a}_1 = \frac{Gm^2}{|\vec{r}_2-\vec{r}_1|^3}(\vec{r}_2-\vec{r}_1) + \frac{Gm^2}{|\vec{r}_3-\vec{r}_1|^3}(\vec{r}_3-\vec{r}_1)$$

$$\therefore \quad \ddot{\vec{a}}_1 = Gm\left(\frac{\vec{r}_2-\vec{r}_1}{|\vec{r}_2-\vec{r}_1|^3} + \frac{\vec{r}_3-\vec{r}_1}{|\vec{r}_3-\vec{r}_1|^3}\right)$$

We can turn this, and perform similar linear momentum balance on (2) and (3), into six first-order vector differential equations:

$$\begin{aligned}
\dot{\vec{r}}_1 &= \vec{V}_1 & \dot{\vec{V}}_1 &= Gm\left(\frac{\vec{r}_2-\vec{r}_1}{|\vec{r}_2-\vec{r}_1|^3} + \frac{\vec{r}_3-\vec{r}_1}{|\vec{r}_3-\vec{r}_1|^3}\right)\\
\dot{\vec{r}}_2 &= \vec{V}_2 & \dot{\vec{V}}_2 &= Gm\left(\frac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|^3} + \frac{\vec{r}_3-\vec{r}_2}{|\vec{r}_3-\vec{r}_2|^3}\right)\\
\dot{\vec{r}}_3 &= \vec{V}_3 & \dot{\vec{V}}_3 &= Gm\left(\frac{\vec{r}_1-\vec{r}_3}{|\vec{r}_1-\vec{r}_3|^3} + \frac{\vec{r}_2-\vec{r}_3}{|\vec{r}_2-\vec{r}_3|^3}\right)
\end{aligned}$$

From plots on subsequent pages, we can see that these initial conditions provide for a very specific displacement function for each mass. If these conditions are modified slightly, as in (c) and (d), the displacement plot is very different.

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```
function Prob1110()
% Problem 11.10 Solution
% April 1, 2008

% VARIABLES
G= 1;
m= 1;

% Initial Conditions
r01= [-0.97000436 0.24308753]'; r02= -r01; r03= [0 0]';
v03= [0.93240737 0.86473146]'; v01= -1/2*v03; v02= -1/2*v03;

z0= [r01; r02; r03; v01; v02; v03]; % pack variables

tspan= [0 10];

[t zarray]= ode45(@rhs,tspan,z0,[],G,m);

% Unpack variables
r1= zarray(:,1:2);
r2= zarray(:,3:4);
r3= zarray(:,5:6);

plot(r1(:,1), r1(:,2), 'r');
hold on;
plot(r2(:,1), r2(:,2),'b--');
plot(r3(:,1), r3(:,2),'g-.');

end

% THE DIFFERENTIAL EQUATIONS (RIGHT HAND SIDE)
function zdot = rhs(t,z,G,m)

% Unpack variables
r1= z(1:2);
r2= z(3:4);
r3= z(5:6);
v1= z(7:8);
v2= z(9:10);
v3= z(11:12);

% The equations
r1dot= v1; r2dot= v2; r3dot= v3;
v1dot= G*m*((r3-r1)/(sqrt(sum((r3-r1).^2)))^3+...
    (r2-r1)/(sqrt(sum((r2-r1).^2)))^3);
v2dot= G*m*((r1-r2)/(sqrt(sum((r1-r2).^2)))^3+...
    (r3-r2)/(sqrt(sum((r3-r2).^2)))^3);
v3dot= G*m*((r1-r3)/(sqrt(sum((r1-r3).^2)))^3+...
    (r2-r3)/(sqrt(sum((r2-r3).^2)))^3);

% Pack the rate of change variables
zdot= [r1dot; r2dot; r3dot; v1dot; v2dot; v3dot];

end
```

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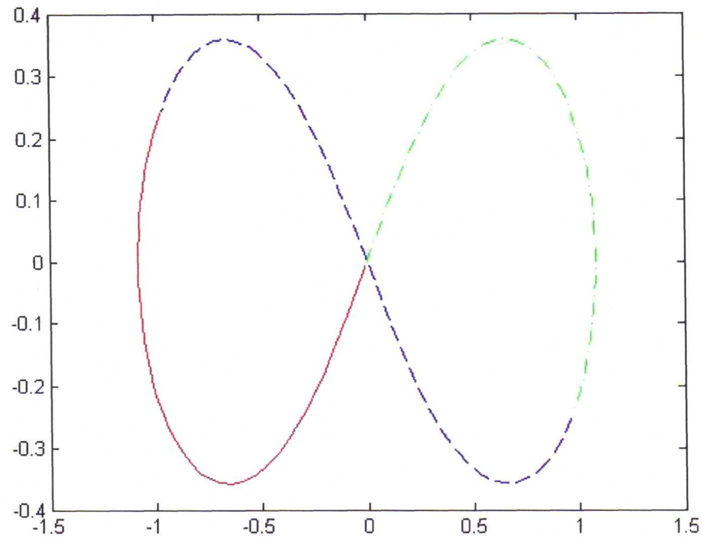
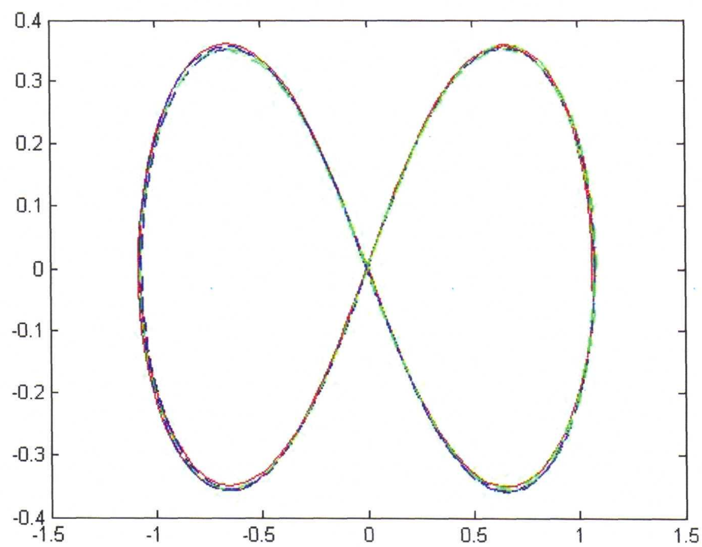

0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-1.5
-1
-0.5
0
0.5
1
1.5

(a)


0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-1.5
-1
-0.5
0
0.5
1
1.5

(b)

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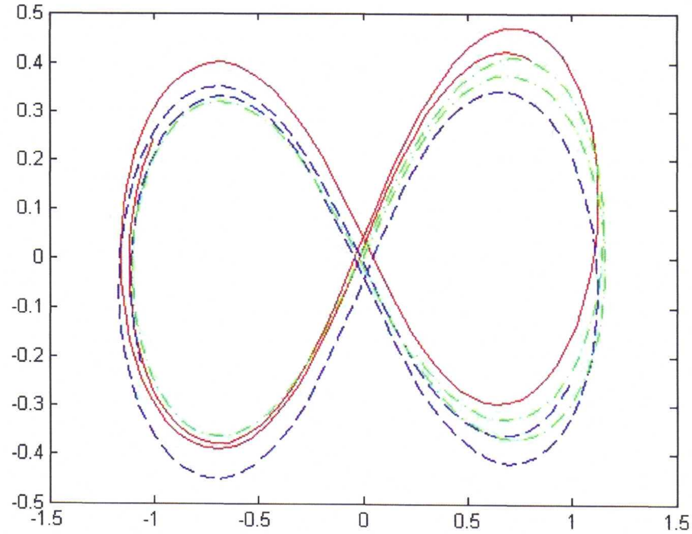
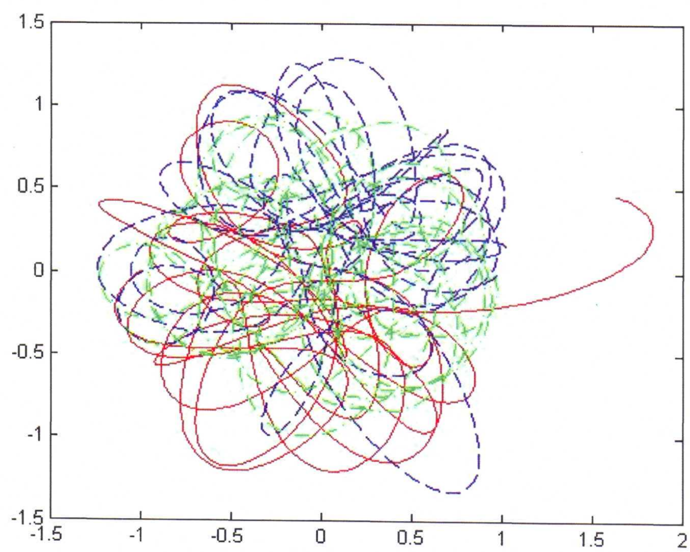
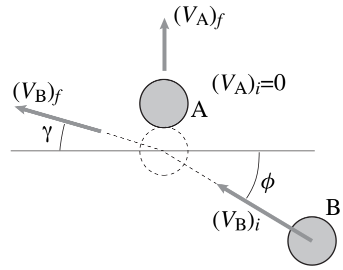

0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-1.5
-1
-0.5
0
0.5
1
1.5

(c)


1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2

(d)

11.2.7 Two frictionless equal-mass pucks sliding on a plane collide as shown below. Puck A is initially at rest. Given that $(V_B)_i = 1.0\,\mathrm{m/s}$, $(V_A)_i = 0$, and $(V_A)_f = 0.5\,\mathrm{m/s}$, find the approach angle ϕ and rebound angle γ. The coefficient of restitution is $e = 0.9$.



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Problem 11.7

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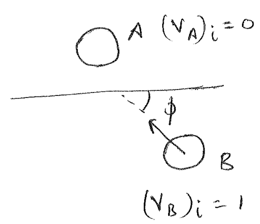
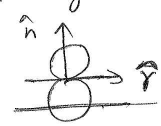
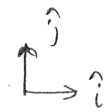
Before Collision

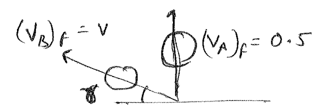


During collision





After Collision



Given: $(V_A)_i = 0$; $(V_B)_i = 1$; $(V_A)_f = 0.5$; $e = 0.9$

Find ϕ, γ

Let $(V_B)_f = V$

$$(\vec{V}_A)_i = 0$$

$$(\vec{V}_B)_i = -(V_B)_i \cos\phi\,\hat{\imath} + (V_B)_i \sin\phi\,\hat{\jmath}$$

$$= -\cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath}$$

$$(\vec{V}_A)_f = 0.5\,\hat{\jmath}$$

$$(\vec{V}_B)_f = -(V_B)_f \cos\gamma\,\hat{\imath} + (V_B)_f \sin\gamma\,\hat{\jmath}$$

$$= -V\cos\gamma\,\hat{\imath} + V\sin\gamma\,\hat{\jmath}$$

Conservation of linear momentum gives

$$m_A(\vec{V}_A)_i + m_B(\vec{V}_B)_i = m_A(\vec{V}_A)_f + m_B(\vec{V}_B)_f$$

Assuming masses are equal $m_A = m_B = m$

$$\Rightarrow \quad m(0) + m(-\cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath}) = m\,0.5\,\hat{\jmath} + m(-v\cos\gamma\,\hat{\imath} + v\sin\gamma\,\hat{\jmath})$$

$$\Rightarrow \quad -\cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath} = (0.5 + v\sin\gamma)\,\hat{\jmath} - v\cos\gamma\,\hat{\imath}$$

$$\{\ \}\cdot\hat{\imath} \qquad -\cos\phi = -v\cos\gamma \qquad \text{— (I)}$$

$$\{\ \}\cdot\hat{\jmath} \qquad \sin\phi = 0.5 + v\sin\gamma \qquad \text{— (II)}$$

We have 2 equation (I), (II) & 3 unknowns, namely, v, ϕ, γ.

Let's use the coefficient of restitution to generate the third equation

$$-e(\vec{v}_A - \vec{v}_B)_i\cdot\hat{n} = (\vec{v}_A - \vec{v}_B)_f\cdot\hat{n}$$

where $\hat{n} = \hat{\jmath}$ {see figure: during collision}

$$\Rightarrow \quad -0.9\left(0 - (-\cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath})\right)\cdot\hat{\jmath} = \left(0.5\,\hat{\jmath} - (-v\cos\gamma\,\hat{\imath} + v\sin\gamma\,\hat{\jmath})\right)\cdot\hat{\jmath}$$

$$\Rightarrow \quad 0.9\sin\phi = 0.5 - v\sin\gamma \qquad \text{— (III)}$$

~~Subtracting~~ Adding (II) and (III)

$$1.9\sin\phi = 1$$

$\therefore \sin\phi = \frac{1}{1.9} = 0.53 \Rightarrow \phi = 32^\circ$

Putting $\phi = 32^\circ$ in I & II gives

$$v\cos\gamma = 0.85 \quad - \text{IV}$$

$$v\sin\gamma = 0.03 \quad - \text{V}$$

Dividing V/IV $\tan\gamma = \frac{0.03}{0.85} \Rightarrow \gamma = 2.02^\circ$ or $\gamma = 182.02^\circ$

square and add V, IV

$$v^2 = 0.7235 \Rightarrow v = \pm 0.85$$

Again from IV, V we observe

$v = 0.85$ $\gamma = 2.02$ is one pair

$v = -0.85$ $\gamma = 182.02$ is the other

But really both solutions above are equivalent.

Thus final answer

$$\boxed{\begin{array}{l} \gamma = 2.02^\circ = 0.036 \text{ rad} \\ \phi = 32^\circ = 0.56 \text{ rad} \end{array}}$$

11.2.10 Solve the general two-particle frictionless collision problem. For example, write computer code that has lines like this near the start :

`m1=3; m2=19`	Set values of masses
`v1zero=[10 20]`	Initial velocity of mass 1
`v2zero=[-5 3]`	Initial velocity of mass 2
`e=.5`	Set coefficient of restitution
`theta=pi/4`	Angle that the normal to contact plane makes, measured CCW from +x axis, in radians

Your program (function, code, script) should calculate the impulse of mass 1 on mass 2, and the velocities of the two masses after the collision. Your program should assume consistent units for all quantities.

a) You should demonstrate that your program works by solving at least 4 different problems for which you can check your answer by simple pencil-and-paper calculations. These problems should have as much variety as possible. Sketch these problems clearly, show their analytic solution, and show that the computer agrees.

b) Solve the problem given in the sample text given in the initial problem statement.

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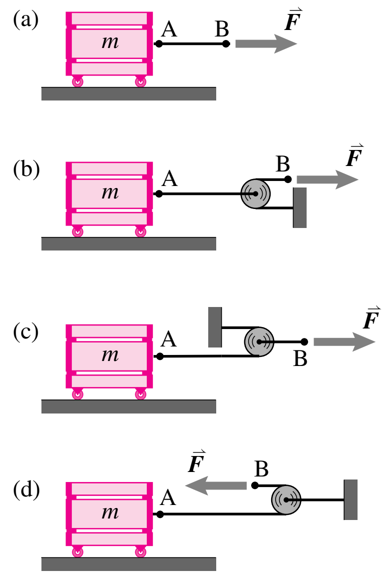
```
% Two-Particle Collisions
% Problem 11.20 Solution
% April 1, 2008

theta = 45;          % angle (degrees) between n and plus x axis
nx = cosd(theta);
ny = sind(theta);
n = [nx ny]';        % impulse direction
v1bef = [10 20]';    % vel of m1 before collision
v2bef = [-5  3]';    % vel of m2 before collision
m1 = 3;  m2 = 19;    % values of two masses
e = .5;              % coefficient of restitution

% Write governing equations in form of Az=b
% where z is a list of unknowns representing
% the particle velocities after the collision
% and the magnitude of the impulse.

A = [ m1    0   m2   0    0        % x comp of lin mom bal
       0   m1    0  m2    0        % y comp of lin mom bal
     -nx  -ny   nx  ny    0        % restitution equation
       0    0   m2   0  -nx        % impulse-momentum for m2, x comp
       0    0    0  m2  -ny];      % impulse-momentum for m2, y comp

b = [m1*v1bef + m2*v2bef;          % x & y comps of lin mom bal for syst
     -e*sum((v2bef-v1bef).*n);     % restitution equation
     m2*v2bef];                    % impulse-momentum for m2, x & y comps

z= A\b;

% Type out the solution (crudely).
disp('   v1xaft     v1yaft    v2xaft     v2yaft      P');
disp(z');

%   ANSWER:
%   v1xaft     v1yaft     v2xaft     v2yaft       P
%  -10.7273   -0.7273   -1.7273    6.2727   87.9384
```

A ball m is thrown horizontally at height h and speed v_0. It then has a sequence of bounces on the horizontal ground. Treating each collision as frictionless with restitution coefficient e how far has the ball travelled horizontally when it just finishes bouncing? Answer in terms of some or all of m, g, h, v_0 and e. A ball m is thrown horizontally at height h and speed v_0. It then has a sequence of bounces on the horizontal ground. Treating each collision as frictionless with restitution coefficient e how far has the ball travelled horizontally when it just finishes bouncing? Answer in terms of some or all of m, g, h, v_0 and e.

For all problems, unless stated otherwise, treat all strings as inextensible, flexible and massless. Treat all pulleys and wheels as round, frictionless and massless. Assume all massive objects are prevented from rotating (e.g., wheels stay on the ground, *etc.*). When numbers are called for use $g = 10\,\mathrm{m/s^2}$ or $g = 32\,\mathrm{ft/s^2}$.

12.1.6 For the various situations pictured, find the acceleration of mass A and point B. Clearly define any variables, coordinates or sign conventions that you use.



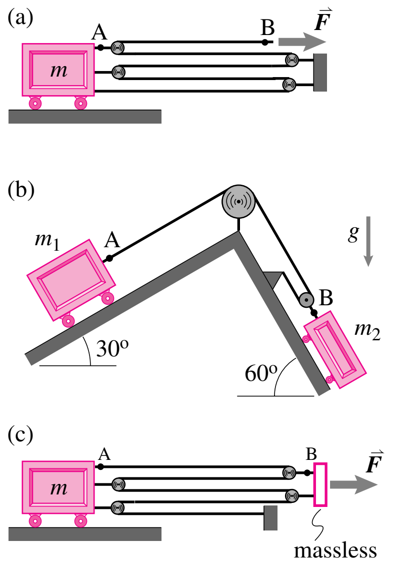
Problem 12.6: Four different ways to pull a mass.

12.1.14 For the situations pictured, find the accelerations of mass A and of point B. Clearly define any variables, coordinates or sign conventions that you use.

a) A single mass and four pulleys.

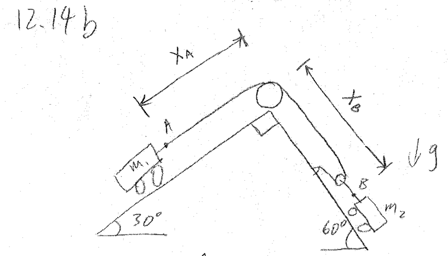
b) Two masses and two pulleys.

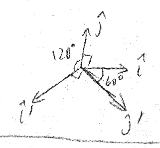
c) A single mass and four pulleys.



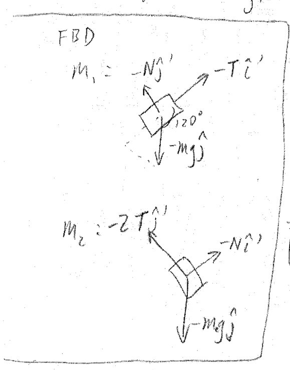
Problem 12.14: Various pulley arrangements.

12.14b





T is the tension force in the cable.



$m_1\ddot{x}_A = -m_1 g\hat{j} - T\hat{i}' - N\hat{j}'$

$\{\}\cdot\hat{i}'$:

$m_1\ddot{x}_A = -m_1 g(\hat{j}\cdot\hat{i}') - T$

$\hat{j}\cdot\hat{i}' = \cos(120) = -\sin(30)$

$\boxed{m_1\ddot{x}_A = m_1 g\sin 30 - T}$ (1)

$m_2\ddot{x}_B = -2T\hat{j}' - N\hat{i}' - m_2 g\hat{j}$

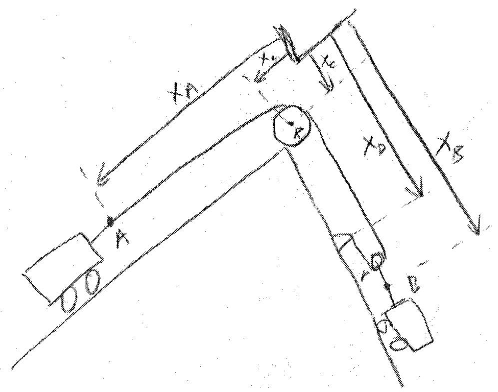
$\{\}\cdot\hat{j}'$:

$m_2\ddot{x}_B = -2T - m_2 g(\hat{j}\cdot\hat{j}')$

$\hat{j}\cdot\hat{j}' = \cos(150) = -\sin 60$

$\boxed{m_2\ddot{x}_B = -2T + m_2 g\sin 60}$ (2)

12.14 b continued



$$l_{tot} = (x_A - x_c) + \frac{\pi R}{2} + (x_B - x_c) + (x_B - x_D) + \pi r$$

$$\ddot{\{\}}: \quad \ddot{x}_A + 2\ddot{x}_B = 0$$

$$\boxed{\ddot{x}_A = -2\ddot{x}_B} \quad (3)$$

We have 3 unknowns : $\ddot{x}_A$, $\ddot{x}_B$, T

(1) $\Rightarrow$ $T = m_1 g \sin 30 - m_1 \ddot{x}_A$

substitute into (2)

$$m_2 \ddot{x}_B = -2(m_1 g \sin 30 - m_1 \ddot{x}_A) + m_2 g \sin 60$$

(3) $\Rightarrow$ $\ddot{x}_A = -2\ddot{x}_B$ substitute into above

$$m_2 \ddot{x}_B = -2 m_1 g \sin 30 - 4 m_1 \ddot{x}_B + m_2 g \sin 60$$

$$(m_2 + 4m_1)\ddot{x}_B = -2 m_1 g \sin 30 + m_2 g \sin 60$$

$$\ddot{x}_B = \frac{-2m_1 g \sin 30^\circ + m_2 g \sin 60^\circ}{m_2 + 4m_1}$$

$$= \frac{-2m_1 + \sqrt{3}m_2}{2m_2 + 8m_1} g$$

$$\ddot{x}_A = -2\ddot{x}_B = \frac{2m_1 - \sqrt{3}m_2}{m_2 + 4m_1} g$$

$\therefore$ Acceleration of point A is

$$\vec{a}_A = \ddot{x}_A \hat{i}' = \frac{2m_1 - \sqrt{3}m_2}{m_2 + 4m_1} g \hat{i}'$$

$$= \frac{2m_1 - \sqrt{3}m_2}{m_2 + 4m_1} g \left(-\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right)$$

Acceleration of point B is

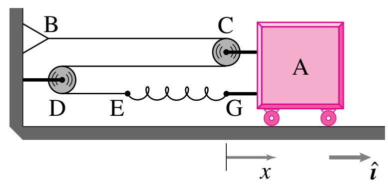
$$\vec{a}_B = \ddot{x}_B \hat{j}' = \frac{\sqrt{3}m_2 - 2m_1}{2m_2 + 8m_1} g \hat{j}'$$

$$= \frac{\sqrt{3}m_2 - 2m_1}{2m_2 + 8m_1} g \left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right)$$

12.1.26 Block A, with mass m_A, is pulled to the right a distance d from the position it would have if the spring were relaxed. It is then released from rest. Assume ideal string, pulleys and wheels. The spring has constant k.

a) What is the acceleration of block A just after it is released (in terms of k, m_A, and d)?

b) What is the speed of the mass when the mass passes through the position where the spring is relaxed? .



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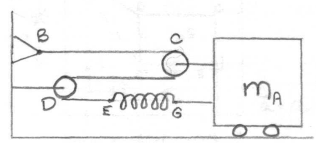
Problem 12.26

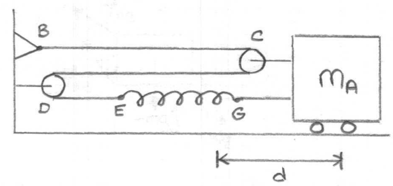
TAM 203 Homework Solution | Due 4/8/08

12.26

ORIGINAL:



PULLED:

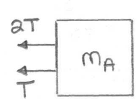


a) Find the stretching of the spring:

We know BC has increased by d, as well as CD and DG. Since the string is inextensible, the spring must have stretched by 3d.

$\therefore$ $F_{spring} = 3kd$, which must equal T_{cable}

FBD:



$\Sigma F = ma$; so $-3T = m_A a$ $T = 3kd$ $\therefore$ $\boxed{a = \frac{-9kd}{m_A}}$

b) We know $\ddot{x} = -\frac{9kx}{m_A}$, so $x(t) = c_1\cos\left(3\sqrt{\frac{k}{m_A}}\,t\right) + c_2\sin\left(3\sqrt{\frac{k}{m_A}}\,t\right)$

$\dot{x}(0) = 0 = -c_1\left(3\sqrt{\frac{k}{m_A}}\right)\sin(0) + c_2\left(3\sqrt{\frac{k}{m_A}}\right)\cos(0)$ $\therefore$ $c_2 = 0$

$x(0) = d = c_1\cos(0)$ $\therefore$ $c_1 = d$

$x(t) = d\cos\left(3\sqrt{\frac{k}{m_A}}\,t\right)$ AND $\dot{x}(t) = -3d\sqrt{\frac{k}{m_A}}\sin\left(3\sqrt{\frac{k}{m_A}}\,t\right)$

$x(t) = 0 = d\cos\left(3\sqrt{\frac{k}{m_A}}\,t\right)$

$\therefore$ $3t\sqrt{\frac{k}{m_A}} = \frac{\pi}{2}$

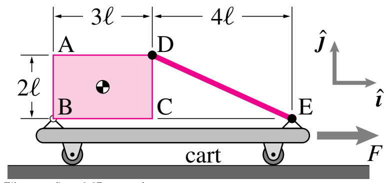
OR $t = \frac{\pi}{6}\sqrt{\frac{m_A}{k}}$

$\dot{x}\left(\frac{\pi}{6}\sqrt{\frac{m_A}{k}}\right) = -3d\sqrt{\frac{k}{m_A}}\sin\left(\frac{\pi}{2}\right)$

$\therefore$ when spring is relaxed,

$\boxed{\dot{x} = -3d\sqrt{k/m_A}}$

12.2.11 Guyed plate on a cart A uniform rectangular plate $ABCD$ of mass m is supported by a rod DE and a hinge joint at point B. The dimensions are as shown. There is gravity. What must the acceleration of the cart be in order for massless rod DE to be in tension?



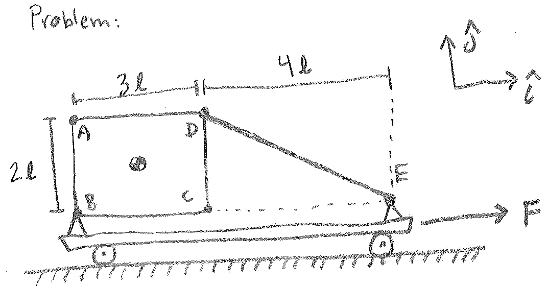
Problem 12.11: Uniform plate supported by a hinge and a cable on an accelerating cart.

TA: Pranav Bhounsule

Alan Argondizza
TAM 2030
HW 10, Due February 24, 2009

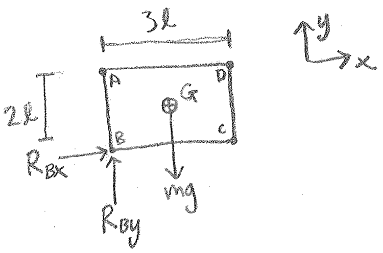
$\boxed{12.40}$ SOLUTION

Problem:



What must the acceleration of the cart be for massless rod DE to be in tension?

Free Body Diagram:



* consider that at some threshold acceleration (as a increases), the rod DE will go from compression to zero-load to tension. Solve the problem where DE carries no load to find the minimum acceleration, past which DE will be in tension. This approach is reflected in the FBD by the absence of forces at point D, and the assumption that the mass is not rotating about the z axis.

LMB:

$\Sigma F_x = ma = R_{Bx}$

$\Sigma F_y = 0 = R_{By} - mg$

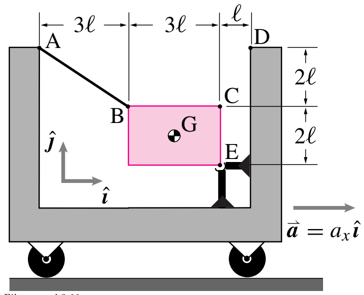
$\Sigma M_G = 0 = \ell \cdot R_{Bx} - 1.5\ell \cdot R_{By}$
(no rotation)

$\rightarrow R_{Bx} = \frac{3}{2} R_{By}$

$ma = \frac{3}{2} mg$

$\rightarrow a = \frac{3}{2} g \longrightarrow$ $\boxed{\text{so } a \text{ must be greater than } \frac{3}{2}g \text{ for DE to be in tension.}}$

12.2.14 A uniform rectangular plate of mass m is supported by an inextensible cable AB and a hinge joint at point E on the cart as shown. The hinge joint is attached to a rigid column welded to the floor of the cart. The cart has acceleration $a_x\hat{\imath}$. There is gravity. Find the tension in cable AB. (What's 'wrong' with this problem? What if instead point B were at the bottom left hand corner of the plate?)



Problem 12.14

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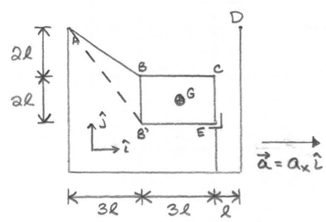
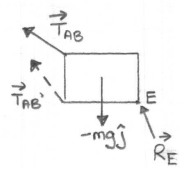
12.43



FBD:



$$\sum \vec{M}_E = \dot{\vec{H}}_E = \vec{r}_{GE} \times (ma_x\hat{\imath})$$

$$\Rightarrow \vec{r}_{GE} \times (-mg\hat{\jmath}) = \vec{r}_{GE} \times (ma_x\hat{\imath}) \qquad (1)$$

The problem with this problem is that the tension acts in a direction through the support E. We cannot determine $\vec{T}_{AB}$ just by summing moments, and unidirectional motion only exists for a particular a_x:

$$\vec{r}_{GE} = -1.5\ell\hat{\imath} + \ell\hat{\jmath}$$

$$\therefore \vec{r}_{GE} \times (-mg\hat{\jmath}) = 1.5mg\ell\hat{k}$$

$$\vec{r}_{GE} \times (ma_x\hat{\imath}) = -ma_x\ell\hat{k}$$

From equation (1), $a_x = -1.5g$ for unidirectional motion. If the cable went from A to B' (dashed above), this problem is avoided:

$$\sum \vec{M}_E = \vec{r}_{B'E} \times (T_{AB'}\hat{\lambda}_{AB'}) + \vec{r}_{GE} \times (-mg\hat{\jmath}) = \vec{r}_{GE} \times (ma_x\hat{\imath})$$

$$\Rightarrow \hat{\lambda}_{AB'} = \frac{3\ell\hat{\imath} - 4\ell\hat{\jmath}}{\sqrt{9\ell^2 + 15\ell^2}} = \frac{3}{5}\hat{\imath} - \frac{4}{5}\hat{\jmath} \quad \text{AND} \quad \vec{r}_{B'E} = 3\ell\hat{\imath}$$

$$\therefore \vec{r}_{B'E} \times (T_{AB'}\hat{\lambda}_{AB'}) = T_{AB'}\left(-\frac{12}{5}\ell\hat{k}\right)$$

$$\text{So, } \left\{-\frac{12}{5}\ell T_{AB'}\hat{k} + 1.5mg\ell\hat{k} = -ma_x\ell\hat{k}\right\} \cdot \hat{k}$$

$$\boxed{T_{AB'} = \frac{5m}{12}\left(a_x + \frac{3}{2}g\right)}$$

12.2.25 Car braking: front brakes versus rear brakes versus all four brakes. What is the peak deceleration of a car when you apply: the front brakes till they skid, the rear brakes till they skid, and all four brakes till they skid? Assume that the coefficient of friction between rubber and road is $\mu = 1$ (about right, the coefficient of friction between rubber and road varies between about .7 and 1.3) and that $g = 10\,\mathrm{m/s}^2$ (2% error). Pick the dimensions and mass of the car, but assume the center of mass height h is greater than zero but is less than half the wheel base w, the distance between the front and rear wheel. Also assume that the CM is halfway between the front and back wheels (*i.e.,* $l_f = l_r = w/2$). The car has a stiff suspension so the car does not move up or down or tip appreciably during braking. Neglect the mass of the rotating wheels in the linear and angular momentum balance equations. Treat this problem as two-dimensional problem; i.e., the car is symmetric left to right, does not turn left or right, and that the left and right wheels carry the same loads. To organize your work, here are some steps to follow.

a) Draw a FBD of the car assuming rear wheel is skidding. The FBD should show the dimensions, the gravity force, what you know *a priori* about the forces on the wheels from the ground (i.e., that the friction force $F_r = \mu N_r$, and that there is no friction at the front wheels), and the coordinate directions. Label points of interest that you will use in your momentum balance equations. (Hint: also draw a free body diagram of the rear wheel.)

b) Write the equation of linear momentum balance.

c) Write the equation of angular momentum balance relative to a point of your choosing. Some particularly useful points to use are:

   - the point above the front wheel and at the height of the center of mass;
   - the point at the height of the center of mass, behind the rear wheel that makes a 45 degree angle line down to the rear wheel ground contact point; and
   - the point on the ground straight under the front wheel that is as far below ground as the wheel base is long.

d) Solve the momentum balance equations for the wheel contact forces and the deceleration of the car. If you have used any or all of the recommendations from part (c) you will have the pleasure of only solving one equation in one unknown at a time.

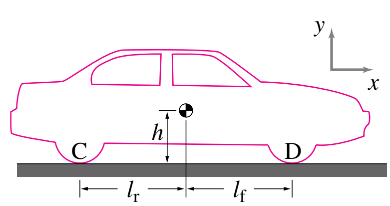
e) Repeat steps (a) to (d) for front-wheel skidding. Note that the advantageous points to use for angular momentum balance are now different. Does a car stop faster or slower or the same by skidding the front instead of the rear wheels? Would your solution to (e) be different if the center of mass of the car were at ground level(h=0)?

f) Repeat steps (a) to (d) for all-wheel skidding. There are some shortcuts here. You determine the car deceleration without ever knowing the wheel reactions (or using angular momentum balance) if you look at the linear momentum balance equations carefully.

g) Does the deceleration in (f) equal the sum of the decelerations in (d) and (e)? Why or why not?

h) What peculiarity occurs in the solution for front-wheel skidding if the wheel base is twice the height of the CM above ground and $\mu = 1$?

i) What impossibility does the solution predict if the wheel base is shorter than twice the CM height? What wrong assumption gives rise to this impossibility? What would really happen if one tried to skid a car this way?



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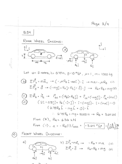
Problem 12.25



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d) From (3), $-\hat{\imath}\times(-mg\hat{\jmath}) + (-2\hat{\imath}+0.75\hat{\jmath})\times(-R_A\hat{\imath}+R_A\hat{\jmath}) = \vec{0}$

$$\{mg\hat{k} - 1.25R_A\hat{k} = 0\hat{k}\}\cdot\hat{k}$$

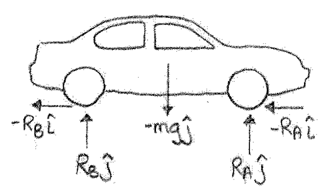
$$\therefore\ R_A = \frac{mg}{1.25} = \frac{10}{1.25} = 8.0\text{ kN}$$

From (2), $R_B = 2.0$ kN

From (1), $a = -R_A/m = \boxed{-8.0\ m/s^2} = \left(\frac{-g}{\omega-h}\right)\cdot\frac{w}{2}$

(f) ALL WHEEL SKIDDING:

a)



b) $\Sigma\vec{F}_x = m\vec{a} \rightarrow -R_B - R_A = ma$ (1)

$\Sigma\vec{F}_y = \vec{0} \rightarrow R_A + R_B = mg$ (2)

Plug (2) into (1): $-mg = ma \quad \therefore\ a = -g = \boxed{-10\ m/s^2}$

(g) No, the acceleration in (f) is not equal to the sum of those found in (d) and (e). The normal forces and friction forces are distributed differently, so there is no reason to believe they would be the same.

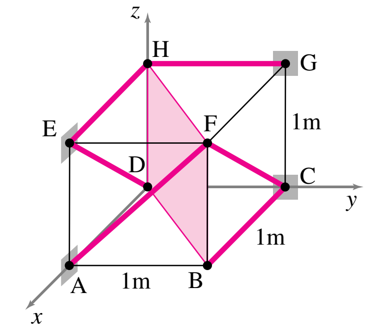
(h) If $\omega = 2h$, with front-wheel skidding, $\vec{r}_{AC} = (-2\hat{\imath}+\hat{\jmath})h$, $\vec{r}_{AC}\times R_A(-\hat{\imath}+\hat{\jmath}) = -R_A\hat{k}$, so $a = -g$

(i) If $\omega < 2h$, with front-wheel skidding, $\vec{r}_{AC} = (-2\hat{\imath}+>1)h$, $\vec{r}_{AC}\times R_A(-\hat{\imath}+\hat{\jmath}) < -R_A\hat{k}$, so $a < -g$ OR $|a| > g$. This is only because we assumed a non-rotating rigid body, which would no longer hold.

12.2.43 The uniform 2 kg plate DBFH is held by six massless rods (AF, CB, CF, GH, ED, and EH) which are hinged at their ends. The support points A, C, G, and E are all accelerating in the x-direction with acceleration $\boldsymbol{a} = 3\,\mathrm{m/s^2}\hat{\imath}$. There is no gravity.

a) What is $\{\sum \vec{F}\}\cdot\hat{\imath}$ for the forces acting on the plate?

b) What is the tension in bar CB?



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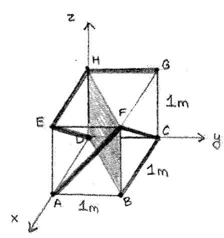
Problem 12.43

TAM 203 Homework Solution | Due 4/10/08

$\boxed{12.72}$



let P be the centroid of plate

Plate DBFH has mass $m = 2$ kg, held by six massless rods:
AF, CB, CF, GH, ED, EH

Points A, C, E + G accelerate with
$\vec{a} = 3\ \mathrm{m/s^2}\ \hat{\imath}$

a) $\{\Sigma \vec{F}\}\cdot\hat{\imath} = m\vec{a}\cdot\hat{\imath} = (2\,\mathrm{kg})(3\,\mathrm{m/s^2}) = \boxed{6\ \mathrm{N}}$

b) To find $\vec{F}_{CB}$, take angular momentum balance about F:

$$\Sigma \vec{M}_{/F} = \dot{\vec{H}}_F = \vec{r}_{P/F}\times m\vec{a} = \tfrac{1}{2}(\hat{\imath}+\hat{\jmath}+\hat{k})\times(6\hat{\imath})$$
$$= 3\left[(\hat{\imath}\times\hat{\imath})+(\hat{\jmath}\times\hat{\imath})+(\hat{k}\times\hat{\imath})\right] = 3(-\hat{\jmath}-\hat{k})$$

$$\therefore\ \Sigma \vec{M}_{/F} = 3\,\mathrm{N\text{-}m}\,(\hat{\jmath}-\hat{k})$$

$$\Sigma \vec{M}_F = \vec{r}_{H/F}\times\vec{T}_{HE} + \vec{r}_{H/F}\times\vec{T}_{HG} + \vec{r}_{D/F}\times\vec{T}_{DE} + \vec{r}_{B/F}\times\vec{T}_{BC}$$

$$= T_{HE}\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 1 & 0\\ 1 & 0 & 0\end{vmatrix} + T_{HG}\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 1 & 0\\ 0 & 1 & 0\end{vmatrix} + \frac{T_{DE}}{\sqrt{3}}\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 1 & 1\\ 1 & 0 & 1\end{vmatrix} + T_{BC}\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 0 & 0 & 1\\ -1 & 0 & 0\end{vmatrix}$$

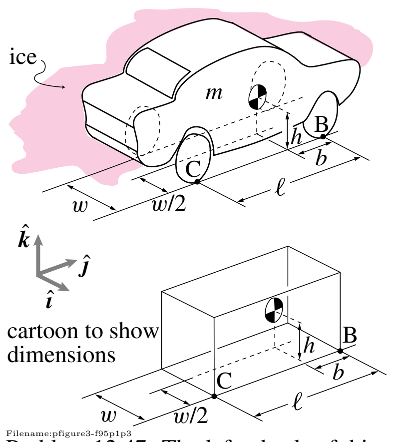
$$= T_{HE}(-\hat{k}) + T_{HG}(\hat{k}) + \frac{T_{DE}}{\sqrt{3}}(\hat{\imath}-\hat{k}) + T_{BC}(-\hat{\jmath})$$

$$\therefore\ \frac{T_{DE}}{\sqrt{3}}\hat{\imath} - T_{BC}\hat{\jmath} + \left(T_{HG}-T_{HE}-\frac{T_{DE}}{\sqrt{3}}\right)\hat{k} = 3\hat{\jmath}-3\hat{k}$$

$\{\}\cdot\hat{\jmath} \longrightarrow\ -T_{BC} = 3 \qquad \therefore\ T_{BC} = -3\mathrm{N}$

OR $\boxed{\vec{T}_{BC} = (-3\mathrm{N})\,\hat{\imath}}$

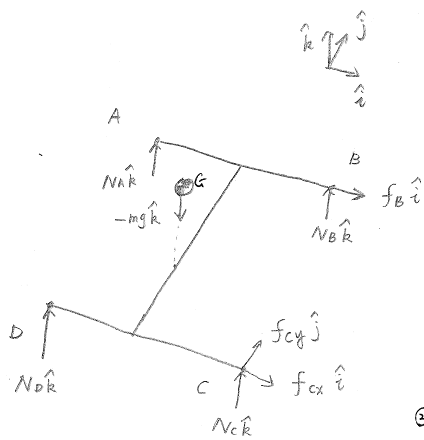
12.2.47 A rear-wheel drive car on level ground. The two left wheels are on perfectly slippery ice. The right wheels are on dry pavement. The negligible-mass front right wheel at B is steered straight ahead and rolls without slip. The right rear wheel at C also rolls without slip and drives the car forward with velocity $\vec{v} = v\hat{\jmath}$ and acceleration $\vec{a} = a\hat{\jmath}$. Dimensions are as shown and the car has mass m . What is the sideways force from the ground on the right front wheel at B? Answer in terms of any or all of m, g, a, b, ℓ, w, and $\hat{\imath}$.



Problem 12.47: The left wheels of this car are on ice.

12.76

FBD



Note:

① Supporting forces $N_i\hat{k}$ on four wheels, but they don't equal.

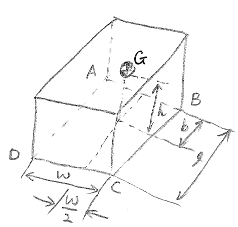
② No friction force on A & D since they are on ice.

③ Side way friction force on B and C.

④ Since C is the driving wheel, there is a driving force (essentially friction force) $f_{cy}\hat{\jmath}$ on C.

$\vec{f} = f_{cx}\hat{\imath} + f_{cy}\hat{\jmath}$ is the friction force acting on C.

Dimensions.



Known: $\vec{V} = v\hat{\jmath}$, $\vec{a} = a\hat{\jmath}$ (Velocity and acceleration)

Want to solve for $f_B\hat{\imath}$.

7 unknowns : f_B, N_B, N_A, N_D, N_c, f_{cx}, f_{cy}

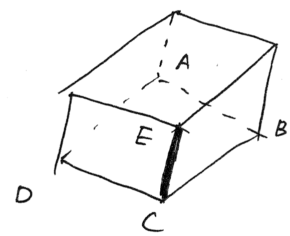
6 equations : LMB (3 equations) , AMB (3 equations)

Can't solve for all forces.

But you can still write down AMB, LMB and manipulate the algebra to get f_B.

However, we can find a short cut for f_B.

Take AMB about vertical line at C: CE



All forces have no moments about CE except $f_B \hat{\imath}$

$\therefore$ AMB about CE $\Rightarrow$

$$(\vec{r}_{B/C} \times f_B \hat{\imath}) \cdot \hat{k} = (\vec{r}_{G/C} \times ma\hat{\jmath}) \cdot \hat{k}$$

$$\Rightarrow (\ell \hat{\jmath} \times f_B \hat{\imath}) \cdot \hat{k} = \left[\left(-\frac{w}{2}\hat{\imath} + (\ell - b)\hat{\jmath} + h\hat{k} \right) \times ma\hat{\jmath} \right] \cdot \hat{k}$$

$$\Rightarrow -f_B \ell = -\frac{maw}{2}$$

$$\therefore \boxed{f_B = \frac{maw}{2\ell}}$$

Sideway force from the ground on B is $\boxed{\frac{maw}{2\ell}\hat{\jmath}}$

Note: if you are famaliar withe moment about a line, you can directly write down

$$\boxed{f_B \ell = \frac{maw}{2}}.$$

13.1.1 A particle goes on a circular path with radius R making the angle $\theta = ct$ measured counter clockwise from the positive x axis. Assume $R = 5\,\text{cm}$ and $c = 2\pi\ \text{s}^{-1}$.

a) Plot the path.

b) What is the angular rate in revolutions per second?

c) Put a dot on the path for the location of the particle at $t = t^* = 1/6$ s.

d) What are the x and y coordinates of the particle position at $t = t^*$? Mark them on your plot.

e) Draw the vectors $\hat{\boldsymbol{e}}_\theta$ and $\hat{\boldsymbol{e}}_R$ at $t = t^*$.

f) What are the x and y components of $\hat{\boldsymbol{e}}_R$ and $\hat{\boldsymbol{e}}_\theta$ at $t = t^*$?

g) What are the R and θ components of $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ at $t = t^*$?

h) Draw an arrow representing both the velocity and the acceleration at $t = t^*$.

i) Find the $\hat{\boldsymbol{e}}_R$ and $\hat{\boldsymbol{e}}_\theta$ components of position $\vec{\boldsymbol{r}}$, velocity $\vec{\boldsymbol{v}}$ and acceleration $\vec{\boldsymbol{a}}$ at $t = t^*$.

j) Find the x and y components of position $\vec{\boldsymbol{r}}$, velocity $\vec{\boldsymbol{v}}$ and acceleration $\vec{\boldsymbol{a}}$ at $t = t^*$. Find the velocity and acceleration two ways:

1. Differentiate the position given as $\vec{\boldsymbol{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$.
2. Differentiate the position give as $\vec{\boldsymbol{r}} = r\hat{\boldsymbol{e}}_r$ and then convert the results to Cartesian coordinates.

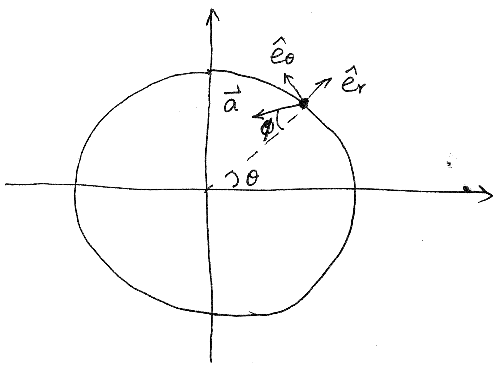
13.1.15 A particle moves in circles so that its acceleration $\vec{a}$ always makes a fixed angle ϕ with the position vector $-\vec{r}$, with $0 \le \phi \le \pi/2$. For example, $\phi = 0$ would be constant rate circular motion. Assume $\phi = \pi/4$, $R = 1$ m and $\dot{\theta}_0 = 1$ rad/ s. How long does it take the particle to reach

a) the speed of sound (≈ 300 m/s)?

b) the speed of light ($\approx 3 \cdot 10^8$ m/s)?

c) ∞?

13.15



Acceleration $\vec{a}$ always makes a fixed angle $\phi = \frac{\pi}{4}$ with $-\hat{e}_r$.

Let $a = |\vec{a}|$

$$\vec{a} = -a\cos\phi\,\hat{e}_r + a\sin\phi\,\hat{e}_\theta$$
$$= -a\cos\tfrac{\pi}{4}\,\hat{e}_r + a\sin\tfrac{\pi}{4}\,\hat{e}_r$$
$$= -\tfrac{\sqrt{2}}{2}a\,\hat{e}_r + \tfrac{\sqrt{2}}{2}a\,\hat{e}_\theta$$
$$= -R\dot{\theta}^2\,\hat{e}_r + R\ddot{\theta}\,\hat{e}_\theta$$

$$\Rightarrow \left.\begin{array}{l} -R\dot{\theta}^2 = -a\frac{\sqrt{2}}{2} \\ R\ddot{\theta} = a\frac{\sqrt{2}}{2} \end{array}\right\} \Rightarrow \cancel{R}\ddot{\theta} = \cancel{R}\dot{\theta}^2$$

$$\boxed{\ddot{\theta} = \dot{\theta}^2} \quad \#$$

Let $\omega = \dot{\theta}$, we have $\boxed{\dot{\omega} = \omega^2}$ (ω : Angular velocity (magnitude))

$$\Rightarrow \frac{d\omega}{dt} = \omega^2 \Rightarrow -\frac{1}{\omega} = t + \underbrace{c}_{\text{constant}}$$

$$\Rightarrow \boxed{\omega = -\frac{1}{t+c}}$$

At $t = 0$, $\dot{\theta}_0 = 1$ rad/s, $\Rightarrow C = -1$ (s)

(Note: in the book, it says $\dot{\phi}_0 = 1$ rad/s, this should be $\dot{\theta}_0 = 1$ rad/s)

$$\therefore \quad \omega = -\frac{1}{t-1} \quad \text{(rad/s)}$$

i) Magnitude of velocity $|\vec{v}| = \omega R = 300$ m/s

$$\Rightarrow -\frac{R}{t-1} = 300 \text{ m/s}$$, since $R = 1$ m, $\boxed{t = 0.99667\ \text{s}}$

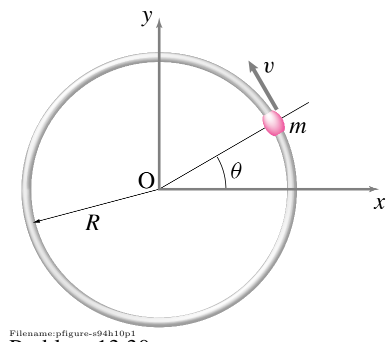
ii) $|\vec{v}| = 3\times10^8$ m/s, similarly to i), we have $-\frac{R}{t-1} = 3\times10^8$ m/s

$$\Rightarrow t = (1 - 0.3333\times10^{-8}) \text{ s}$$

13.2.30 Bead on a hoop with friction. A bead slides on a rigid, stationary, circular wire. The coefficient of friction between the bead and the wire is μ. The bead is loose on the wire (not a tight fit but not so loose that you have to worry about rattling). Assume gravity is negligible.

a) Given v, m, R, & μ; what is $\dot{v}$?

b) If $v(\theta = 0) = v_0$, how does v depend on θ, μ, v_0 and m?

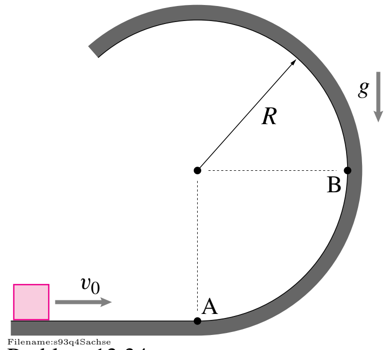


Problem 13.30

13.2.34 A block with mass m is moving to the right at speed v_0 when it reaches a circular frictionless portion of the ramp.

a) What is the speed of the block when it reaches point B? Solve in terms of R, v_0, m and g.

b) What is the force on the block from the ramp just after it gets onto the ramp at point A? Solve in terms of R, v_0, m and g. Remember, force is a vector.



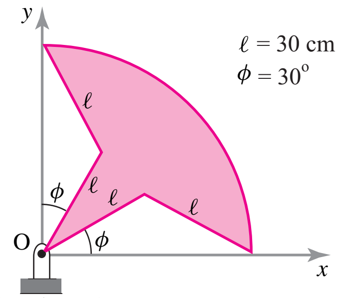
Problem 13.34

13.3.8 Write a computer program to animate the rotation of an object. Your input should be a set of x and y coordinates defining the object (such that `plot y vs x` draws the object on the screen) and the rotation angle θ. The output should be the rotated coordinates of the object.

a) From the geometric information given in the figure, generate coordinates of enough points to define the given object.

b) Using your program, plot the object at $\theta = 20°, 60°, 100°, 160°$, and $270°$.

c) Assume that the object rotates with constant angular speed $\omega = 2\,\text{rad/s}$. Find and plot the position of the object at $t = 1\,\text{s},\ 2\,\text{s},\ \text{and} 3\,\text{s}$.



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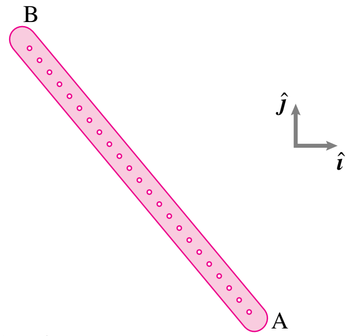
Problem 13.8

13.4.14 A $0.4\,\mathrm{m}$ long rod AB has many holes along its length such that it can be pegged at any of the various locations. It rotates counter-clockwise at a constant angular speed about a peg whose location is not known. At some instant t, the velocity of end B is $\vec{v}_B = -3\,\mathrm{m/s}\,\hat{\jmath}$. After $\frac{\pi}{20}$ s, the velocity of end B is $\vec{v}_B = -3\,\mathrm{m/s}\,\hat{\imath}$. If the rod has not completed one revolution during this period,

a) find the angular velocity of the rod, and

b) find the location of the peg along the length of the rod.



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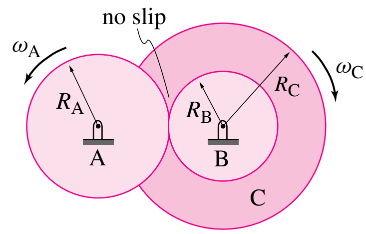
Problem 13.14

13.4.22 2-D constant rate gear train. The angular velocity of the input shaft (driven by a motor not shown) is a constant, $\omega_{\text{input}} = \omega_A$. What is the angular velocity $\omega_{\text{output}} = \omega_C$ of the output shaft and the speed of a point on the outer edge of disc C, in terms of R_A, R_B, R_C, and ω_A?



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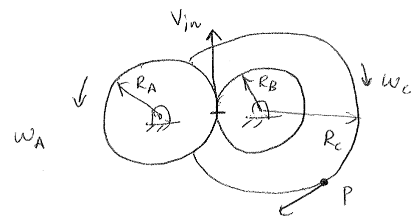
Problem 13.22: Gear B is welded to C and engages with A.

13.81



Given: R_A, R_B, R_C, ω_A

Find ω_c, V_p ?

⇒ For ~~non~~ slip between the gear R_A, R_B

$$V_{in} = \omega_A R_A = \omega_B R_B$$

$$\Rightarrow \omega_B = \omega_A \frac{R_A}{R_B}$$

Also $\omega_c = \omega_B$

Thus $\boxed{\omega_c = \omega_A \cdot \frac{R_A}{R_B}}$

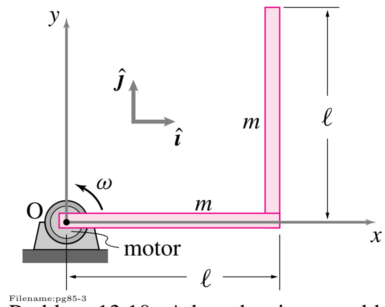
$$\Rightarrow V_p = \omega_c R_c$$

$$\boxed{V_p = \omega_A \frac{R_A R_c}{R_B}}$$

13.6.10 Motor turns a bent bar. Two uniform bars of length ℓ and uniform mass m are welded at right angles. One end is attached to a hinge at O where a motor keeps the structure rotating at a constant rate ω (counterclockwise). What is the net force and moment that the motor and hinge cause on the structure at the instant shown.

a) neglecting gravity

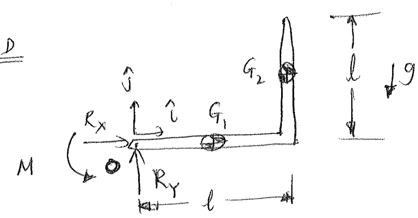
b) including gravity.



Problem 13.10: A bent bar is rotated by a motor.

13.112

FBD



Given : $m, \ell, g, \omega, \alpha = 0$

Find : R_x, R_Y, M

We will solve the general case involving gravity and than reduce to no gravity case by putting gravity $= 0$.

AMB/O $\quad \vec{M}_{/O} = \dot{\vec{H}}_{/O}$

$$\vec{r}_{G_1/O} \times mg(-\hat{j}) + \vec{r}_{G_2/O} \times mg(-\hat{j}) + M\hat{k} = \vec{r}_{G_1/O} \times m\vec{a}_{G_1} + \vec{r}_{G_2/O} \times m\vec{a}_{G_2}$$

$$\therefore \frac{\ell}{2}\hat{\imath} \times mg(-\hat{j}) + (\ell\hat{\imath} + \frac{\ell}{2}\hat{j}) \times mg(-\hat{j}) + M\hat{k} = \vec{0}$$

why ?

{Because $a_{G_1} = -\omega^2 r_{G_1/O}$
$a_{G_2} = -\omega^2 r_{G_2/O}$ }

$\{AMB_{/O}\} \cdot k$

$$-\frac{3}{2} mg\ell + M = 0$$

$$\Rightarrow \quad M = \frac{3}{2} mg\ell \qquad \text{— (I)}$$

LMB

$$R_x\hat{\imath} + R_y\hat{\jmath} - mg\hat{\jmath} - mg\hat{\jmath} = m\vec{a}_{G_1} + m\vec{a}_{G_2}$$

But $\vec{a}_{G_1} = \vec{a}_{G_1/0} = -\omega^2 r_{G_1/0} = -\omega^2\left\{\frac{l}{2}\hat{\imath}\right\}$

$$\vec{a}_{G_2} = \vec{a}_{G_2/0} = -\omega^2 r_{G_2/0} = -\omega^2\left\{l\hat{\imath} + \frac{l}{2}\hat{\jmath}\right\}$$

Thus

$$R_x\hat{\imath} + \{R_y - 2mg\}\hat{\jmath} = m\left\{-\frac{3\omega^2 l}{2}\hat{\imath} - \frac{\omega^2 l}{2}\hat{\jmath}\right\}$$

$\{LMB\}\cdot\hat{\imath}$

$$R_x = -\frac{3m\omega^2 l}{2} \qquad \text{— II}$$

$\{LMB\}\cdot\hat{\jmath}$

$$R_y = 2mg - \frac{m\omega^2 l}{2} \qquad \text{— III}$$

a) Neglect gravity

Put $g = 0$ in I, II, III

$$\boxed{\begin{aligned} M &= 0 \\ R_x &= -\frac{3m\omega^2 l}{2} \\ R_y &= -\frac{m\omega^2 l}{2} \end{aligned}}$$

b) Including gravity

From I, II, III

$$\boxed{\begin{aligned} M &= \frac{3}{2}mgl \\ R_x &= -3m\omega^2 l/2 \\ R_y &= 2mg - \frac{3m\omega^2 l}{2} \end{aligned}}$$

13.6.20 At the input to a gear box a 100 lbf force is applied to gear A. At the output, the machinery (not shown) applies a force of F_B to the output gear. Gear A rotates at constant angular rate $\omega = 2\,\text{rad/s}$, clockwise.

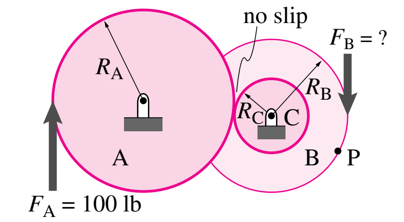
a) What is the angular speed of the right gear?

b) What is the velocity of point P?

c) What is F_B?

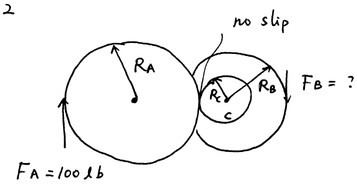
d) If the gear bearings had friction, would F_B have to be larger or smaller in order to achieve the same constant velocity?

e) If instead of applying a 100 lbf to the left gear it is driven by a motor (not shown) at constant angular speed ω, what is the angular speed of the right gear?

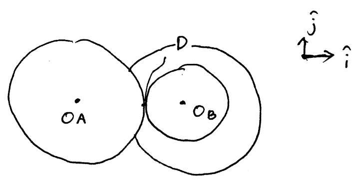


Problem 13.20: Two gears with end loads.

13.122



(a)



$\hat{i}$ is along the line connecting O_A and O_B. D is the point of contact

$\therefore\quad \vec{r}_{D/O_A} = R_A\hat{i}\ ,\qquad \vec{r}_{D/O_B} = -R_c\,\hat{i}$

No slip condition:

$$\vec{V}_D\big|_{\text{on gear A}} = \vec{V}_D\big|_{\text{on gear B}}$$

$$\cancel{\vec{V}_{O_A}}^{\,0} + \vec{\omega}_A \times \vec{r}_{D/O_A} = \cancel{\vec{V}_{O_B}}^{\,0} + \vec{\omega}_B \times \vec{r}_{D/O_B}$$

Gear A rotates clockwise with $\omega = 2\,\text{rad/s}$.

$\therefore\quad \vec{\omega}_A = -\omega\hat{k}$

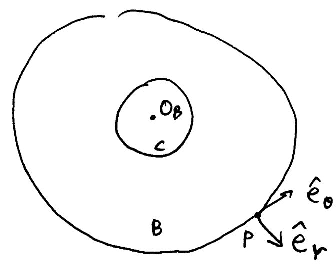
Assume $\vec{\omega}_B = \omega_B\hat{k}$

$\Rightarrow\quad -\omega\hat{k} \times R_A\hat{i} = \omega_B\hat{k} \times (-R_c\hat{i})$

$\Rightarrow\quad \boxed{\omega_B = \omega\dfrac{R_A}{R_c}}\ ,\quad \boxed{\omega_B > 0}$ means the right gear rotates counterclockw

$\therefore$ gear B,C rotates counterclockwise with ~~spee~~ angular velocity $\omega_B = \omega\dfrac{R_A}{R_c}$

b.).



From (a), we know the angular velocity of B, $\vec{\omega}_B = \omega \frac{R_A}{R_C} \hat{k}$

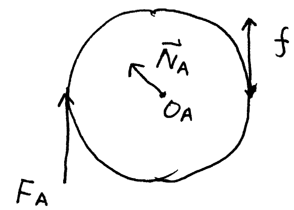
$\therefore \ \vec{V}_P = \vec{V}_{OB} + \vec{\omega}_B \times \vec{r}_{P/OB} = \omega \frac{R_A}{R_C} R_B \ \hat{e}_\theta$

$\therefore$ The velocity of P is $\boxed{\frac{\omega R_A R_B}{R_C}}$ and is in $\hat{e}_\theta$ direction.

c). Assume friction less bearing.

FBD $\quad \hat{j}, \hat{i}$

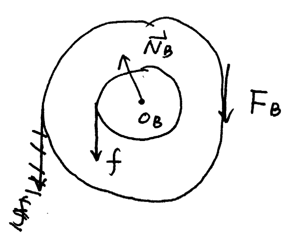
Left gear:



f: friction frorce at contact point

$\vec{N}_A$: reaction force at gear bearing

Right gear:



AMB of left gear about OA

$$\Rightarrow \quad \Sigma \vec{M}_{/OA} = \dot{\vec{H}}_{/OA} = \frac{d}{dt}(I_{OA} \underbrace{\vec{\omega}_A}_{\text{constant angular velocity}}) = 0$$

$$\Rightarrow \ -F_A R_A + f R_A = 0$$

$$\Rightarrow \ F_A = f$$

AMB of right gear about O_B

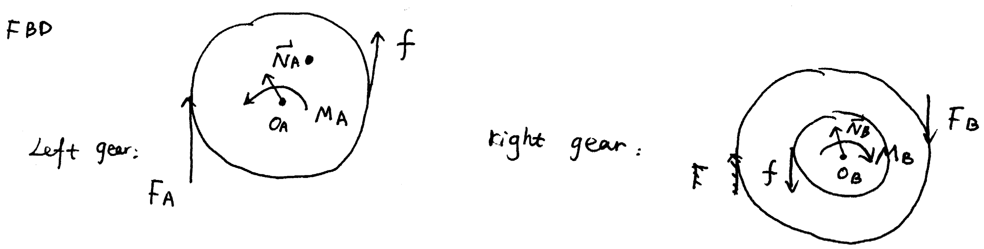
$$\Rightarrow \sum \vec{M}_{/O_B} = \dot{\vec{H}}_{/O_B} = \frac{d}{dt}(I_{O_B}\dot{\vec{\omega}}_B) = 0$$

$$\Rightarrow -F_B R_B + f R_c = 0 \qquad \Rightarrow F_B = \frac{f R_c}{R_B}$$

$$\therefore \boxed{F_B = \frac{F_A R_c}{R_B}}$$

d). If gear bearing had friction,

FBD



then there will be moment from the bearing resisting rotation of the gears.

$\because$ A is rotating clockwise, so $\vec{M}_A = M_A\hat{k}$, $M_A > 0$.
i.e., the moment is counterclockwise

B, C is rotating counterclockwise, so $\vec{M}_B = -M_B\hat{k}$, $M_B > 0$
i.e., the moment on B,C. is clockwise.

Use the same argument as in (c).,

Left gear: $\sum \vec{M}_{/O_A} = 0$

$$\Rightarrow -F_A R_A + f R_A + M_A = 0$$

$$\Rightarrow f = F_A - \frac{M_A}{R_A}$$

right gear: $\sum \vec{M}_{/O_B} = 0$

$$\Rightarrow -F_B R_B + f R_c - M_B = 0 \quad \Rightarrow F_B = f\frac{R_c}{R_B} - \frac{M_B}{R_B}$$

$$\therefore \boxed{F_B = \frac{F_A R_c}{R_B} - \frac{M_A R_c}{R_A R_B} - \frac{M_B}{R_B}} < \frac{F_A R_c}{R_B} \quad \text{since } M_A, M_B > 0$$

$\therefore$ F_B is smaller because of friction

e). If the left gear is driven by a motor, angular speed of the right gear is $\boxed{\text{still} \quad \omega_B = \omega \frac{R_A}{R_C}}$, counter clockwise.

Because this result comes from the kinematic constraint that there is no slip between left gear and right gear. It doesn't depend on how the left gear is driven.

13.6.34 A pegged compound pendulum. A uniform bar of mass m and length ℓ hangs from a peg at point C and swings in the vertical plane about an axis passing through the peg. The distance d from the center of mass of the rod to the peg can be changed by putting the peg at some other point along the length of the rod.

a) Find the angular momentum of the rod about point C.

b) Find the rate of change of angular momentum of the rod about C.

c) How does the period of the pendulum vary with d? Show the variation by plotting the period against $\frac{d}{\ell}$. [Hint, you must first find the equations of motion, linearize for small θ, and then solve.]

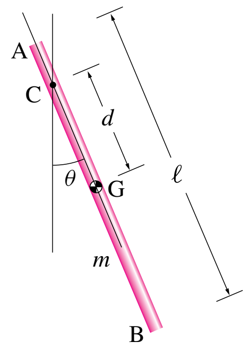
d) Find the total energy of the rod (using point C as a datum for potential energy).

e) Find $\ddot{\theta}$ when $\theta = \pi/6$.

f) Find the reaction force on the rod at C, as a function of m, d, ℓ, θ, and $\dot{\theta}$.

g) For the given rod, what should be the value of d (in terms of ℓ) in order to have the fastest pendulum?

h) **Test of Schuler's pendulum.** The pendulum with the value of d obtained in (g) is called the Schuler's pendulum. It is not only the fastest pendulum but also the "most accurate pendulum". The claim is that even if d changes slightly over time due to wear at the support point, the period of the pendulum does not change much. Verify this claim by calculating the percent error in the time period of a pendulum of length $\ell = 1\,\text{m}$ under the following three conditions: (i) initial $d = 0.15\,\text{m}$ and after some wear $d = 0.16\,\text{m}$, (ii) initial $d = 0.29\,\text{m}$ and after some wear $d = 0.30\,\text{m}$, and (iii) initial $d = 0.45\,\text{m}$ and after some wear $d = 0.46\,\text{m}$. Which pendulum shows the least error in its time period? What is the connection between this result and the plot obtained in (c)?



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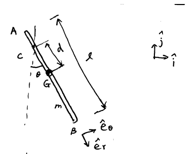
Problem 13.34

13.136.



a). $\vec{H}_{/C} = I_C\,\vec{\omega} = (I_G + m d^2)(\dot{\theta}\hat{k}) = \boxed{\left(\frac{m\ell^2}{12} + m d^2\right)\dot{\theta}\hat{k}}$

or $\vec{H}_{/C} = \vec{r}_{G/C} \times m\vec{v}_G + I_G\,\vec{\omega} = d\hat{e}_r \times m d\dot{\theta}\hat{e}_\theta + \frac{m\ell^2}{12}\dot{\theta}\hat{k}$

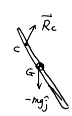
$= \left(\frac{m\ell^2}{12} + m d^2\right)\dot{\theta}\hat{k}$

b). $\dot{\vec{H}}_{/C} = \frac{d}{dt}(\vec{H}_{/C}) = \frac{d}{dt}\Big[\underbrace{\left(\frac{m\ell^2}{12} + m d^2\right)}_{\text{constant}}\dot{\theta}\underbrace{\hat{k}}_{\text{constant}}\Big]$

$= \boxed{\left(\frac{m\ell^2}{12} + m d^2\right)\ddot{\theta}\hat{k}}$

c). AMB about C.

FBD



$\vec{R}_C$: reaction force at the hinge.

$\Sigma\vec{M}_{/C} = \dot{\vec{H}}_{/C}$

$\vec{r}_{G/C} \times (-mg\hat{j}) = \left(\frac{m\ell^2}{12} + m d^2\right)\ddot{\theta}\hat{k}$

$\Rightarrow \left(\frac{m\ell^2}{12} + m d^2\right)\ddot{\theta} + mgd\sin\theta = 0$

$\Rightarrow \ddot{\theta} + \frac{12gd}{12d^2 + \ell^2}\sin\theta = 0$

For small θ, $\sin\theta \approx \theta$, $\Rightarrow$ $\boxed{\ddot{\theta} + \frac{12gd}{12d^2+\ell^2}\theta = 0}$

The period $\boxed{T = \frac{2\pi}{\sqrt{\frac{12gd}{12d^2+\ell^2}}}} = 2\pi\sqrt{\frac{12d^2+\ell^2}{12gd}} = \frac{2\pi\sqrt{\ell}}{\sqrt{g}}\sqrt{D + \frac{1}{12D}}$

where $D = \frac{d}{\ell}$.

Can plot $\underbrace{\frac{T}{2\pi}\sqrt{\frac{g}{\ell}}}_{\text{normalized } T} \sim D$ to ~~th~~ show the variation

normalized T : $\frac{T}{2\pi}\sqrt{\frac{g}{\ell}} = \sqrt{D + \frac{1}{12D}}$

d). Use the height of point c as a datum for potential energy.
At a position with angle θ.

$$E_p = -mgd\cos\theta$$

$$E_k = \frac{1}{2}mV_G^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}I_c\omega^2 = \frac{1}{2}(md^2+\frac{ml^2}{12})\dot{\theta}^2$$

$$\therefore E_T = E_k + E_p = \boxed{\frac{1}{2}(md^2+\frac{ml^2}{12})\dot{\theta}^2 - mgd\cos\theta}$$

e). $\theta = \frac{\pi}{6} \Rightarrow \sin\theta = \frac{1}{2}$

$\because \ddot{\theta} + \frac{12gd}{12d^2+l^2}\sin\theta = 0$ is satisfied all the time

$$\therefore \ddot{\theta} = -\frac{12gd}{12d^2+l^2}\sin\theta = \boxed{-\frac{6gd}{12d^2+l^2}} \quad \text{if } \theta = \frac{\pi}{6}$$

f). Use LMB

$$\Sigma\vec{F} = m\vec{a}_G \Rightarrow \vec{R}_c - mg\hat{j} = m\vec{a}_G$$

where $\vec{a}_G = \ddot{\theta}d\hat{e}_\theta - \dot{\theta}^2 d\hat{e}_r = (-\frac{12gd}{12d^2+l^2}\sin\theta\,\hat{e}_\theta) - \dot{\theta}^2 d\hat{e}_r$

$\because \hat{e}_\theta = \cos\theta\hat{i} + \sin\theta\hat{j}$, $\hat{e}_r = \sin\theta\hat{i} - \cos\theta\hat{j}$

$$\Rightarrow \vec{a}_G = -(\frac{12gd}{12d^2+l^2}\cos\theta\sin\theta + \dot{\theta}^2 d\sin\theta)\hat{i} - (\frac{12gd}{12d^2+l^2}\sin^2\theta - \dot{\theta}^2 d\cos\theta)\hat{j}$$

$\therefore$ The reaction force

$$\boxed{\vec{R}_c = -m(\frac{12gd}{12d^2+l^2}\cos\theta\sin\theta + \dot{\theta}^2 d\sin\theta)\hat{i} + (mg - \frac{12mgd}{12d^2+l^2}\sin^2\theta + m\dot{\theta}^2 d\cos\theta)\hat{j}}$$

g). $\because T = 2\pi\sqrt{\frac{12d^2+l^2}{12gd}}$

To find the minimum T, set $\frac{\partial T}{\partial d} = 0$

$$\because \frac{\partial T}{\partial d} = 2\pi\sqrt{\frac{12gd}{12d^2+l^2}}\left(\frac{1}{g} - \frac{l^2}{12gd^2}\right) = 0$$

$$\Rightarrow \frac{1}{g} - \frac{l^2}{12gd^2} = 0 \Rightarrow \boxed{d_m = \sqrt{\frac{1}{12}}\,l} \simeq 0.2887l$$

when $d < d_m$, $\frac{\partial T}{\partial d} < 0$; when $d > d_m$, $\frac{\partial T}{\partial d} > 0$

$\therefore \; d_m = \sqrt{\frac{1}{12}}\, \ell$ is the minimum point for T.

h). $\ell = 1\,m$, $g = 9.8\,m/s^2$, $\quad T = 2\pi \sqrt{\dfrac{12d^2+\ell^2}{12gd}} = \dfrac{2\pi\sqrt{\ell}}{\sqrt{g}} \sqrt{D + \frac{1}{12D}}$ $\quad \boxed{D = \frac{d}{\ell}}$

i). ~~initial~~ initial $d_o = 0.15\,m$

$T_o = 1.6859\ s$

after some wear, $d = 0.16\ m$, $T = 1.6561\ s$

error $\left|\dfrac{T-T_o}{T_o}\right| = 1.767\ \%$

ii) initial $d_o = 0.29\ m$, $T_o = 1.5251\ s$

after some wear $d = 0.30\ m$, $T = 1.5256\ s$

error $\left|\dfrac{T-T_o}{T_o}\right| = 0.0343\ \%$

iii) initial $d_o = 0.45\ m$, $T_o = 1.5996\ s$

after some wear $d = 0.46\,m$, $T = 1.6071\,s$

error $\left|\dfrac{T-T_o}{T_o}\right| = 0.470\ \%$

The second case where $d_o = 0.29\,m$ shows the least error.

Since $d_o = 0.29\,m$ is close to $d_m = \sqrt{\frac{1}{12}}\,\ell \approx 0.2887\,\ell$,

and

$$\Delta T = |T - T_o| \approx \left| \frac{\partial T}{\partial d}\bigg|_{d=d_o} (d - d_o) \right|$$

For all the 3 cases, $d - d_o = 0.1\,m$. However, $d_o = 0.29\,m$ is close to $d_m \approx 0.2887\,\ell$ where $\frac{\partial T}{\partial d} = 0$. So $\left|\frac{\partial T}{\partial d}\Big|_{d=d_o}\right|$ is the least when $d_o = 0.29\,m$.

From the graph given in c). one can see the slope near $d_m = \sqrt{\frac{1}{12}}\,\ell$ is very small, that is, the curve is flat near d_m.

14.1.1 A disk of radius R is hinged at point O at the edge of the disk, approximately as shown. It rotates counterclockwise with angular velocity $\dot\theta = \vec{\omega}$. A bolt is fixed on the disk at point P at a distance r from the center of the disk. A frame $x'y'$ is fixed to the disk with its origin at the center C of the disk. The bolt position P makes an angle ϕ with the x'-axis. At the instant of interest, the disk has rotated by an angle θ.

a) Write the position vector of point P relative to C in the $x'y'$ coordinates in terms of given quantities.

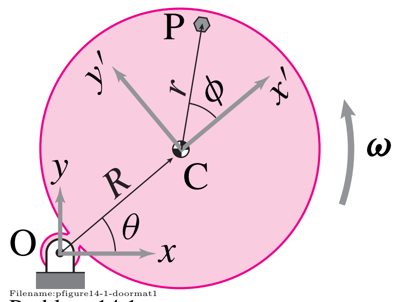
b) Write the position vector of point P relative to O in the xy coordinates in terms of given quantities.

c) Write the expressions for the rotation matrix $R(\theta)$ and the angular velocity matrix $\mathcal{S}(\vec{\omega})$.

d) Find the velocity of point P relative to C using $R(\theta)$ and the angular velocity matrix $\mathcal{S}(\vec{\omega})$.

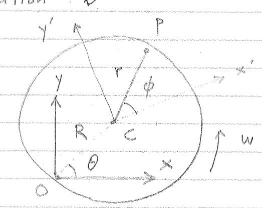
e) Using $R = 30\,\text{cm}$, $r = 25\,\text{cm}$, $\theta = 60°$, and $\phi = 45°$, find $[\vec{r}_{C/0}]_{xy}$, and $[\vec{r}_{P/0}]_{xy}$ at the instant shown.

f) Assuming that the angular speed is $\omega = 10\,\text{rad/s}$ at the instant shown, find $[\vec{v}_{C/0}]_{xy}$ and $[\vec{v}_{P/0}]_{xy}$ taking other quantities as specified above.



Problem 14.1

14.1 Solution ✓



a) $\boxed{[\vec r_{P/C}]_{x'y'} = \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}}$

b) $[\vec r_{P/O}]_{xy} = [\vec r_{C/O}]_{xy} + [R(\theta)][\vec r_{P/C}]_{x'y'}$

$$= \boxed{\begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}}$$

c) $\boxed{R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}$ Rotation matrix

Book ⇒ Pg. 664 $S(\vec w) = \dot R R^{-1} = \begin{bmatrix} -\dot\theta\sin\theta & -\dot\theta\cos\theta \\ \dot\theta\cos\theta & -\dot\theta\sin\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$= \begin{bmatrix} \dot\theta(-\sin\theta\cos\theta + \sin\theta\cos\theta) & -\dot\theta(\sin^2\theta + \cos^2\theta) \\ \dot\theta(\cos^2\theta + \sin^2\theta) & \dot\theta(\sin\theta\cos\theta - \sin\theta\cos\theta) \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 0 & -\dot\theta \\ \dot\theta & 0 \end{bmatrix}}$$

d) $$[\dot{\vec{r}}_{P/C}]_{x'y'} = S(\vec{w})\, R(\theta)\, [\vec{r}_{P/C}]_{x'y'}$$

$$= \begin{bmatrix} 0 & -\dot{\theta} \\ \dot{\theta} & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$= \begin{bmatrix} -\dot{\theta}\sin\theta & -\dot{\theta}\cos\theta \\ \dot{\theta}\cos\theta & -\dot{\theta}\sin\theta \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -\dot{\theta}\,(r\cos\phi\sin\theta + r\cos\phi\cos\theta) \\ \dot{\theta}\,(r\cos\phi\cos\theta - r\sin\phi\sin\theta) \end{bmatrix}}$$

e) $R = 30\,\text{cm}$, $r = 25\,\text{cm}$, $\theta = 60^\circ$, $\phi = 45^\circ$

$$[\vec{r}_{C/O}]_{xy} = \begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix} = \begin{bmatrix} 30\cos 60^\circ \\ 30\sin 60^\circ \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 15 \\ 15\sqrt{3} \end{bmatrix} [\text{cm}]}$$

$$[\vec{r}_{P/O}]_{xy} = \begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 15\sqrt{3} \end{bmatrix} + \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 25\cos 45^\circ \\ 25\sin 45^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 15\sqrt{3} \end{bmatrix} + 25 \begin{bmatrix} \sqrt{2}/4 - \sqrt{6}/4 \\ \sqrt{6}/4 + \sqrt{2}/4 \end{bmatrix} [\text{cm}]$$

$$= \boxed{\begin{bmatrix} 8.53 \\ 50.13 \end{bmatrix} [\text{cm}]}$$

f) $\omega = 10\ \text{rad/s}$

$$[\vec{v}_{C/O}]_{xy} = [\dot{\vec{r}}_{C/O}]_{xy} = \begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix}^{\cdot} = \dot{\theta}\begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix}$$

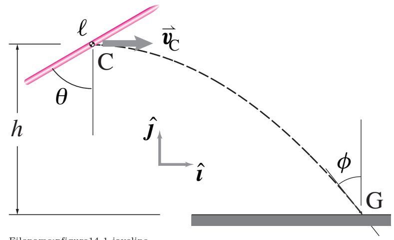
$$= \dot{\theta}\,[\vec{r}_{C/O}]_{xy} = (10)\begin{bmatrix} 15 \\ 15\sqrt{3} \end{bmatrix} = \boxed{\begin{bmatrix} 150 \\ 150\sqrt{3} \end{bmatrix} [\text{cm/s}]}$$

$$[\vec{v}_{P/O}]_{xy} = \dot{\theta}\,[\vec{r}_{P/O}]_{xy} = (10)\begin{bmatrix} 8.53 \\ 50.13 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 85.3 \\ 500.13 \end{bmatrix} [\text{cm/s}]}$$

14.1.12 The center of mass of a javelin travels on a more or less parabolic path while the javelin rotates during its flight. In a particular throw, the velocity of the center of mass of a javelin is measured to be $\vec{v}_C = 10\,\mathrm{m/s}\hat{\imath}$ when the center of mass is at its highest point $h = 6\,\mathrm{m}$. As the javelin lands on the ground, its nose hits the ground at G such that the javelin is almost tangent to the path of the center of mass at G. Neglect the air drag and lift on the javelin.

a) Given that the javelin is at an angle $\theta = 45°$ at the highest point, find the angular velocity of the javelin. Assume the angular velocity isconstant during the flight and that the javelin makes less than a full revolution.



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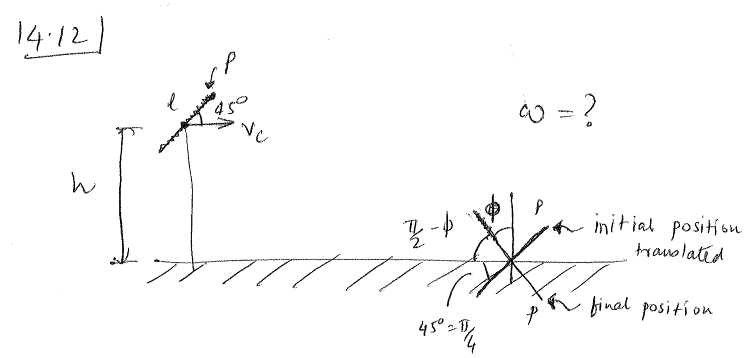
Problem 14.12

14.12)



— Ignore the length of the javelin in calculation

— Consider linear motion, solve for ϕ and time to hit ground (t)

— Solve for angular speed using ϕ and t.

For projectile motion:

$$h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 6}{10}}$$

$$\Rightarrow t = 1.1\ s$$

$$\phi = \tan^{-1}\left(\frac{V_x}{V_y}\right) = \tan^{-1}\left(\frac{V_c}{gt}\right) = \tan^{-1}\left(\frac{10}{10 \times 1.1}\right)$$

$$\Rightarrow \phi = 0.7378$$

Total angle turned from figure above $= \frac{\pi}{4} + \left(\frac{\pi}{2} - \phi\right)$

$= 1.62$

Angular velocity, $\omega = \dfrac{\text{Angle turned}}{\text{time taken}} = \dfrac{1.62}{1.1}$

$$\boxed{\omega = 1.47\ \text{rad/s}}$$

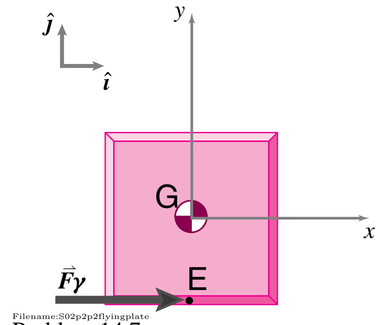
14.2.7 A uniform 1kg plate that is one meter on a side is initially at rest in the position shown. A constant force $\vec{F} = 1\,\mathrm{N}\hat{\imath}$ is applied at $t = 0$ and maintained henceforth. If you need to calculate any quantity that you don't know, but can't do the calculation to find it, assume that the value is given.

a) Find the position of G as a function of time (the answer should have numbers and units).

b) Find a differential equation, and initial conditions, that when solved would give θ as a function of time. θ is the counterclockwise rotation of the plate from the configuration shown.

c) Write computer commands that would generate a drawing of the outline of the plate at $t = 1\,\mathrm{s}$. You can use hand calculations or the computer for as many of the intermediate commands as you like. Hand work and sketches should be provided as needed to justify or explain the computer work.

d) Run your code and show clear output with labeled plots. Mark output by hand to clarify any points.



Problem 14.7

10:10AM Wed section 205
TA: Pranav Bhounsule

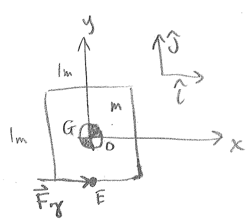
Alan Argondizza
TAM 2030
HW 17 Due 3/26/09

14.12, 14.19, 14.21, 14.31

10/10

$\boxed{14.12}$ See $\boxed{\text{SOLUTION}}$

$\boxed{14.19}$ ✓



$\vec{F} = 1\mathrm{N}\hat{\imath}$
$m = 1\,\mathrm{kg}$

a) LMB: $\Sigma \vec{F} = m\vec{a}_G$

$\vec{a}_G = \dfrac{\vec{F}}{m}$

$\left\{ \vec{a}_G = \dfrac{1N\hat{\imath}}{1kg} = a_x\hat{\imath} + a_y\hat{\jmath} \right\}$

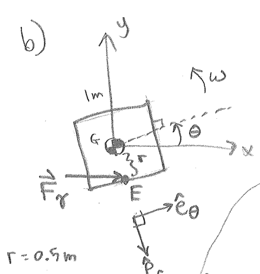
$\{.\}\cdot\hat{\imath} \Rightarrow 1\,m/s^2 = a_x = \ddot{x}$

Position:

$\vec{r}_{G/O} = x\hat{\imath} = (\cancel{x_o} + \cancel{\dot{x}_o} + \tfrac{1}{2}\ddot{x}t^2)\hat{\imath}$

$$\boxed{\vec{r}_{G/O} = \frac{t^2}{2}\hat{\imath}\ m}$$

b)



Pretend G is fixed:

AMB $\Sigma \vec{M}_{/G} = \dot{\vec{H}}_{/G}$

$I_{cm} = \frac{m}{12}(a^2+b^2) = \frac{m}{12}(1^2+1^2)$
$= \frac{m}{6} = \frac{1}{6} = I_{cm}$

$r(\vec{F}_\gamma\cdot\hat{e}_\theta)\,\hat{k} = I^{cm}\dot{\vec{\omega}} + \cancel{\vec{\omega}\times(I^{cm}\vec{\omega})}$

$\{ rF_\gamma\cos\theta\,\hat{k} = I_{cm}\ddot{\theta}\,\hat{k} \}$

$\{.\}\cdot\hat{k} \Rightarrow rF_\gamma\cos\theta = I_{cm}\ddot{\theta} \Rightarrow (\tfrac{1}{2})(1)\cos\theta = \tfrac{1}{6}\ddot{\theta}$

$$\boxed{\ddot{\theta} = 3\cos\theta} \qquad \boxed{\theta|_{t=0} = 0 \quad \dot{\theta}|_{t=0} = 0}$$

OR

$\vec{r}_{E/G}\times\vec{F}_\gamma$
$= r\hat{e}_r\times\{F_\gamma\cos\theta\,\hat{e}_\theta + F_\gamma\sin\theta\,\hat{e}_r\}$
$= rF_\gamma\cos\theta\,\hat{k}$

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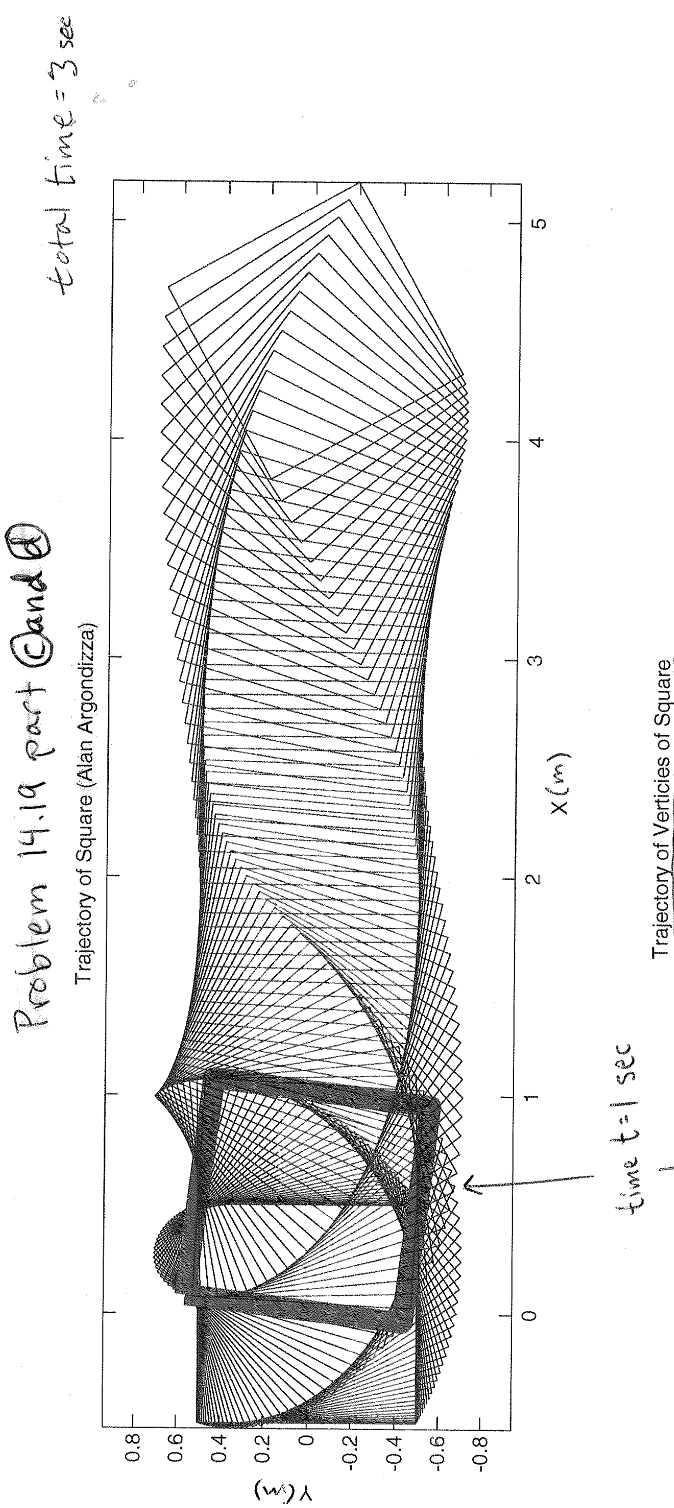
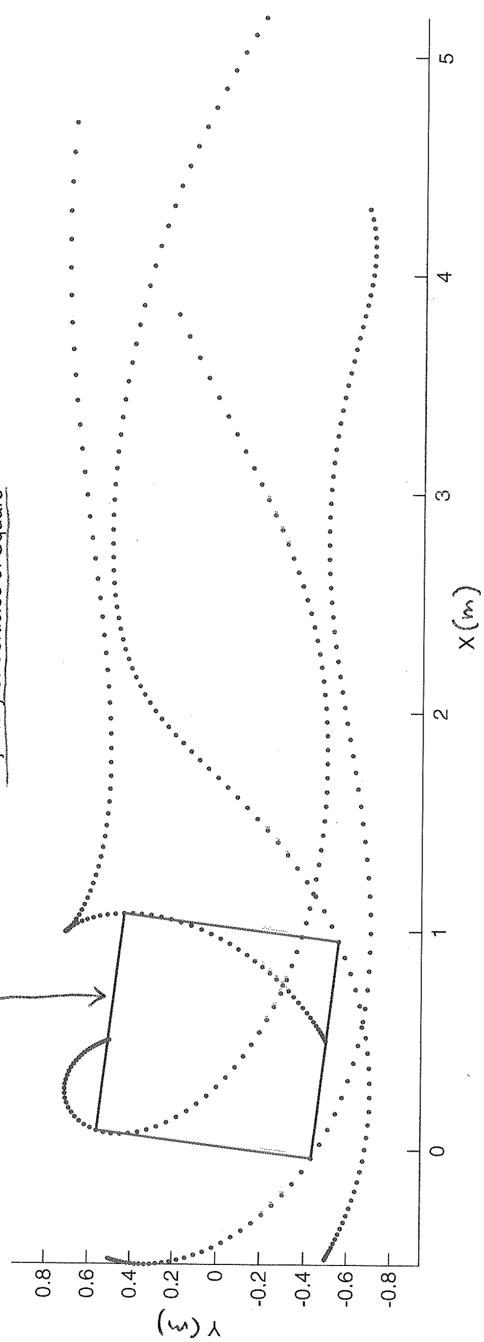
```
function square_plate
% Alan Argondizza
% Solution to 14.19 part c and d

% Initial conditions and time span
time= 3;
tspan= linspace(0,time,101);  %Integrate for time seconds
z0 = [0,0]';                  % initial [angle,omega] both zero

% solve the ODE:
[t,z] = ode45(@rhs, tspan, z0);

% Unpack the variables
theta = z(:,1);      %first column of z
thetadot = z(:,2);   %second column of z

clf
tag=0

%plot using a loop:
for i= 1:1:length(t)

    %entire square:
    subplot(2,1,1)
    %create initial square:
    square= [.5,-.5,-.5,.5,.5;.5,.5,-.5,-.5,.5];
    %create rotation matrix:
    R= [cos(theta(i)),-sin(theta(i));...
        sin(theta(i)),cos(theta(i))];
    %determine displacement of G:
    xdisp= .5*t(i)^2;
    rotatedsquare= R*square + [xdisp,xdisp,xdisp,xdisp,xdisp;0,0,0,0,0];

    plot(rotatedsquare(1,:),rotatedsquare(2,:));

    %this conditional marks the square at time t= 1 second:
    if floor(t(i)) == 1
        if tag ~= 55
            line(rotatedsquare(1,:),rotatedsquare(2,:),'LineWidth',10,'Color','red');
        end
    end
    title('Trajectory of Square (Alan Argondizza)');
    xlabel('X');
    ylabel('Y');
    axis('equal');
    hold on

    %verticicies of square:
    subplot(2,1,2)
    line(rotatedsquare(1,:),rotatedsquare↙
(2,:),'LineStyle','none','Color','red','Marker','.');
```

```
    %this conditional marks the square at time t= 1 second:
    if floor(t(i)) == 1
        if tag ~= 55
            line(rotatedsquare(1,:),rotatedsquare(2,:),'LineWidth',1,'Color','red');
        end
        tag=55;
    end
    title('Trajectory of Verticies of Square');
    xlabel('X');
    ylabel('Y');
    axis('equal');
    hold on
end
end

function zdot = rhs(t,z)

theta = z(1);       % unpack z into readable variables
thetadot = z(2);

%RHS:
omega = thetadot;
omegadot = 3*cos(theta);
% pack up the derivatives:
z1dot = omega;
z2dot = omegadot;
%function return:
zdot = [z1dot z2dot]';
end
```


total time = 3 sec
Problem 14.19 part c and d
Trajectory of Square (Alan Argondizza)
X (m)
Y (m)
time t = 1 sec

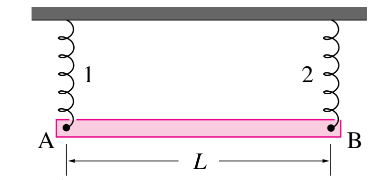

Trajectory of Verticies of Square
X (m)
Y (m)

14.2.9 A uniform slender bar AB of mass m is suspended from two springs (each of spring constant K) as shown. Immediately after spring 2 breaks, determine

a) the angular acceleration of the bar,

b) the acceleration of point A, and

c) the acceleration of point B.



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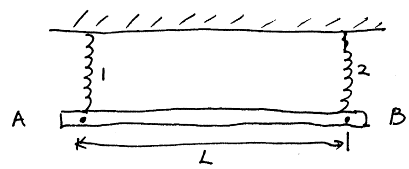
Problem 14.9

14.21



1, 2 : two springs with spring const k

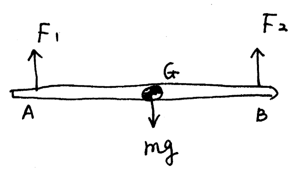
If spring 2 breaks, determine at that instant,

(a) angular acceleration of the bar

(b) the acceleration of point A

(c) the acceleration of point B.

Before "2" breaks, the bar is in equilibrium.



It's easy to get

$F_1 = F_2 = \frac{mg}{2}$ from LMB, AMB $/_G$.

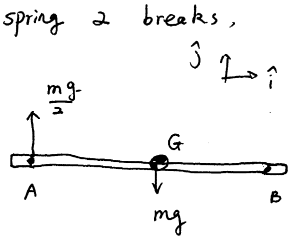
After "2" breaks,

The distance between A ~~bet~~ and the ceiling ~~is~~ at that instant is the same as that before "2" breaks.

∴ The stretch of spring 1 reemains unchanged. So the tension force on spring is still $\frac{mg}{2}$.

But spring 2 breaks, so $F_2 = 0$

∴



AMB $/_G$ ⇒

$\sum \vec{M}_{/G} = I_G \dot{\vec{\omega}}$

$\Rightarrow -\frac{mgL}{4}\hat{k} = \frac{mL^2}{12}\dot{\vec{\omega}}$

$\Rightarrow \boxed{\dot{\vec{\omega}} = -\frac{3g}{L}\hat{k}}$

∴ angular acceleration of the bar is $-\frac{3g}{L}\hat{k}$ at that instant.

LMB $\Rightarrow$

$$-mg\hat{j} + \frac{mg}{2}\hat{j} = m\vec{a}_G$$

$$\Rightarrow \quad \vec{a}_G = -\frac{g}{2}\hat{j}$$

$$\therefore \quad \vec{a}_A = \vec{a}_G + \dot{\vec{\omega}} \times \vec{r}_{A/G} - \omega^2 \vec{r}_{A/G}$$

At this instant $\omega = 0$

$$\therefore \quad \vec{a}_A = -\frac{g}{2}\hat{j} + \left(-\frac{3g}{L}\hat{k}\right) \times \left(-\frac{L}{2}\hat{i}\right)$$

$$= -\frac{g}{2}\hat{j} + \frac{3g}{2}\hat{j} = g\hat{j}$$

Similarly

$$\vec{a}_B = \vec{a}_G + \dot{\vec{\omega}} \times \vec{r}_{B/G} - \overset{0}{\cancel{\omega^2}}\,\vec{r}_{B/G}$$

$$= -\frac{g}{2}\hat{j} + \left(-\frac{3g}{L}\hat{k}\right) \times \left(\frac{L}{2}\hat{i}\right)$$

$$= -\frac{g}{2}\hat{j} - \frac{3g}{2L}\hat{j}$$

$$= -2g\hat{j}$$

$\therefore$ accelerations of point A, B at that instant are

$$\boxed{\vec{a}_A = g\hat{j}}$$

$$\boxed{\vec{a}_B = -2g\hat{j}}$$

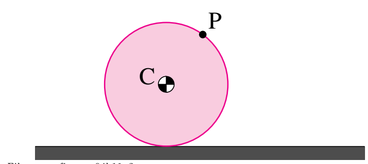
The next several problems concern Work, power and energy

14.3.3 Rolling at constant rate. A round disk rolls on the ground at constant rate. It rolls $1\frac{1}{4}$ revolutions over the time of interest.

a) **Particle paths.** Accurately plot the paths of three points: the center of the disk C, a point on the outer edge that is initially on the ground, and a point that is initially half way between the former two points. [Hint: Write a parametric equation for the position of the points. First find a relation between ω and v_C. Then note that the position of a point is the position of the center plus the position of the point relative to the center.] Draw the paths on the computer, make sure x and y scales are the same.

b) **Velocity of points.** Find the velocity of the points at a few instants in the motion: after $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1 revolution. Draw the velocity vector (by hand) on your plot. Draw the direction accurately and draw the lengths of the vectors in proportion to their magnitude. You can find the velocity by differentiating the position vector or by using relative motion formulas appropriately. Draw the disk at its position after one quarter revolution. Note that the velocity of the points is perpendicular to the line connecting the points to the ground contact.

c) **Acceleration of points.** Do the same as above but for acceleration. Note that the acceleration of the points is parallel to the line connecting the points to the center of the disk.



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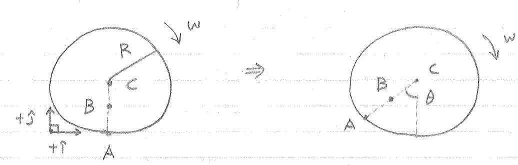
Problem 14.3

For this problem, I looked at the sample derivation done on page 767 in textbook.

14.31



a) point C : $\vec{r}_C = \boxed{R\theta\,\hat{\imath} + R\hat{\jmath}}$

point A : $\vec{r}_A = \vec{r}_C + \vec{r}_{A/C}$

$= \vec{r}_C + (-R\sin\theta\,\hat{\imath} - R\cos\theta\,\hat{\jmath})$

$= R\theta\,\hat{\imath} + R\hat{\jmath} - R\sin\theta\,\hat{\imath} - R\cos\theta\,\hat{\jmath}$

$= \boxed{R(\theta - \sin\theta)\,\hat{\imath} + R(1-\cos\theta)\,\hat{\jmath}}$

point B : $\vec{r}_B = \vec{r}_C + \vec{r}_{B/C}$

$= \vec{r}_C + (-\frac{1}{2}R\sin\theta\,\hat{\imath} - \frac{1}{2}R\cos\theta\,\hat{\jmath})$

$= R\theta\,\hat{\imath} + R\hat{\jmath} - \frac{1}{2}R\sin\theta\,\hat{\imath} - \frac{1}{2}R\cos\theta\,\hat{\jmath}$

$= \boxed{R(\theta - \frac{1}{2}\sin\theta)\,\hat{\imath} + R(1 - \frac{1}{2}\cos\theta)\,\hat{\jmath}}$

Plot of path on separate sheets.

b) point C: $\vec{v}_C = \dot{\vec{r}}_C = R\dot{\theta}\hat{\imath}$

$\frac{1}{4}$ rev. $(\theta = \frac{\pi}{2}) \Rightarrow \vec{v}_C = \boxed{R\dot{\theta}\hat{\imath}}$

$\frac{1}{2}$ rev. $(\theta = \pi) \Rightarrow \vec{v}_C = \boxed{R\dot{\theta}\hat{\imath}}$

$\frac{3}{4}$ rev. $(\theta = \frac{3\pi}{2}) \Rightarrow \vec{v}_C = \boxed{R\dot{\theta}\hat{\imath}}$

1 rev. $(\theta = 2\pi) \Rightarrow \vec{v}_C = \boxed{R\dot{\theta}\hat{\imath}}$

point A: $\vec{v}_A = \vec{v}_C + \vec{\omega} \times \vec{r}_{A/C}$

$\frac{1}{4}$ rev. $\Rightarrow \vec{v}_A = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times R(-\hat{\imath})$

$= R\dot{\theta}\hat{\imath} + R\dot{\theta}\hat{\jmath} = \boxed{R\dot{\theta}(\hat{\imath}+\hat{\jmath})}$

$\frac{1}{2}$ rev. $\Rightarrow \vec{v}_A = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times R(+\hat{\jmath})$

$= R\dot{\theta}\hat{\imath} + R\dot{\theta}\hat{\imath} = \boxed{2R\dot{\theta}\hat{\imath}}$

$\frac{3}{4}$ rev. $\Rightarrow \vec{v}_A = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times R(+\hat{\imath})$

$= R\dot{\theta}\hat{\imath} - R\dot{\theta}\hat{\jmath} = \boxed{R\dot{\theta}(\hat{\imath}-\hat{\jmath})}$

1 rev. $\Rightarrow \vec{v}_A = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times R(-\hat{\jmath})$

$= R\dot{\theta}\hat{\imath} - R\dot{\theta}\hat{\imath} = \boxed{\vec{0}}$

✓

point B: $\vec{v}_B = \vec{v}_C + \vec{\omega} \times \vec{r}_{B/C}$

$\frac{1}{4}$ rev. $\Rightarrow$ $\vec{v}_B = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times \frac{1}{2}R(-\hat{\imath})$

$= \boxed{R\dot{\theta}\,(\hat{\imath} + \frac{1}{2}\hat{\jmath})}$

$\frac{1}{2}$ rev. $\Rightarrow$ $\vec{v}_B = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times \frac{1}{2}R(+\hat{\jmath})$

$= \boxed{\frac{3}{2}R\dot{\theta}\hat{\imath}}$

$\frac{3}{4}$ rev. $\Rightarrow$ $\vec{v}_B = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times \frac{1}{2}R(+\hat{\imath})$

$= \boxed{R\dot{\theta}(\hat{\imath} - \frac{1}{2}\hat{\jmath})}$

1 rev. $\Rightarrow$ $\vec{v}_B = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times \frac{1}{2}R(-\hat{\jmath})$

$= \boxed{\frac{1}{2}R\dot{\theta}\hat{\imath}}$

velocity vectors shown on separate sheets. $\rightarrow$

c) point C: $\vec{a}_C = \dot{\vec{v}}_C = R\ddot{\theta}\hat{\imath} = \boxed{\vec{0}}$ $(\ddot{\theta} = 0)$

point A: $\vec{a}_A = \vec{a}_C + \vec{a}_{A/C}$

$$= \vec{a}_C + \cancelto{0}{\dot{\vec{\omega}} \times \vec{r}_{A/C}} - \omega^2 \vec{r}_{A/C}$$

$\frac{1}{4}$ rev. $\Rightarrow$ $\vec{a}_A = \vec{0} - \dot{\theta}^2 R(-\hat{\imath}) = \boxed{R\dot{\theta}^2\hat{\imath}}$

$\frac{1}{2}$ rev. $\Rightarrow$ $\vec{a}_A = \vec{0} - \dot{\theta}^2 R(+\hat{\jmath}) = \boxed{-R\dot{\theta}^2\hat{\jmath}}$

$\frac{3}{4}$ rev. $\Rightarrow$ $\vec{a}_A = \vec{0} - \dot{\theta}^2 R(+\hat{\imath}) = \boxed{-R\dot{\theta}^2\hat{\imath}}$

1 rev. $\Rightarrow$ $\vec{a}_A = \vec{0} - \dot{\theta}^2 R(-\hat{\jmath}) = \boxed{R\dot{\theta}^2\hat{\jmath}}$

point B: $\vec{a}_B = \vec{a}_C + \vec{a}_{B/C}$

$$= \vec{a}_C + \cancelto{0}{\dot{\vec{\omega}} \times \vec{r}_{B/C}} - \omega^2 \vec{r}_{B/C}$$

$\frac{1}{4}$ rev. $\Rightarrow$ $\vec{a}_A = \boxed{\frac{1}{2}R\dot{\theta}^2\hat{\imath}}$

$\frac{1}{2}$ rev. $\Rightarrow$ $\vec{a}_A = \boxed{-\frac{1}{2}R\dot{\theta}^2\hat{\jmath}}$

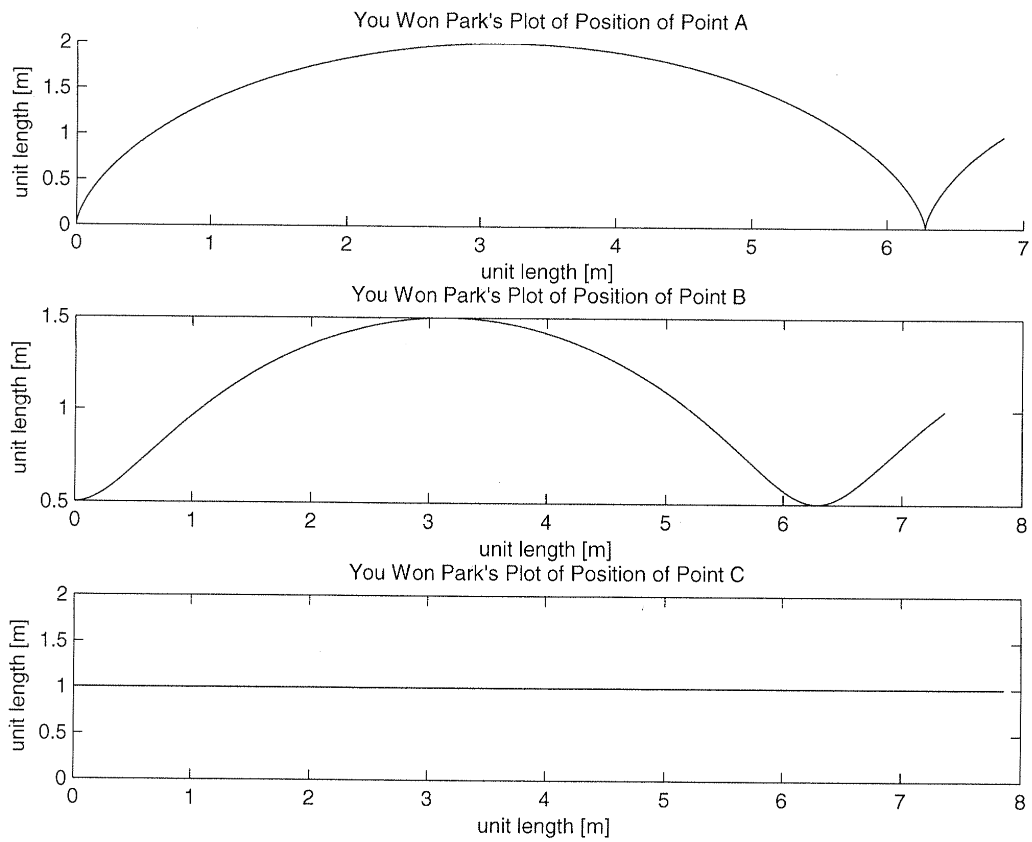
$\frac{3}{4}$ rev. $\Rightarrow$ $\vec{a}_A = \boxed{-\frac{1}{2}R\dot{\theta}^2\hat{\imath}}$

1 rev. $\Rightarrow$ $\vec{a}_A = \boxed{\frac{1}{2}R\dot{\theta}^2\hat{\jmath}}$

$\boxed{\text{acceleration vectors shown on separate sheets.}}$

✓

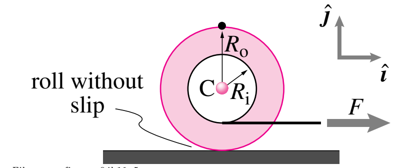
```
function prob1431
% You Won Park's solution to problem 14.31 in HW 17
% Due Mar. 26, 2009

% Constants, initial conditions
R = 1;      % Radius of disk [m]

% Angle interval
angspan = linspace(0,5*pi/2,1001);

% Point C coordinates (center of disk)
rc_x = R*angspan;   % x coord. of C
rc_y = R;           % y coord. of C

% Point A coordinates (ground contact)
ra_x = R*(angspan-sin(angspan));     % x coord. of A
ra_y = R*(1-cos(angspan));           % y coord. of A

% Point B coordinates (halfway)
rb_x = R*(angspan-.5*sin(angspan));     % x coord. of B
rb_y = R*(1-.5*cos(angspan));           % y coord. of B

% Plot positions of A,B,C
figure(1)
subplot(3,1,1)
hold on
plot(ra_x,ra_y,'k')     % Position of A
title('You Won Park''s Plot of Position of Point A')
xlabel('unit length [m]')
ylabel('unit length [m]')

subplot(3,1,2)
plot(rb_x,rb_y,'k')     % Position of B
title('You Won Park''s Plot of Position of Point B')
xlabel('unit length [m]')
ylabel('unit length [m]')

subplot(3,1,3)
plot(rc_x,rc_y,'k')     % Position of C
title('You Won Park''s Plot of Position of Point C')
xlabel('unit length [m]')
ylabel('unit length [m]')

end
```


You Won Park's Plot of Position of Point A
unit length [m]
unit length [m]
You Won Park's Plot of Position of Point B
unit length [m]
unit length [m]
You Won Park's Plot of Position of Point C
unit length [m]
unit length [m]

14.4.6 Spool Rolling without Slip and Pulled by a Cord. The light-weight spool is nearly empty but a lead ball with mass m has been placed at its center. A force F is applied in the horizontal direction to the cord wound around the wheel. Dimensions are as marked. Coordinate directions are as marked.

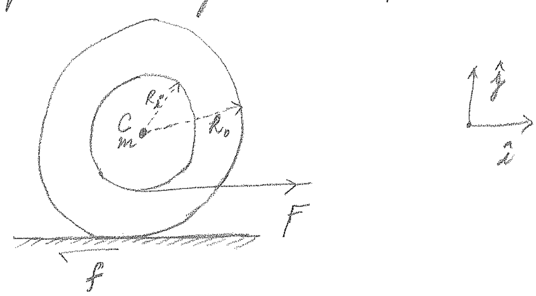
a) What is the acceleration of the center of the spool?

b) What is the horizontal force of the ground on the spool?



Problem 14.6

14.39 Spool rolling w/o slip



* We need to determine the acceleration of the center of mass $\vec{a}_{cm}$, and the horizontal force $\vec{f}$ acting on the lowest pt. as shown.

* Newton's II Law immediately yields:

LMB: $F\hat{\imath} - f\hat{\imath} = m\vec{a}_{cm}$ —(i)

& AMB: $R_i(-\hat{\jmath}) \times F(\hat{\imath}) + R_o(-\hat{\jmath}) \times f(-\hat{\imath}) = I^c \vec{\alpha}$ —(ii)

* Since the spool is "massless", $I^c \to 0$

$\Rightarrow$ (ii) yields $R_i F = R_o f$

Substituting in (i) $\boxed{\vec{a}_{cm} = \frac{F}{m}\left(1 - \frac{R_i}{R_o}\right)\hat{\imath}}$

Also, $\boxed{-\left(\frac{R_i}{R_o}\right) F\hat{\imath} = \vec{f}}$

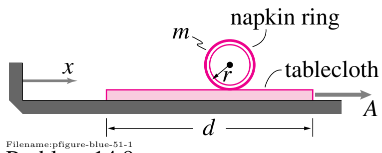
14.4.9 A napkin ring lies on a thick velvet tablecloth. The thin ring (of mass m, radius r) rolls without slip as a mischievous child pulls the tablecloth (mass M) out with acceleration A. The ring starts at the right end ($x = d$). You can make a reasonable physical model of this situation with an empty soda can and a piece of paper on a flat table.

a) What is the ring's acceleration as the tablecloth is being withdrawn?

b) How far has the tablecloth moved to the right from its starting point $x = 0$ when the ring rolls off its left-hand end?

c) Clearly describe the subsequent motion of the ring. Which way does it end up rolling at what speed?

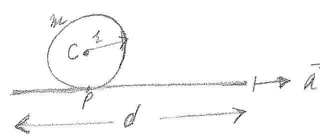
d) Would your answer to the previous question be different if the ring slipped on the cloth as the cloth was being pulled out?



Problem 14.9

14.4.2 "Napkin ring" problem.

(a) The lowest pt on the ring will have the same acceleration as the cloth.



$\alpha = \dot{\omega}$

$$\vec{a}_p = a\hat{\imath} = a_{cm}\hat{\imath} + \alpha\hat{k} \times r(-\hat{\jmath}) \qquad \text{— (1)}$$

Since friction acts at the lowest pt, then

$$\vec{M}_p = \dot{\vec{H}}_{/p} = r\hat{\jmath} \times m\, a_{cm}\hat{\imath} + I^c_{zz}\alpha\hat{k} = \vec{0}$$

$$\Rightarrow \quad m\, a_{cm}\, r = I^c_{zz}\,\alpha$$

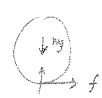
$$\cancel{m}\, a_{cm}\, \cancel{r} = \cancel{m}\, r^{\cancel{2}}\,\alpha$$

$$\Rightarrow \quad \boxed{a_{cm} = r\alpha}$$

Using (1) $\boxed{a_{cm} = \frac{a}{2}}$

FBD

on cloth



on table (before rolling)

(b) The ring moves 'd' in time 't' relative to the cloth with acceleration $a/2$

$$\Rightarrow \quad d = \frac{1}{2}\frac{a}{2}t^2 \quad \Rightarrow \quad t = 2\sqrt{d/a}$$

In the same time, the cloth moves $\frac{1}{2}a\left(2\sqrt{d/a}\right)^2 = \boxed{2d}$

(C) When the ring leaves the cloth, the lowest pt has velocity $\vec{v}_p = 2\sqrt{ad}\ \hat{\imath}$ $(v = v_0 + at)$

Now two cases are possible:

(i) Table is smooth: In this all velocities (angular, linear) are conserved. The ring coasts with the same velocity it has when it leaves the cloth.

(ii) Table has friction: In this case, friction will act until lowest pt. comes to rest. Since lowest pt. is moving to the right — friction will be in the ~~m~~ $-\hat{\imath}$ direction. Friction will cause a clockwise torque about C.M.

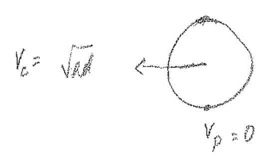
Assuming friction (μ), one can calculate the time 't_R' when lowest pt will come to rest: $t_R = \dfrac{2\sqrt{ad}}{\mu g}$ $(v = at)$

Now turn attention to the center-of-mass: As it leaves the cloth, the center-of-mass will have velocity $v_c = \sqrt{ad}\ \hat{i}$.

On the table, because of friction, it will deaccelerate. By the time the lowest pt. comes to rest (in time 't_s'), the cm will have velocity

$$\sqrt{ad} - \frac{2\sqrt{ad}\,\cancel{\mu g}}{\cancel{\mu g}} = -\sqrt{ad}\ \hat{i}$$

At t_s, we have a situation like this:



Now on there is no work from friction. So it stays rolling like above. (Obviously $v_c = \omega R$).

(d) Of course, if there is sliding initially on the cloth, we don't expect the same v_c as above. But it remains ~~to~~ true the ring will roll in the same direction.

14.4.23 A disk rolls in a cylinder. For all of the problems below, the disk rolls without slip and rocks back and forth due to gravity.

a) **Sketch.** Draw a neat sketch of the disk in the cylinder. The sketch should show all variables, coordinates and dimension used in the problem.

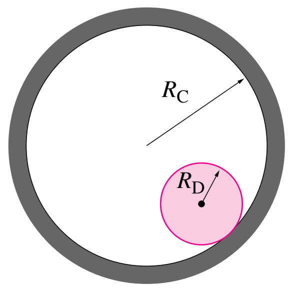
b) **FBD.** Draw a free body diagram of the disk.

c) **Momentum balance.** Write the equations of linear and angular momentum balance for the disk. Use the point on the cylinder which touches the disk for the angular momentum balance equation. Leave as unknown in these equations variables which you do not know.

d) **Kinematics.** The disk rolling in the cylinder is a *one*-degree-of-freedom system. That is, the values of only *one* coordinate and its derivatives are enough to determine the positions, velocities and accelerations of all points. The angle that the line from the center of the cylinder to the center of the disk makes from the vertical can be used as such a variable. Find all of the velocities and accelerations needed in the momentum balance equation in terms of this variable and it's derivative. [Hint: you'll need to think about the rolling contact in order to do this part.]

e) **Equation of motion.** Write the angular momentum balance equation as a single second order differential equation.

f) **Simple pendulum?** Does this equation reduce to the equation for a pendulum with a point mass and length equal to the radius of the cylinder, when the disk radius gets arbitrarily small? Why, or why not?



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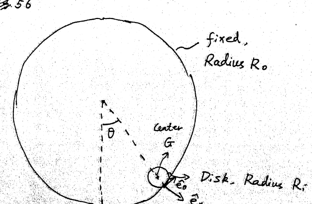
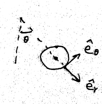
Problem 14.23: A disk rolls without slip inside a bigger cylinder.

14
~~13~~.56





G is doing a circular motion with a Radius $(R_o - R_i)$

$$\vec{V}_G = \dot{\theta}(R_o - R_i)\hat{e}_\theta$$

$$\vec{a}_G = \ddot{\theta}(R_o - R_i)\hat{e}_\theta - \dot{\theta}^2(R_o - R_i)\hat{e}_r$$

No slip condition

$\Rightarrow$ Angular velocity of the disk

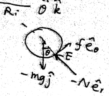
$$\vec{V}_G = -\omega R_i \hat{e}_\theta \qquad (\omega > 0 \text{ if counter clockwise})$$

$$\omega = -\frac{R_o - R_i}{R_i}\dot{\theta}$$

Angular acceleration

$$\dot{\omega} = -\frac{R_o - R_i}{R_i}\ddot{\theta}, \quad \vec{\dot{\omega}} = \dot{\omega}\hat{k} = -\frac{R_o - R_i}{R_i}\ddot{\theta}\hat{k}$$

Take AMB about the contact point E.



$$\dot{\vec{H}}_{/E} = \vec{r}_{G/E} \times m\vec{a}_G + I_G \vec{\dot{\omega}}$$

$$= (-R_i\hat{e}_r) \times m(\ddot{\theta}(R_o - R_i)\hat{e}_\theta - \dot{\theta}^2(R_o - R_i)\hat{e}_r) + \frac{1}{2}mR_i^2\left(-\frac{R_o - R_i}{R_i}\ddot{\theta}\right)\hat{k}$$

$$= -m\ddot{\theta}R_i(R_o - R_i)\hat{k} - \frac{1}{2}m(R_o - R_i)R_i\ddot{\theta}\hat{k}$$

$$= -\frac{3}{2} m R_i (R_o - R_i)\ddot{\theta}\hat{k}$$

$$\Sigma \vec{M}_{/E} = \vec{r}_{G/E} \times (-mg\hat{j})$$

$$= (-R_i\hat{e}_r) \times (-mg\hat{j}) = mg R_i \sin\theta \hat{k}$$

AMB : $\Sigma \vec{M}_{/E} = \dot{\vec{H}}_{/E} \Rightarrow mgR_i \sin\theta\hat{k} = -\frac{3}{2} mR_i(R_o - R_i)\ddot{\theta}\hat{k}$

$$\Rightarrow \frac{3}{2} m R_i (R_o - R_i)\ddot{\theta} + mg R_i \sin\theta = 0$$

$$\Rightarrow \boxed{\ddot{\theta} + \frac{2g}{3(R_o - R_i)} \sin\theta = 0}$$

If the disk radius gets arbitrarily small, then $R_i = 0$

The equation of motion becomes

$$\ddot{\theta} + \frac{2g}{3R_o} \sin\theta = 0$$

This equation does not reduce to the simple pendulum equation,

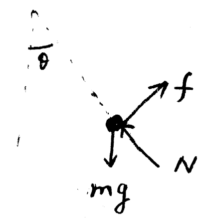
which should be $\ddot{\theta} + \frac{g}{R_o} \sin\theta = 0$

Reason: As the radius of the disk, R_i, goes to 0, our problem is NOT the same as a simple pendulum.

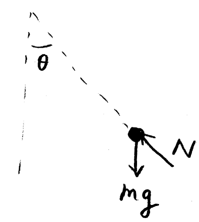
To enforce no slip condition, there must be friction acting the the object. However, for a simple pendulum, the reaction force should be in normal direction.

The difference can be illustated in the FBD's below

Our problem when $R_i \to 0$:



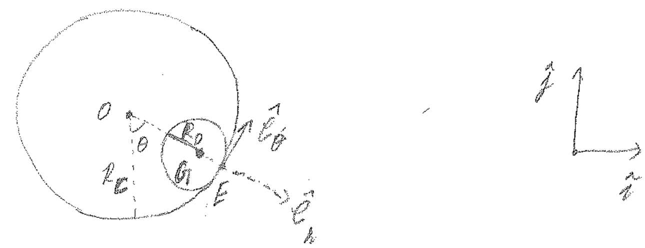
Simple pendulum.



The existence of friction force in our problem makes it different from simple pendulum problem

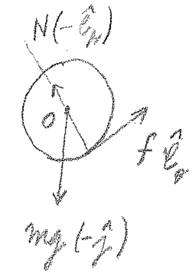
14.56

(a)



(b) During rolling, there is no kinetic friction but there is static friction.

FBD:



(c) LMB: $mg(-\hat{j}) + f(\hat{e}_\theta) + N(-\hat{e}_r) = m\vec{a}_G$

AMB: $\sum \vec{M}_{/E} = \dot{\vec{H}}_{/E} = \vec{r}_{G/E} \times m\vec{a}_G + I\dot{\omega}\hat{k}$

(d) Kinematics: G is moving in a circle of radius $(R_C - R_D)$

$$\vec{V}_G = \dot{\theta}(R_C - R_D)\hat{e}_\theta \quad \text{— (i)}$$

$$\Rightarrow \vec{a}_G = \ddot{\theta}(R_C - R_D)\hat{e}_\theta - \dot{\theta}^2(R_C - R_D)\hat{e}_r \quad \text{— (ii)}$$

(Remember: $\dot{\theta} \neq \omega$. Here, ω refers to rotation of the disc.)

Rolling w/o slip :

$$\vec{v}_G = -\omega(\hat{k}) \times R_D(\hat{e}_r) = -\omega R_D \hat{e}_\theta$$

using (i) : $\omega = -\frac{R_C - R_D}{R_D}\dot{\theta}$

(e) Now substitute $\vec{a}_G$ from (ii) in AMB,

& $\vec{r}_{G/E} = R_D(-\hat{e}_R)$

& $\sum M_{/E} = \vec{r}_{G/E} \times mg(-\hat{j}) = mgR_D \sin\theta\,\hat{k}$

(after some algebra ....)

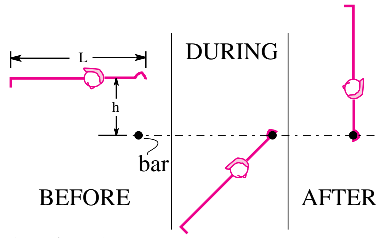
$$\boxed{\ddot{\theta} + \frac{2g}{3(R_c - R_D)}\sin\theta = 0}$$

(f) If $R_D \to 0$, $\ddot{\theta} = -\frac{2}{3}\frac{g}{R_c}\sin\theta$

which is not the same as $\ddot{\theta} = -\frac{g}{R_c}\sin\theta$, the equation for simple pendulum.

This is because even at small R_c, there is rolling! The function for $\ddot{\theta}$ will not abruptly jump by continuously varying R_D.

14.5.8 An acrobat modeled as a rigid body with uniform rigid mass m of length l. She falls without rotation in the position shown from height h where she was stationary. She then grabs a bar with a firm but slippery grip. What is h so that after the subsequent motion the acrobat ends up in a stationary handstand? [Hint: What quantities are preserved in what parts of the motion?]



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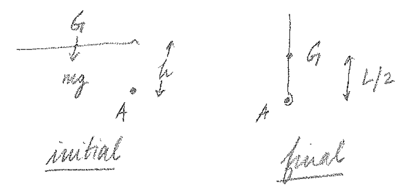
Problem 14.8

14.65



* NOTE: Cannot simply use conservation of energy because there is loss upon impact. However, angular momentum is conserved during impact.

$\vec{H}^-_{/A} = m\sqrt{2gh}\,\frac{L}{2}\,\hat{k}$ (just before impact)

$\vec{H}^+_{/A} = I_A^{zz}\,\omega_1\,\hat{k}$ (just after impact)

$= \frac{mL^2}{3}\,\omega_1\,\hat{k}$

$\Rightarrow$ (just after impact) $\omega_1 = \frac{3}{2}\frac{\sqrt{2gh}}{L}$

Since the pivot is "slippery", energy is conserved during the swing:

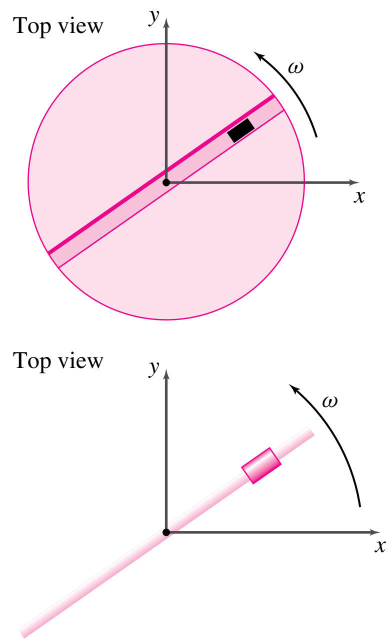
$E_i = E_f$

$\frac{1}{2} I_A^{zz}\omega_1^2 = mg\frac{L}{2}$

$\Rightarrow \frac{mL^2}{3}\left(\frac{3}{2}\frac{\sqrt{2gh}}{L}\right)^2 = \frac{mgL}{2}$

$\boxed{h = \frac{2}{3}\,l}$

15.1.5 Picking apart the polar coordinate formula for velocity. This problem concerns a small mass m that sits in a slot in a turntable. Alternatively you can think of a small bead that slides on a rod. The mass always stays in the slot (or on the rod). Assume the mass is a little bug that can walk as it pleases on the rod (or in the slot) and you control how the turntable/rod rotates. Name two situations in which one of the terms is zero but the other is not in the two term polar coordinate formula for velocity, $\dot{R}\hat{e}_R + R\dot{\theta}\hat{e}_\theta$. You should thus gain some insight into the meaning of each of the two terms in that formula.



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Problem 15.5

15.5

$$\vec{V} = \dot{R}\,\hat{e}_R + R\dot{\theta}\,\hat{e}_\theta$$

If $\dot{R} = 0$

$$\vec{V} = R\,\dot{\theta}\,\hat{e}_\theta$$

which means velocity only depends on R and the angular speed of the turnable/rod in $\hat{e}_\theta$

If $R = 0$ or the small mass in center of rotation.

$$\vec{V} = \dot{R}\,\hat{e}_R$$

which means velocity only depends on the rate of change of R in the direction of $\hat{e}_R$

15.1.6 Picking apart the polar coordinate formula for acceleration. Reconsider the configurations in problem 15.1.5. This time, name four situations in which all of the terms, but one, in the four term polar coordinate formula for acceleration, $\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{e}_R + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{e}_\theta$, are zero. Each situation should pick out a different term. You should thus gain some insight into the meaning of each of the four terms in that formula.

15.6

$$\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{e}_R + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{e}_\theta$$

If $R=0$, $\dot{\theta}=0$, $\dot{R}=0$, $\ddot{\theta}=0$

$\vec{a} = \ddot{R}\hat{e}_R$

which means when mass is in center of rotation, angular speed is zero and angular acceleration is also zero, the acceleration of mass only depends on $\ddot{R}$.

If $\ddot{R}=0$, $\dot{R}=0$, $\ddot{\theta}=0$

$\vec{a} = -R\dot{\theta}^2\hat{e}_R$

which means when R stays constant and angular acceleration is zero, the acceleration of mass only depends on angular velocity and R.

If $\ddot{R}=0$, $R=0$, $\ddot{\theta}=0$

$\vec{a} = 2\dot{R}\theta\hat{e}_\theta$

which means when mass is in center of rotation, $\ddot{R}=0$, and angular velocity is zero, the acceleration of mass only depends on rate of change of R and angular velocity

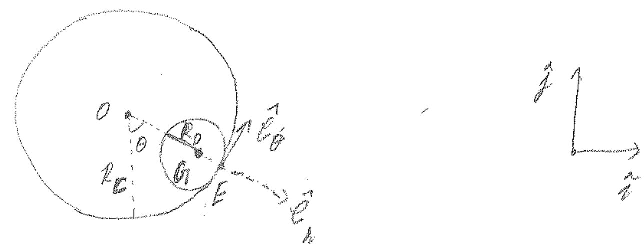
If $\ddot{R}=0$, $\dot{\theta}=0$, $\dot{R}=0$

$\vec{a} = R\ddot{\theta}\hat{e}_\theta$

which means when R stays constant and angular velocity is zero, the acceleration of mass only depends on angular acceleration

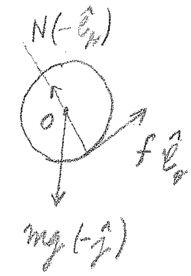
14.56

(a)



(b) During rolling, there is no kinetic friction but there is static friction.

FBD :



(c) LMB : $mg(-\hat{j}) + f(\hat{e}_\theta) + N(-\hat{e}_r) = m\,\vec{a}_G$

AMB: $\sum \vec{M}_{/E} = \dot{\vec{H}}_{/E} = \vec{r}_{G/E} \times m\,\vec{a}_G + I\dot{\omega}\,\hat{k}$

(d) Kinematics : G is moving in a circle of radius $(R_C - R_D)$

$$\vec{V}_G = \dot{\theta}\,(R_C - R_D)\,\hat{e}_\theta \qquad \text{— (i)}$$

$$\Rightarrow \vec{a}_G = \ddot{\theta}\,(R_C - R_D)\,\hat{e}_\theta - \dot{\theta}^2\,(R_C - R_D)\,\hat{e}_r \qquad \text{— (ii)}$$

(Remember : $\dot{\theta} \neq \omega$. Here, ω refers to rotation of the disc.)

Rolling w/o slip :

$$\vec{V}_G = -\omega(\hat{k}) \times R_D(\hat{e}_r) = -\omega R_D \hat{e}_\theta$$

using (i) : $\omega = -\dfrac{R_C - R_D}{R_D}\dot{\theta}$

(e) Now substitute $\vec{a}_G$ from (ii) in AMB,

& $\vec{r}_{G/E} = R_D(-\hat{e}_R)$

& $\sum M_{/E} = \vec{r}_{G/E} \times mg(-\hat{j}) = mgR_D \sin\theta\, \hat{k}$

(after some algebra ....)

$$\boxed{\ddot{\theta} + \frac{2g}{3(R_c - R_D)} \sin\theta = 0}$$

(f) If $R_D \to 0$, $\ddot{\theta} = -\dfrac{2}{3}\dfrac{g}{R_c}\sin\theta$

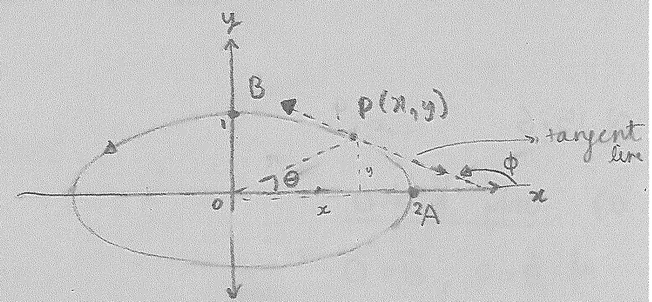
which is not the same as $\ddot{\theta} = -\dfrac{g}{R_c}\sin\theta$, the equation for simple pendulum.

This is because even at small R_c, there is rolling! The function for $\ddot{\theta}$ will not abruptly jump by continuously varying R_D.

15.1.10 A particle travels at non-constant speed on an elliptical path given by $y^2 = b^2(1 - \frac{x^2}{a^2})$. Carefully sketch the ellipse for particular values of a and b. For various positions of the particle on the path, sketch the position vector $\vec{r}(t)$; the polar coordinate basis vectors $\hat{e}_r$ and $\hat{e}_\theta$; and the path coordinate basis vectors $\hat{e}_n$ and $\hat{e}_t$. At what points on the path are $\hat{e}_r$ and $\hat{e}_n$ parallel(or $\hat{e}_\theta$ and $\hat{e}_t$ parallel)?

15.10 $\quad y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$

- let $a = 2 \quad b = 1$
  then ellipse is as shown



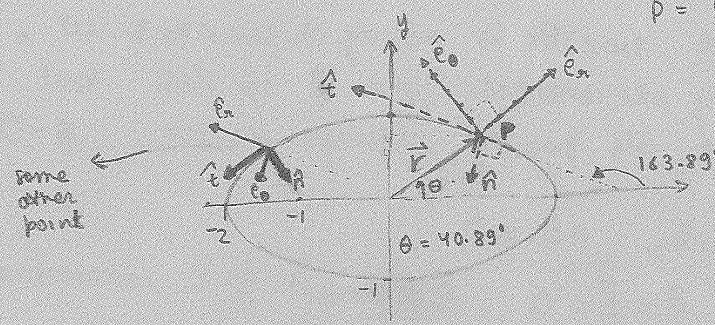
for a point P(x,y) as shown

- $y = b\sqrt{1 - \frac{x^2}{a^2}} = \frac{b}{a}\sqrt{a^2 - x^2} = \frac{1}{2}\sqrt{4 - x^2}$
- $\tan\theta = \frac{y}{x} = \frac{1}{2x}\sqrt{4 - x^2}$ = slope of $\hat{e}_r$
- slope of tangent line = slope of $\hat{t}$ = $\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{2}\frac{-2x}{\sqrt{4 - x^2}} = \frac{-x}{2\sqrt{4 - x^2}} = \tan\phi$

sketch

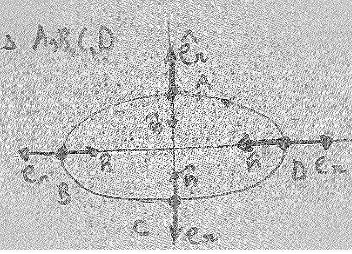
- let $P = \left(1, \frac{1}{2}\sqrt{4 - 1^2}\right) = \left(1, \frac{\sqrt{3}}{2}\right)$
- $\tan\theta = \frac{\sqrt{3}}{2} \implies \theta = 40.89°$
- $\tan\phi = -\frac{1}{2\sqrt{4-1}} = \frac{-1}{2\sqrt{3}} \implies \phi = 163.89°$

now we can draw $\qquad P = \left(1, \frac{\sqrt{3}}{2}\right)$



$\hat{e}_r \parallel \hat{e}_n$ or $\hat{e}_r \perp \hat{e}_t$

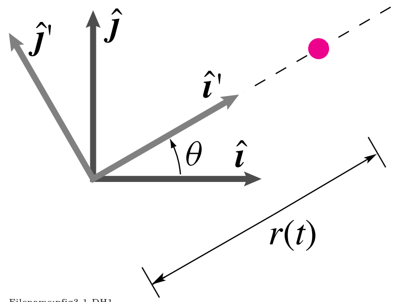
- intuitively they are parallel at following 4 points A, B, C, D
  (in mind: just try to span on the whole of ellipse)
  $\vec{e}_r$ and $\hat{e}_t$ and see where they are ⊥)
  ie see when line to origin is ⊥ to tangent)



15.2.5 Given that $\vec{r}(t) = ct^2\hat{\imath}'$ and that $\theta(t) = d\,\sin(\lambda t)$, find $\vec{v}(t)$

a) in terms of $\hat{\imath}$ and $\hat{\jmath}$,

b) in terms of $\hat{\imath}'$ and $\hat{\jmath}'$.



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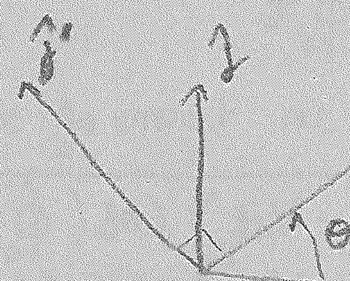
Problem 15.5

Q15.15



b) $\vec{r}(t) = ct^2\,\hat{\imath}'$

$$\vec{v}(t) = \dot{\vec{r}}(t) = 2ct\,\hat{\imath}' + ct^2\,\dot{\hat{\imath}}'$$

$$= 2ct\,\hat{\imath}' + ct^2\,\dot{\theta}\,\hat{\jmath}'$$

$$\boxed{\vec{V} = 2ct\,\hat{\imath}' + d\lambda ct^2\cos(\lambda t)\,\hat{\jmath}'}$$

a) now $\hat{\imath}' = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$

$\hat{\jmath}' = -\sin\theta\,\hat{\imath} \quad \cos\theta\,\hat{\jmath}$

using these in above I get

$$\boxed{\vec{V} = (2ct\cos\theta - d\lambda ct^2\sin\theta)\,\hat{\imath} + (2c}$$

$\boxed{\text{where } \theta = d\sin(\lambda t)}$

15.3.2 Actual path of bug trying to walk a straight line. A straight line is inscribed on a horizontal turntable. The line goes through the center. Let ϕ be angle of rotation of the turntable which spins at constant rate $\dot{\phi}_0$. A bug starts on the outside edge of the turntable of radius R and walks towards the center, passes through it, and continues to the opposite edge of the turntable. The bug walks at a constant speed v_A, as measured by how far her feet move per step, on the line inscribed on the table. Ignore gravity.

a) **Picture.** Make an accurate drawing of the bug's path as seen in the room (which is not rotating with the turntable). In order to make this plot, you will need to assume values of v_A and $\dot{\phi}_0$ and initial values of R and ϕ. You will need to write a parametric equation for the path in terms of variables that you can plot (probably x and y coordinates). You will also need to pick a range of times. Your plot should include the instant at which the bug walks through the origin. Make sure your x and y- axes are drawn to the same scale. A computer plot would be nice.

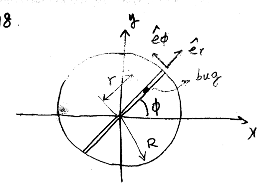
b) Calculate the radius of curvature of the bug's path as it goes through the origin.

c) Accurately draw (say, on the computer) the osculating circle when the bug is at the origin on the picture you drew for (a) above.

d) **Force.** What is the force on the bugs feet from the turntable when she starts her trip? Draw this force as an arrow on your picture of the bug's path.

e) **Force.** What is the force on the bugs feet when she is in the middle of the turntable? Draw this force as an arrow on your picture of the bug's path.

15.18



Assume at time t, the distance between the bug and the center is r(t), and the table has turned an angle of $\phi(t)$

Initially, $r(t=0) = R$
$\phi(t=0) = 0$

a). The position of bug at time t is

$$\begin{cases} x(t) = r\cos\phi \\ y(t) = r\sin\phi \end{cases} \quad \text{where} \quad \begin{cases} r(t) = R - V_A t \\ \phi(t) = \dot{\phi}_o t \end{cases}$$

So
$$\begin{cases} x(t) = (R - V_A t)\cos(\dot{\phi}_o t) \\ y(t) = (R - V_A t)\sin(\dot{\phi}_o t) \end{cases}$$

Let $R = 1\,m$, $V_A = 0.2\,m/s$, $\dot{\phi}_o = 1\,rad/s$. If we choose a sequence of number, then we can get a sequence of (x, y) using the parametric equation above and can plot the trajectory. See attached Matlab code.

b). The velocity of the bug is

$$\vec{V} = \dot{\vec{r}} = \frac{d}{dt}(r(t)\,\hat{e}_r) = \dot{r}(t)\,\hat{e}_r + r(t)\,\dot{\hat{e}}_r$$

$$= -V_A\,\hat{e}_r + (R - V_A t)\,\vec{\omega}\times\hat{e}_r$$

$$= -V_A\,\hat{e}_r + (R - V_A t)\,\dot{\phi}_o\,\hat{e}_\phi$$

($\hat{e}_r$, $\hat{e}_\phi$ are polar coordinate unit vector shown in the picture)

The acceleration is ·

$$\vec{a} = \dot{\vec{v}} = -2V_A\dot{\phi}_o\hat{e}_\phi - (R - V_A t)\dot{\phi}_o^2\hat{e}_r$$

when the bug goes through the center. $r = R - V_A t = 0$, $t = \frac{R}{V_A}$

∴ at that instant

$$\vec{v} = -V_A\hat{e}_r, \quad \vec{a} = -2V_A\dot{\phi}_o\hat{e}_\phi$$

compare this to the path coordinates expression

$$\vec{v} = v\hat{e}_t, \quad \vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

where ρ is the radius of curvature.

we get when the bug is at the origin

$$\begin{cases} v = |\vec{v}| = V_A \\ \hat{e}_t = -\hat{e}_r \\ \dot{v} = 0 \quad \text{(since } \vec{a} \perp \vec{v} \text{ at that instant)} \\ \frac{v^2}{\rho} = 2V_A\dot{\phi}_o \\ \hat{e}_n = -\hat{e}_\phi \end{cases}$$

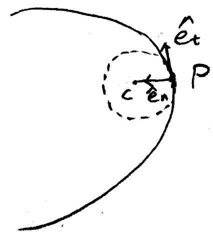
$$\Rightarrow \quad \rho = \frac{v^2}{2V_A\dot{\phi}_o} = \frac{V_A^2}{2V_A\dot{\phi}_o} = \frac{V_A}{2\dot{\phi}_o}$$

∴ The radius of curvature at origin $\boxed{\rho = \frac{V_A}{2\dot{\phi}_o}}$

c). To draw the osculating circle, we need to the center and radius. The radius is given in b).

Now we want to figure out the center.

Generally, Let's say c is the center of the osculating circle at point P.



$\therefore \quad \vec{r}_{c/P} = \rho \hat{e}_n$

In our case, P is at the origin, $\hat{e}_n = -\hat{e}_\phi$

$\therefore \quad (x_c - x_P)\hat{\imath} + (y_c - y_P)\hat{\jmath} = -\rho \hat{e}_\phi$

where $x_P = y_P = 0$

$\rho = \frac{V_A}{2\dot{\phi}_o}$, $\hat{e}_\phi = -\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath}$ (when the bug is at the origin, $\phi = \dot{\phi}_o t = \dot{\phi}_o \frac{R}{V_A}$)

$= -\sin\left(\dot{\phi}_o \frac{R}{V_A}\right)\hat{\imath} + \cos\left(\dot{\phi}_o \frac{R}{V_A}\right)\hat{\jmath}$

$$\Rightarrow \begin{cases} x_c = \frac{V_A}{2\dot{\phi}_o}\sin\left(\dot{\phi}_o \frac{R}{V_A}\right) \\ y_c = -\frac{V_A}{2\dot{\phi}_o}\cos\left(\dot{\phi}_o \frac{R}{V_A}\right) \end{cases}$$

With the position of c, we can then draw the circle. See Matlab code.

d). At the beginning, $t=0$, using the expression derived in b).

$\vec{a}_1 = -2V_A\dot{\phi}_o\hat{e}_\phi - R\dot{\phi}_o^2\,\hat{e}_r$

and $\hat{e}_r = \hat{\imath}$, $\hat{e}_\phi = \hat{\jmath}$ at this time,

$\therefore \quad \vec{a}_1 = -2V_A\dot{\phi}_o\hat{\jmath} - R\dot{\phi}_o^2\hat{\imath}$

Use LMB

$$\boxed{\vec{F}_1 = m\vec{a}_1 = -m\left(R\dot{\phi}_o^2\hat{\imath} + 2V_A\dot{\phi}_o\hat{\jmath}\right)}$$

is the force acting on the bug at the beginning.

e). When the bug is at the origin, $t = \frac{R}{V_A}$

$$\vec{a}_2 = -2V_A\dot{\phi}_o\,\hat{e}_\phi$$

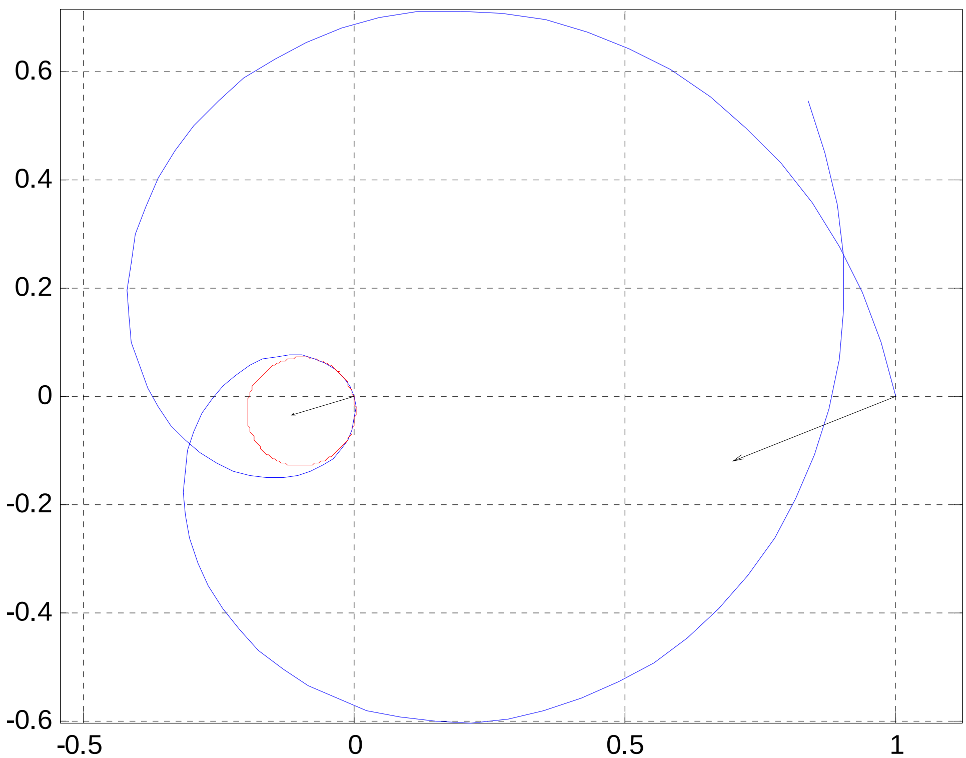
$$= -2V_A\dot{\phi}_o\left(-\sin\left(\dot{\phi}_o\frac{R}{V_A}\right)\hat{\imath} + \cos\left(\dot{\phi}_o\frac{R}{V_A}\right)\hat{\jmath}\right)$$

$$= 2V_A\dot{\phi}_o\sin\left(\dot{\phi}_o\frac{R}{V_A}\right)\hat{\imath} - 2V_A\dot{\phi}_o\cos\left(\dot{\phi}_o\frac{R}{V_A}\right)\hat{\jmath}$$

So the force on the bug at the origin is

$$\vec{F}_2 = m\vec{a}_2 = 2mV_A\dot{\phi}_o\left(\sin\left(\dot{\phi}_o\frac{R}{V_A}\right)\hat{\imath} - \cos\left(\dot{\phi}_o\frac{R}{V_A}\right)\hat{\jmath}\right)$$

```
function path1518()
%%%    draw path
R=1;    % radius of the turntable
va=0.2; % velocity of the bug on the turntable
phidot=1; % angular velocity of the turntable
t=[0:0.1:10];
x=(R-va*t).*cos(phidot*t);
y=(R-va*t).*sin(phidot*t);
plot(x,y);
axis equal;

grid on;

%%%% draw osculating circle when bug goes through the center
rau=va/(2*phidot);    % radius of curvature of the path at the origin
xc= va*sin(phidot*R/va)/(2*phidot);
yc= -va*cos(phidot*R/va)/(2*phidot); % position of the center
                                      %draw the circle;
theta=[0:0.01:2*pi];
circle1=xc+rau*cos(theta);
circle2=yc+rau*sin(theta);
hold on;
plot(circle1,circle2,'r');

%%%% draw force vector
m=1;            %mass of the bug
scale=0.3;      % scale for graphics

f1x=-m*R*phidot;
f1y=-m*2*va*phidot;
quiver(1,0,f1x,f1y,scale,'k'); % draw force at the beginning;

f2x=2*m*va*phidot*sin(phidot*R/va);
f2y=-2*m*va*phidot*cos(phidot*R/va);
quiver(0,0,f2x,f2y,scale,'k'); %draw force at the origin
```


0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.5
0
0.5
1

Q 15.18

a)

$\vec{R} = r\hat{e}_r$

$\vec{V} = \dot{r}\hat{e}_r + r\dot{\phi}\hat{e}_\theta$

. let initially $\phi = 0$, $r = R$

. $\dot{r} = -V_A$ initially $r = R$

(NOTE: by convention r is c

. to be positive, here using

origin ---- but lets no wo

.correct result) (?)

Hence . $r = R - V_A t$

$\phi = \dot{\phi}_0 t$

- writing equation in terms of x, y ($x = r\cos\theta$, $y = r\sin\theta$)

$$\therefore \boxed{\begin{aligned} x(t) &= (R - V_A t)\cos(\dot{\phi}_0 t) \\ y(t) &= (R - V_A t)\sin(\dot{\phi}_0 t) \end{aligned}}$$

- now we can plot these in MATLAB
  - with $R = 1\,m$ $\dot{\phi}_0 = 1$ rad/sec $V_A = 0.2$ m/s
  - time scale $t = 0$ to $t = 10$ sec (it will cross origin at $t=$ when $R - V_A t = ($
- see the figure and code below.

DEFINE / NOTE : $\hat{e}_r \hat{e}_\theta$ is a frame moving with the turntable

b) we have velocity of bug

- $\vec{V} = \underbrace{0}_{\text{velocity of origin of moving frame}} + \vec{\omega} \times \vec{r} + \vec{V}_{\text{rel to moving frame}}$

and acceleration using 5 term formula

- $\vec{a} = \underbrace{0}_{\vec{a}\text{ of origin of moving frame}} - \underbrace{\omega^2}_{\dot{\phi}_0} \underbrace{\vec{r}}_{r\hat{e}_r} + \cancel{\vec{\alpha}}^{\,0} \times \vec{r} + 2\underbrace{\vec{\omega}}_{\dot{\phi}_0} \times \vec{V}_{\text{rel to moving frame}} + \vec{a}_{\text{rel moving frame}}$

$\alpha = 0$

$\dot{\phi} = $ constant

- now

→ $\vec{V}_{\text{rel to moving frame}} = -V_A \hat{e}_r$

→ $\vec{a}_{\text{rel to moving frame}} = 0$ (V_A = constant)

→ $\vec{\omega} = \dot{\phi}_0 \hat{k}$

- doing the math

$$\vec{V} = -V_A \hat{e}_r + r\dot{\phi}_0 \hat{e}_\theta \qquad (1)$$

$$\vec{a} = -\dot{\phi}_0^2 r \hat{e}_r - 2\dot{\phi}_0 V_A \hat{e}_\theta \qquad (2)$$

when bug goes through origin $(r=0)$

$\vec{V} = -V_A \hat{e}_r \longrightarrow (3)$

$\vec{a} = -2\dot{\phi}_0 V_A \hat{e}_\theta \longrightarrow (4)$

→ we know . $\vec{V} = |\vec{V}|\hat{t}$ $\therefore \hat{t} = -\hat{e}_r$

. $\vec{a} = \dot{V}\hat{t} + \frac{V^2}{\rho}\hat{n} = -2\dot{\phi}_0 V_A \hat{e}_\theta$

→ $\hat{n}$ is $\perp$ to $\hat{t}$ (ie $-\hat{e}_r$) $\therefore$ $\hat{n} = \hat{e}_\theta$ or $-\hat{e}_\theta$, lets figure out bel

$\therefore \quad -2\dot{\phi}_0 V_A \hat{e}_\theta = -\dot{V}\hat{e}_r + \frac{V^2}{\rho}\hat{n}$

$\}\cdot\hat{e}_r \Rightarrow \dot{V} = 0$

$\}\cdot\hat{e}_\theta \Rightarrow \underbrace{\frac{V^2}{\rho}}_{\text{always +ve}} = 2\dot{\phi}_0 V_A$ and $\hat{n} = -\hat{e}_\theta$

Finally

just when bug crosses origin. (only at that instant!)

$$\boxed{\begin{aligned}\hat{n} &= -\hat{e}_\theta \\ \rho &= \frac{V_A^2}{2\dot{\phi}_0 V_A} = \frac{V_A}{2\dot{\phi}_0}\end{aligned}} \longrightarrow \text{radius of curvature}$$

$\boxed{\rho = .1\text{ m}}$ using $V_A = .2$, $\dot{\phi}_0 = 1$

c) . radius of circle is ρ

. centre of oscul. circle is along $\hat{n}$, at distance ρ from the bug (the centre of turntable) (drawn below)

$\therefore$ centre $= -\rho\,\hat{e}_\theta$

$\boxed{\text{centre} = (-.09589\text{m}, -.2837\text{m})}$

- $-\hat{e}_\theta = \sin\phi\,\hat{\imath} - \cos\phi\,\hat{\jmath}$
- at centre of turntable
- $t=5$ $\phi = 5$ radians
- $\sin\phi = -.9589$
- $\cos\phi = .2837$

d) by LMB $\vec{F} = m\vec{a} = m\left(-\dot{\phi}_0^2 r\, \hat{e}_r - 2\dot{\phi}_0 V_A\, \hat{e}_\theta\right) \xrightarrow{\text{from (2)}}$

at start $t=0$, $r=R=1m$, $\dot{\phi}_0 = 1$ rad/s, $V_A = .2$ m/s, $\phi = 0$

$\vec{e}_r = \cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath} = \hat{\imath}$

$\hat{e}_\theta = -\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath} = \hat{\jmath}$

also let $m = 1$ kg (too much for a bug)

$\therefore \boxed{\vec{F} = -1\,\hat{\imath} - .4\,\hat{\jmath}}$ N

e) again by LMB $\vec{F} = m\left(-\dot{\phi}_0^2 r\, \hat{e}_r - 2\dot{\phi}_0 V_A\, \hat{e}_\theta\right)$

at centre $t=5$, $r=0$, $\dot{\phi}_0 = 1$, $V_A = .2$, $\phi = 5$ rad

$\vec{F} = -2\times.2\,\hat{e}_\theta = \boxed{-.4\,\hat{e}_\theta}$ N

$\hat{e}_\theta = +.9589\,\hat{\imath} + .2837\,\hat{\jmath}$

$\therefore \boxed{\vec{F} = -.3836\,\hat{\imath} - .1135\,\hat{\jmath} \text{ N}}$

both are ~~draw~~ drawn in figure.

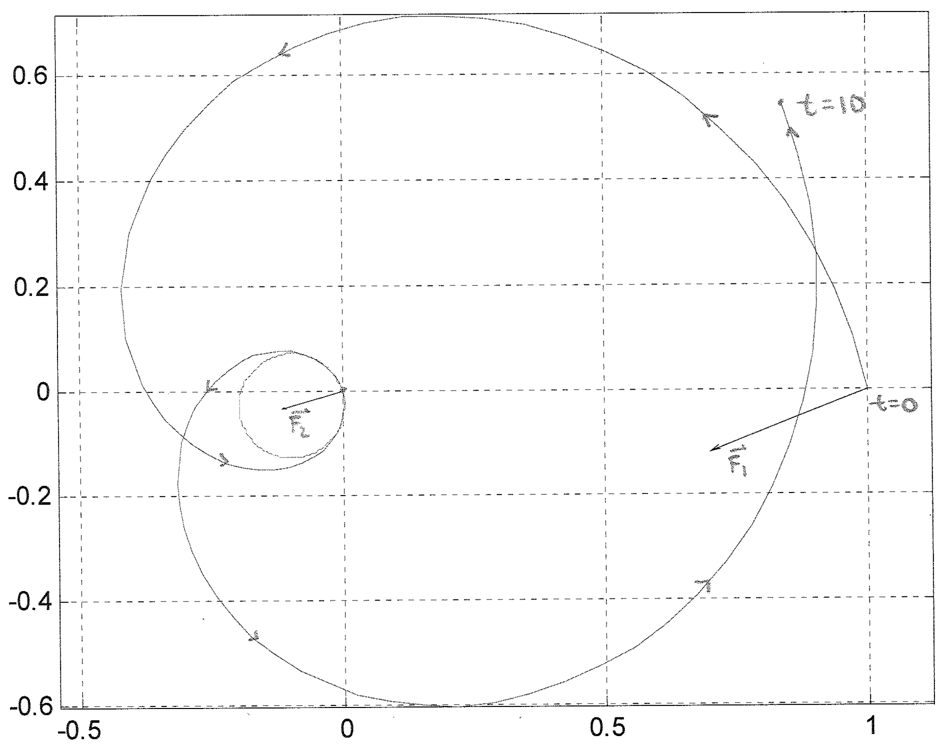
CODE

```
function path1518()
%%%    draw path
R=1;    % radius of the turntable
va=0.2; % velocity of the bug on the turntable
phidot=1; % angular velocity of the turntable
t=[0:0.1:10];
x=(R-va*t).*cos(phidot*t);
y=(R-va*t).*sin(phidot*t);
plot(x,y);
axis equal;

grid on;

%%%% draw osculating circle when bug goes through the center
rau=va/(2*phidot);    % radius of curvature of the path at the origin
xc= va*sin(phidot*R/va)/(2*phidot);
yc= -va*cos(phidot*R/va)/(2*phidot); % position of the center
                                     %draw the circle:
theta=[0:0.01:2*pi];
circle1=xc+rau*cos(theta);
circle2=yc+rau*sin(theta);
hold on;
plot(circle1,circle2,'r');

%%%% draw force vector
m=1;            %mass of the bug
scale=0.3;      % scale for graphics

f1x=-m*R*phidot;
f1y=-m*2*va*phidot;
quiver(1,0,f1x,f1y,scale,'k'); % draw force at the beginning:

f2x=2*m*va*phidot*sin(phidot*R/va);
f2y=-2*m*va*phidot*cos(phidot*R/va);
quiver(0,0,f2x,f2y,scale,'k'); %draw force at the origin
```

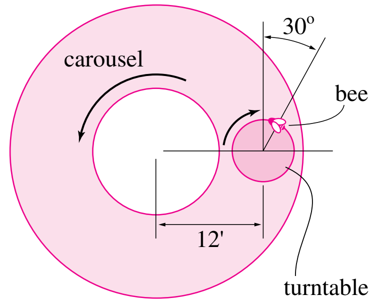
% $\frac{R}{va}$ is time taken to reach origin

% $phidot \cdot \frac{R}{va}$ = angle of table when bug is at origin

PLOTS



15.3.11 A honeybee, sensing that it can get a cheap thrill, alights on a phonograph turntable that is being carried by a carnival goer who is riding on a carousel. The situation is sketched below. The carousel has angular velocity of 5 rpm, which is increasing (accelerating) at 10 rev/min^2; the phonograph rotates at a constant 33 1/3 rpm. The honeybee is at the outer edge of the phonograph record in the position shown in the figure; the radius of the record is 7 inches. Calculate the magnitude of the acceleration of the honeybee.



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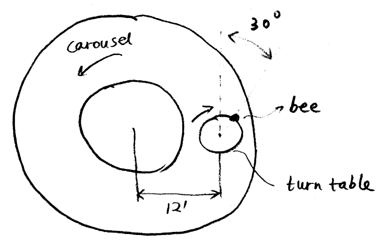
Problem 15.11

15.27

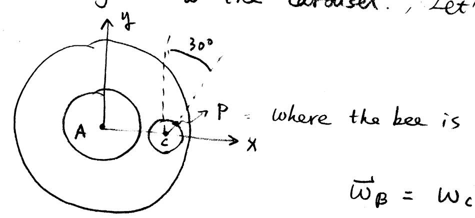


At this instant, the carousel has angular velocity of $\omega_c = 5\,rpm = 10\pi$ rad/min

angular acceleration $\alpha_c = 10\ r/min^2 = 20\pi$ rad/min^2

The turntable rotates at $\omega_t = 33\frac{1}{3}\ rpm = 66\frac{2}{3}\pi$ rad/min, $\alpha_t = 0$

Use the moving frame glued to the carousel., Let's call it $\mathcal{B}$



$\vec{\omega}_{\mathcal{B}} = \omega_c \hat{k} = 10\pi$ rad/min

$\dot{\vec{\omega}}_{\mathcal{B}} = \alpha_c \hat{k} = 20\pi$ rad/min^2

$$\vec{a}_P = \vec{a}_A + \vec{a}_{P/\mathcal{B}} - \omega_{\mathcal{B}}^2 \vec{r}_{P/A} + \dot{\vec{\omega}}_{\mathcal{B}} \times \vec{r}_{P/A} + 2\vec{\omega}_{\mathcal{B}} \times \vec{v}_{P/\mathcal{B}}$$

i) $\vec{a}_A = 0$ since A is fixed

ii) $\vec{a}_{P/\mathcal{B}} = -\omega_t^2 \vec{r}_{P/C} = -(66\frac{2}{3}\pi)^2\, 7\, (\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j})$ in/min^2

Since P rotates with the turntable at a constant angular velocity.

$\therefore\ \vec{a}_{P/\mathcal{B}} = (-1.5353\times 10^5\,\hat{i} - 2.6952\times 10^5\,\hat{j})$ in/min^2

iii) $-\omega_{\mathcal{B}}^2 \vec{r}_{P/A} = -\omega_{\mathcal{B}}^2 (\vec{r}_{P/C} + \vec{r}_{C/A})$

$$= -(10\pi)^2\left(\frac{7}{2}\hat{i} + \frac{7\sqrt{3}}{2}\hat{j} + 12\times 12\,\hat{i}\right)$$

$$= -1.4558\times 10^5\,\hat{i} - 5.9831\times 10^3\,\hat{j} \quad \text{in/min}^2.$$

iv) $\dot{\vec{\omega}}_B \times \vec{r}_{P/A}$

$$= 20\pi\,\hat{k} \times \left[\left(\frac{7}{2} + 144\right)\hat{i} + \frac{7\sqrt{3}}{2}\hat{j}\right]$$

$$= -380.898\,\hat{i} + 9.2677\times 10^3\,\hat{j} \quad \text{in/min}^2$$

v) $2\vec{\omega}_B \times \vec{V}_{P/B}$

$$\vec{V}_{P/B} = (-\omega_t\,\hat{k}) \times \vec{r}_{P/C}$$

$$\therefore\ 2\vec{\omega}_B \times \vec{V}_{P/B} = 2\,\omega_c\,\omega_t\,\vec{r}_{P/C}$$

$$= 2\times(10\pi)\times\left(66\tfrac{2}{3}\pi\right)\times\left(\frac{7}{2}\hat{i} + \frac{7\sqrt{3}}{2}\hat{j}\right)$$

$$= 4.6058\times 10^4\,\hat{i} + 7.9775\times 10^4\,\hat{j} \quad \text{in/min}^2$$

Sum all the five terms up.

$$\vec{a}_P = \left(-2.5343\times 10^5\,\hat{i} - 1.8646\times 10^5\,\hat{j}\right) \quad \text{in/min}^2$$

$\therefore$ the magnitude of acceleration

$$\boxed{\begin{aligned} |\vec{a}_P| &= 3.1464\times 10^5 \ \text{in/min}^2 \\ &= 87.4 \ \text{in/sec}^2 \end{aligned}}$$

15.4.1 Slider crank kinematics (No FBD required!). 2-D . Assume $R, \ell, \theta, \dot{\theta}, \ddot{\theta}$ are given. The crank mechanism parts move on the xy plane with the x direction being along the piston. Vectors should be expressed in terms of $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ components.

a) What is the angular velocity of the crank OA?

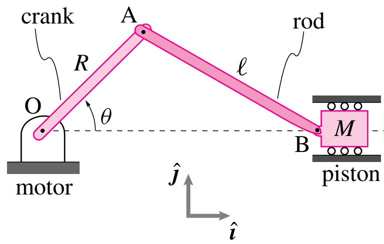
b) What is the angular acceleration of the crank OA?

c) What is the velocity of point A?

d) What is the acceleration of point A?

e) What is the angular velocity of the connecting rod AB? [Geometry fact: $\vec{r}_{AB} = \sqrt{\ell^2 - R^2 \sin^2\theta}\ \hat{\imath} - R\sin\theta\,\hat{\jmath}$]

f) For what values of θ is the angular velocity of the connecting rod AB equal to zero (assume $\dot{\theta} \neq 0$)? (you need not answer part (e) correctly to answer this question correctly.)



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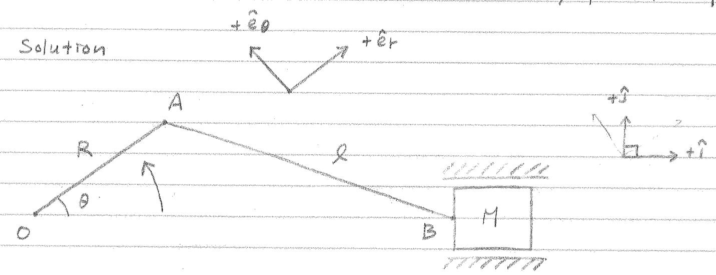
Problem 15.1

Sec 205

You Won Park
TAM 2030
10:10 AM Sec 205
TA: Pranav Bhounsule
HW 22, Apr. 14, 2009

15.29 Solution



$R, \ell, \theta, \dot{\theta}, \ddot{\theta}$ given

a) Angular velocity of OA?

$$\boxed{\vec{\omega}_{OA} = \dot{\theta}\hat{k}}$$

b) Angular acceleration of OA?

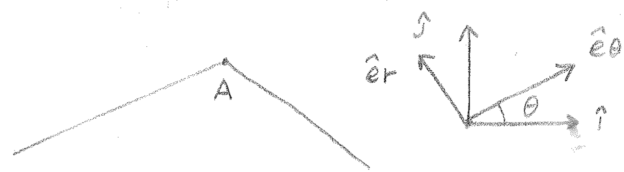
$$\boxed{\dot{\vec{\omega}}_{OA} = \ddot{\theta}\hat{k}}$$

c) Velocity of A?

$$\vec{v}_A = \dot{\theta}R\,\hat{e}_\theta = \boxed{\dot{\theta}R(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath})}$$

d) Acceleration of A?

$$\vec{a}_A = \dot{\vec{v}}_A = \boxed{(-\ddot{\theta}R\sin\theta - \dot{\theta}^2R\cos\theta)\hat{\imath} + (\ddot{\theta}R\cos\theta - \dot{\theta}^2R\sin\theta)\hat{\jmath}}$$



e) Angular velocity of AB?

$$\vec{r}_{AB} = \sqrt{\ell^2 - R^2\sin^2\theta}\,\hat{\imath} - R\sin\theta\,\hat{\jmath}$$

$$\vec{v}_A = \vec{r}_{AB} \times \vec{\omega}_{AB}$$

$$= \left(\sqrt{\ell^2 - R^2\sin^2\theta}\,\hat{\imath} - R\sin\theta\,\hat{\jmath}\right) \times \omega_{AB}\hat{k}$$

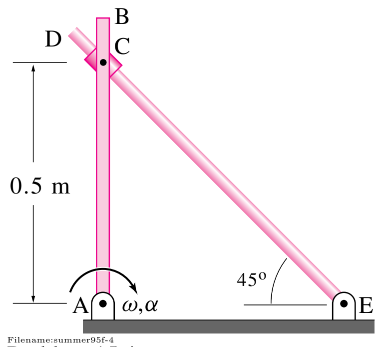
$$\left\{ -R\dot\theta\sin\theta\,\hat{\imath} + R\dot\theta\cos\theta\,\hat{\jmath} = -\omega_{AB}\sqrt{\ell^2 - R^2\sin^2\theta}\,\hat{\jmath} - \omega_{AB}R\sin\theta\,\hat{\imath} \right\}$$

$$\{\}\cdot\hat{\jmath} \Rightarrow R\dot\theta\cos\theta = -\omega_{AB}\sqrt{\ell^2 - R^2\sin^2\theta}$$

$$\therefore \boxed{\vec{\omega}_{AB} = -\frac{R\dot\theta\cos\theta}{\sqrt{\ell^2 - R^2\sin^2\theta}}\hat{k}}$$

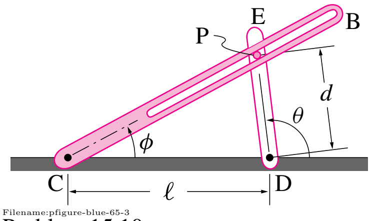
f) for $\boxed{\theta = \pm 90^\circ}$

15.4.4 The two rods AB and DE, connected together through a collar C, rotate in the vertical plane. The collar C is pinned to the rod AB but is free to slide on the frictionless rod DE. At the instant shown, rod AB is rotating clockwise with angular speed $\omega = 3\,\text{rad/s}$ and angular acceleration $\alpha = 2\,\text{rad/s}^2$. Find the angular velocity of rod DE.



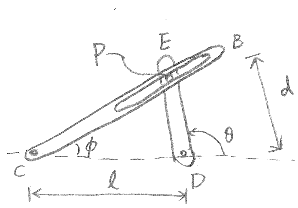
Problem 15.4

15.4.10 The slotted link CB is driven in an oscillatory motion by the link ED which rotates about D with constant angular velocity $\dot{\theta} = \omega_D$. The pin P is attached to ED at fixed radius d and engages the slot on CB as shown. Find the angular velocity and acceleration $\dot{\phi}$ and $\ddot{\phi}$ of CB when $\theta = \pi/2$.



Problem 15.10

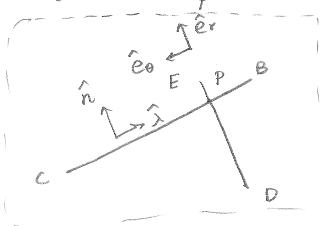
15.38



P is pinned on DE and can slide along CB , $\dot{\theta} = \omega_D$

Find $\dot{\phi}$, $\ddot{\phi}$ when $\theta = \frac{\pi}{2}$

i) Find $\dot{\phi}$



Build up two local coordinate system $\hat{e}_\theta, \hat{e}_r$ and $\hat{\lambda}, \hat{n}$.

Call the moving frame attaching to CB , $\mathcal{B}$

the moving frame attaching to DE , $\mathcal{D}$

(a)

$$\vec{V}_P = \vec{V}_P$$

$$\cancelto{0}{\vec{V}_C} + \vec{V}_{P/\mathcal{B}} + \vec{\omega}_{\mathcal{B}} \times \vec{r}_{P/C} = \cancelto{0}{\vec{V}_D} + \cancelto{0}{\vec{V}_{P/\mathcal{D}}} + \vec{\omega}_{\mathcal{D}} \times \vec{r}_{P/D}$$

$$\Rightarrow \quad V_{P/\mathcal{B}}\,\hat{\lambda} + \dot{\phi}\,\hat{k} \times r_{P/C}\,\hat{\lambda} = \omega_D\,\hat{k} \times d\,\hat{e}_r$$

when $\theta = \frac{\pi}{2}$, $\quad r_{P/C} = \frac{\ell}{\cos\phi}$

$$\Rightarrow \left\{ V_{P/\mathcal{B}}\,\hat{\lambda} + \dot{\phi}\,\frac{\ell}{\cos\phi}\,\hat{n} = \omega_D\, d\, \hat{e}_\theta \right\}$$

$$\{\ \} \cdot \hat{n} \quad \Rightarrow \quad \dot{\phi}\,\frac{\ell}{\cos\phi} = \omega_D\, d\ (\hat{e}_\theta \cdot \hat{n})$$

$$\{\ \} \cdot \hat{\lambda} \quad \Rightarrow \quad V_{P/\mathcal{B}} = \omega_D\, d\,(\hat{e}_\theta \cdot \hat{\lambda})$$

when $\theta = \frac{\pi}{2}$, $\quad \hat{e}_\theta \cdot \hat{n} = \sin\phi$

$$\hat{e}_\theta \cdot \hat{\lambda} = -\cos\phi$$

$$\therefore \quad \dot{\phi} = \omega_D \frac{d}{\ell} \cos\phi \sin\phi \quad , \quad \vec{V}_{P/B} = -\omega_D d \cos\phi \,\hat{\lambda}$$

$$\because \quad \cos\phi = \frac{\ell}{\sqrt{d^2+\ell^2}} \,, \quad \sin\phi = \frac{d}{\sqrt{d^2+\ell^2}} \quad \text{when } \theta = \frac{\pi}{2}, \quad \Rightarrow \quad \boxed{\dot{\phi} = \omega_D \frac{d^2}{d^2+\ell^2}}$$

(b)
$$\vec{a}_P = \vec{a}_P$$

(Note: $\dot{\vec{\omega}}_D = 0$ because $\dot{\theta} = \omega_D =$ constant)

$$\cancel{\vec{a}_C}^{\,0} + \vec{a}_{P/B} + \dot{\vec{\omega}}_B \times \vec{r}_{P/C} - \omega_B^2 \vec{r}_{P/C} + 2\vec{\omega}_B \times \vec{V}_{P/B} = \cancel{\vec{a}_D}^{\,0} + \cancel{\vec{a}_{P/D}}^{\,0} + \cancel{\dot{\vec{\omega}}_D \times \vec{r}_{P/D}}^{\,0} - \omega_D^2 \vec{r}_{P/D} + \cancel{2\vec{\omega}_D \times \vec{V}_{P/D}}^{\,0}$$

$\Rightarrow$

$$a_{P/B}\,\hat{\lambda} + \ddot{\phi}\,\hat{k} \times \frac{\ell}{\cos\phi}\hat{\lambda} - \dot{\phi}^2 \frac{\ell}{\cos\phi}\hat{\lambda} + 2\dot{\phi}\,\hat{k} \times (-\omega_D d \cos\phi)\hat{\lambda} = -\omega_D^2 d\, \hat{e}_r$$

$$\Rightarrow \quad \left\{ a_{P/B}\,\hat{\lambda} + \ddot{\phi}\frac{\ell}{\cos\phi}\hat{n} - \dot{\phi}^2\frac{\ell}{\cos\phi}\hat{\lambda} - 2\omega_D \dot{\phi} d \cos\phi\, \hat{n} = -\omega_D^2 d\, \hat{e}_r \right\}$$

$$\{ \ \} \cdot \hat{n} \quad \Rightarrow \quad \ddot{\phi}\frac{\ell}{\cos\phi} - 2\omega_D \dot{\phi} d \cos\phi = -\omega_D^2 d\,(\hat{e}_r \cdot \hat{n})$$

$$\because \quad \hat{e}_r \cdot \hat{n} = \cos\phi$$

$$\therefore \quad \ddot{\phi} = \frac{2\omega_D \dot{\phi} d \cos^2\phi}{\ell} - \omega_D^2 \frac{d}{\ell}\cos^2\phi$$

$$= \frac{2\omega_D d}{\ell} \frac{\ell^2}{\ell^2+d^2}\, \omega_D \frac{d^2}{d^2+\ell^2} - \omega_D^2 \frac{d}{\ell}\frac{\ell^2}{\ell^2+d^2}$$

$$= \omega_D^2 \frac{d\ell(d^2-\ell^2)}{(\ell^2+d^2)^2}$$

$\therefore$ Angular acceleration of CB when $\theta = \frac{\pi}{2}$, is $\ddot{\phi} = \omega_D^2 \dfrac{d\ell(d^2-\ell^2)}{(\ell^2+d^2)^2}$